GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

EE3230
Solution to Spectrum Analysis Project

Date Assigned: November 6, 1998
Date Due: November 25, 1998

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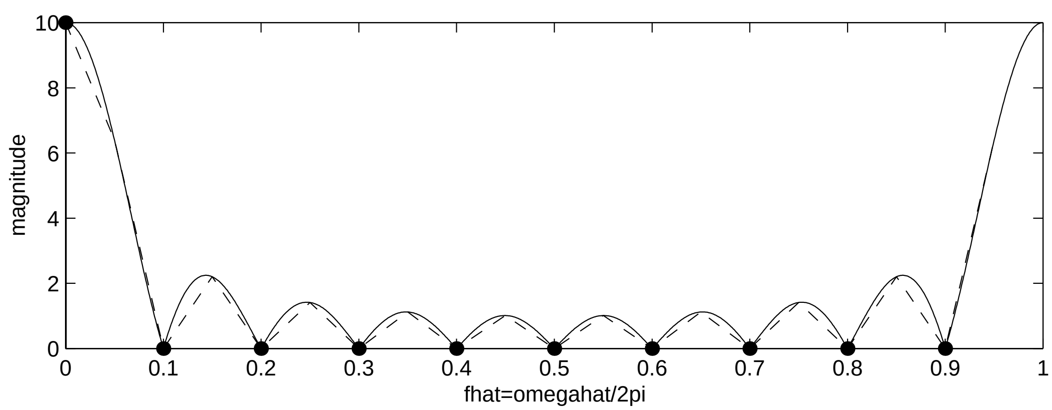
1 Using the DFT to Plot the DTFT of a Sequence

Problem 1: Run Program A and observe the plot. Now change N to 20 and repeat the experiment. Repeat for $N = 1000$. How does the plotted curve change as N increases? How should N be chosen in the DFT if we want the resulting plot to show the detailed variations of the DTFT of a sequence? At what frequencies is $|W_R(e^{j\hat{\omega}})| = 0$? Does your answer agree with Eq. (4)?

A combined plot is shown below:



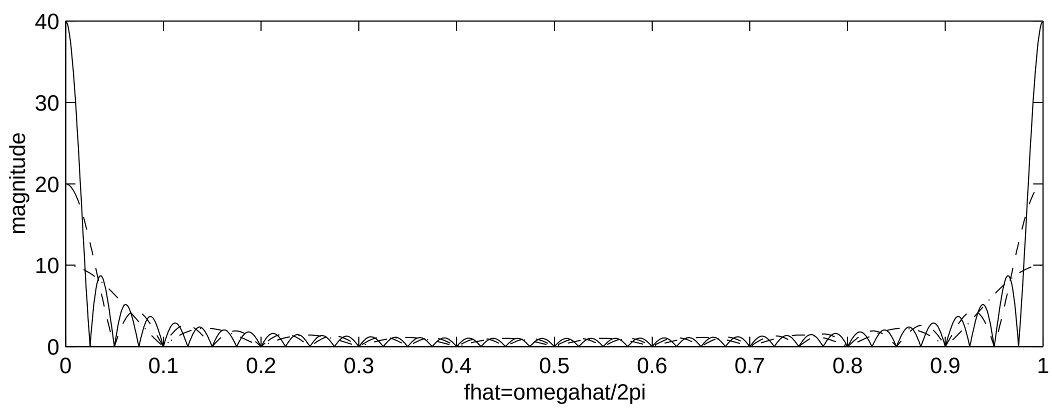
This figure shows that the DFT for $N = 10$ gives only the 10 values of the DTFT of the rectangular window shown by the solid dots in the plot below. When $N = 20$, we compute the DTFT at the original 10 points plus another set of 10 in-between points. These are shown connected by the dashed line. Finally, when $N = 1000$, we evaluate the DTFT at frequencies $2\pi k/1000$ for $k = 0, 1, \ldots, 999$; i.e., at 1000 points between 0 and 2π. These points, when connected by straight lines give a smooth curve that is a faithful representation of the DTFT at all frequencies $0 \leq \hat{\omega} < 2\pi$.

Note that in Eq. (4), the numerator $\sin(\hat{\omega}M/2)$ is zero when $\hat{\omega}M/2 = \pi\ell$ where ℓ is an integer. This means that $W_R(e^{j\hat{\omega}}) = 0$ for $\hat{\omega}_\ell = 2\pi\ell/M$ or $\hat{f}_\ell = \hat{\omega}_\ell/(2\pi) = \ell/M$. Since $M = 10$ in the test case, the zeros should occur at $\hat{f}_\ell = 0.1\ell$, which is what we see in the above figure.

Problem 2: Modify the MATLAB code of Program A so that $N = 1000$ and then change the `plot( )` command to plot the three cases $M = 10$, $M = 20$, and $M = 40$ on the same graph using different line styles for the three graphs. Hand in a hard copy of your plot. What is the width of the "main lobe" of the Fourier transform in each case? How is this width related to the window length, M?

```
%     PROGRAM A (modified)
      M=10;w=ones(1,M);
      N=1000;W=fft(w,N);
      W20=fft(ones(1,20),N);W40=fft(ones(1,40),N);
      fhat=(0:N-1)/N;
      plot(fhat,abs(W),'-.',fhat,abs(W20),'--',fhat,abs(W40),'-'); Ax=axis;
      axis([0,1,Ax(3),Ax(4)])
      xlabel('fhat');ylabel('magnitude')
```

The resulting plot is shown below:

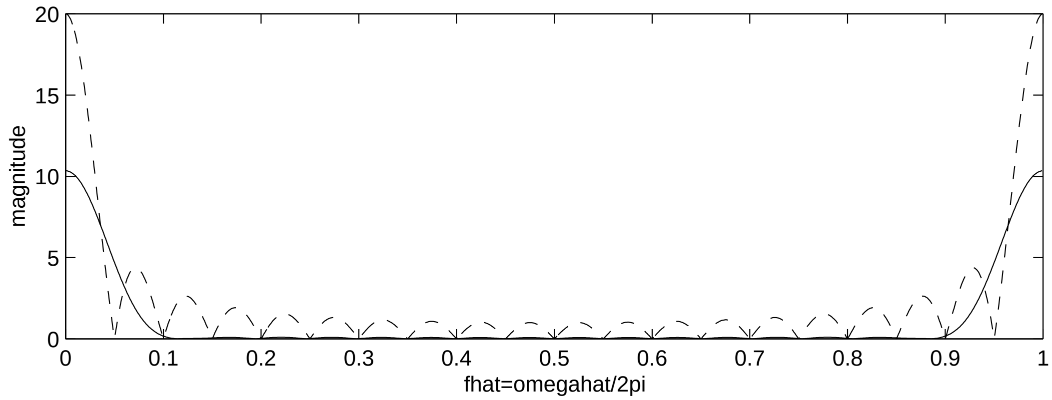


The width of the main lobe would be the frequency difference between the first zero on the negative frequency side (or $-2\pi/M$) to the first zero on the positive frequency side (or $+2\pi/M$); i.e., $\Delta\hat{\omega} = 4\pi/M$. On the normalized scale $\Delta\hat{f} = \Delta\hat{\omega}/(2\pi)$, this would be 0.2 for $M = 10$, 0.1 for $M = 20$, and 0.05 for $M = 40$. Thus, doubling M will halve the width of the main lobe of the DTFT of the window.

Problem 3: Modify the MATLAB code of Program A to plot the magnitude of the Fourier transforms of both the rectangular window and the Hamming window on the same graph for the case $M = 20$ and $N = 1000$. For this value of M, how do the main lobe widths compare?

```
%     PROGRAM A (second modification)
      N=1000;W=fft(ones(1,20),N);
      WH=fft(hamming(20),N);
      fhat=(0:N-1)/N;
      plot(fhat,abs(W),'--',fhat,abs(WH20),'-'); Ax=axis;
      axis([0,1,Ax(3),Ax(4)])
      xlabel('fhat=omegahat/2pi');ylabel('magnitude')
```

The resulting plot is shown below:



It is clear from this plot that the main lobe width of the Hamming window is twice the width of the same length rectangular window.

2 Spectrum Analysis of Continuous-Time Signals

Problem 4: How should $w[n]$ in Figure 2 be chosen so that the systems of Figures 1 and 2 are equivalent?

It is obvious that Figures 1 and 2 will produce the same output $V[k]$ if $w[n] = w_c(nT)$.

```
%    PROGRAM B
     A=10;phi=pi/3;f0=100;f1=130;                       %signal parameters
     T=1/1000;                                          %sampling period
     M=32;w=ones(M,1);                                  %window
     n=0:M-1;
     x=A*cos(2*pi*f0*n*T+phi)+0.5*A*cos(2*pi*f1*n*T);   %sampled signal
     v=w.*x';                                           %windowed signal
     N=32;
     V=plotdft(v,N);                                    %compute DFT and plot it
```

Problem 5: Write an equation for the signal $x_c(t)$ that is sampled in Program B. What is the sampling rate?

$$x_c(t) = 10\cos(200\pi t + \pi/3) + 5\cos(260\pi t)$$

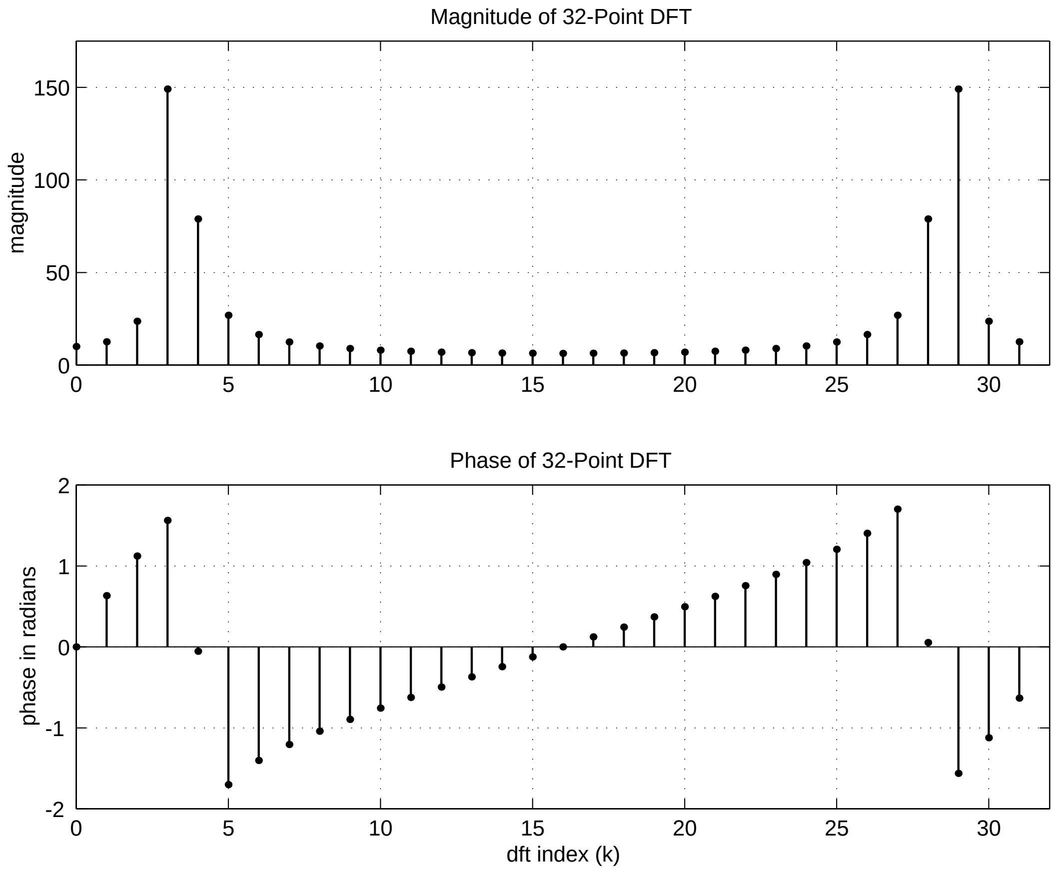
The sampling rate is $f_s = 1/T = 1000$ samples/second.

Problem 6: Run Program B. Observe the plots that are produced. From the plots, what symmetries do you observe for DFT values? Verify that $V[N-k] = V^*[k]$ for $k = 0, 1, \ldots, N-1$. This condition will always be true if $v[n]$ is real. Save a hardcopy of the magnitude and phase plots to hand in and for future reference.

The general proof of this symmetry is

$$\begin{aligned} V[N-k] &= \sum_{n=0}^{M-1} v[n]e^{-j(2\pi/N)(N-k)n} = \sum_{n=0}^{M-1} v[n]e^{j(2\pi/N)kn}e^{j(2\pi/N)Nn} \\ &= \left(\sum_{n=0}^{M-1} v[n]e^{-j(2\pi/N)kn}\right)^* = V^*[k] \end{aligned}$$

if $v[n]$ is real since $e^{j(2\pi/N)Nn} = e^{j2\pi n} = 1$. The plot requested is shown below.



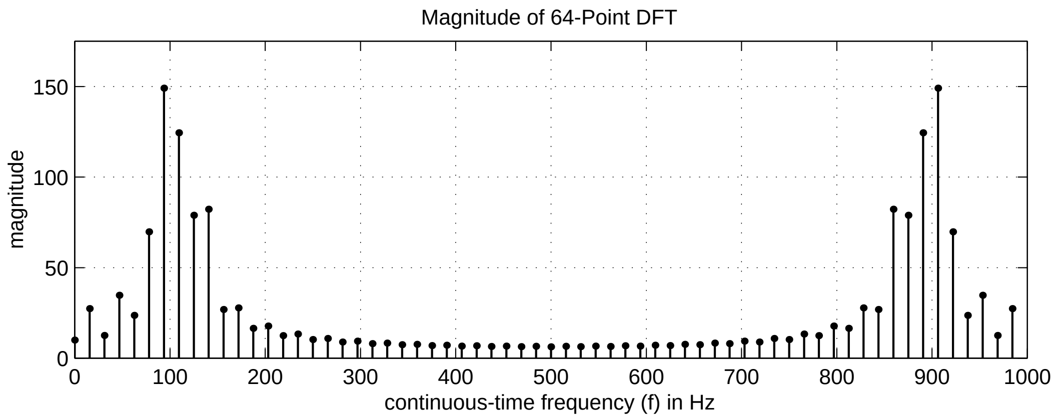
Note that $|V[32-k]| = |V[k]|$ and $\angle V[32-k] = -\angle V[k]$, or $V[32-k] = V^*[k]$.

Problem 7: Modify the function `plotdft( )` to take a third argument, T, the sampling period. Inside the function, use T to calibrate the frequency axis for continuous-time cyclic frequency. This should require only a simple modification of the statement `k=0:N-1`. (You may also want to remove the plotting of the real and imaginary parts and the phase, since those will not be needed.)

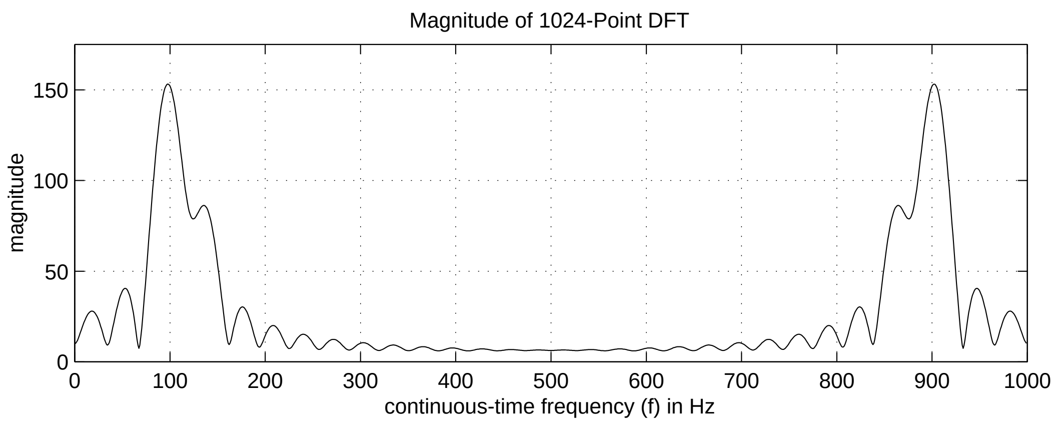
See program in Section 4.

Problem 8: Now with the frequency axis calibrated in terms of continous-time frequency, keep $M = 32$ and change N first to 64 and then to 1024. What differences did you observe in the DTFT magnitude as you went from $N = 32$ to 64 to 1024? Next, increase M to 64 and repeat with $N = 64$ and then 1024. What do you conclude about the effects of M and N on the ability to resolve two sinusoids whose frequencies are about the same?

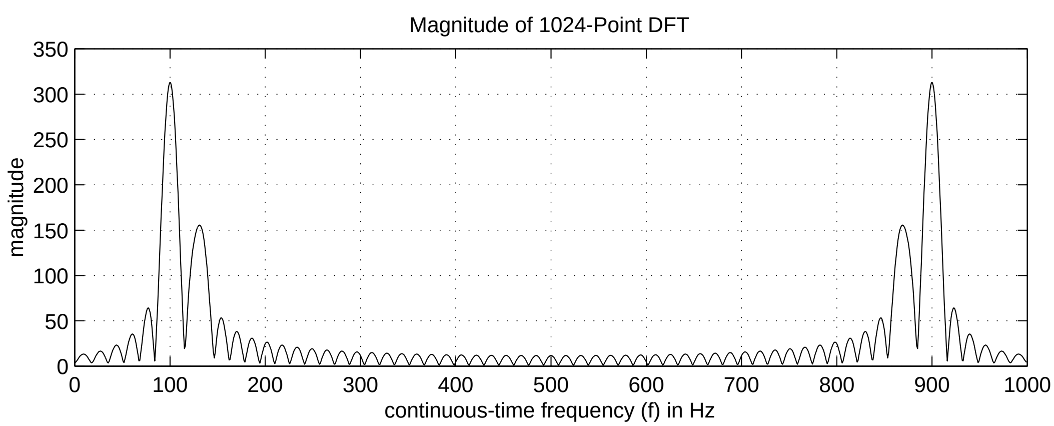
Here is the magnitude and phase plot for $M = 32$ and $N = 64$. You can see that there is a hint of two frequency components, but we could not be sure, and we do not have enough refinement of the graph to estimate the frequencies.



If we increase N to 1024 and keep $M = 32$, we get the following plot.



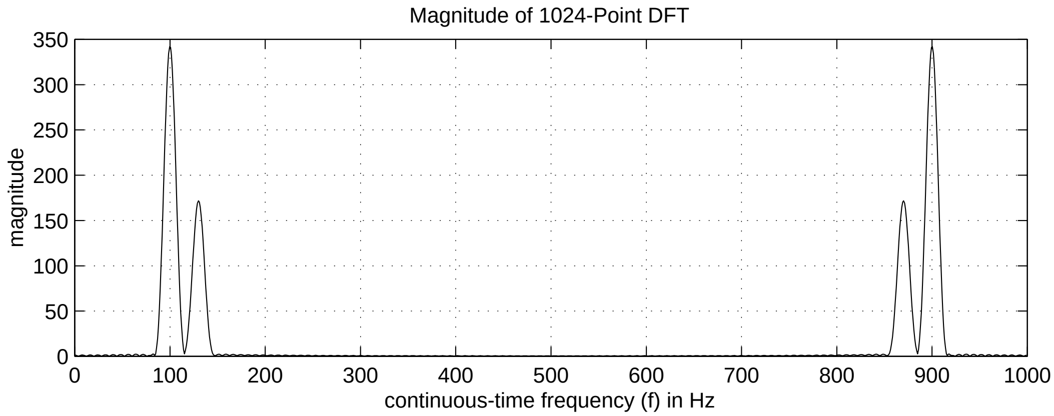
In this plot, the main peak is at 100 Hz as expected, and there is a hint of the other component whose frequency was 130 Hz. Now if we increase the window length to $M = 64$ and maintain $N = 1024$, we get the following plot in which both the frequencies are clearly in evidence.



Note that the peaks are higher because the Fourier transform of the window is twice as large.

Problem 9: Repeat Problem 8 with a Hamming window instead of the rectangular window. How large must M be in order to see two distinct peaks in the DTFT magnitude when $N = 1024$?

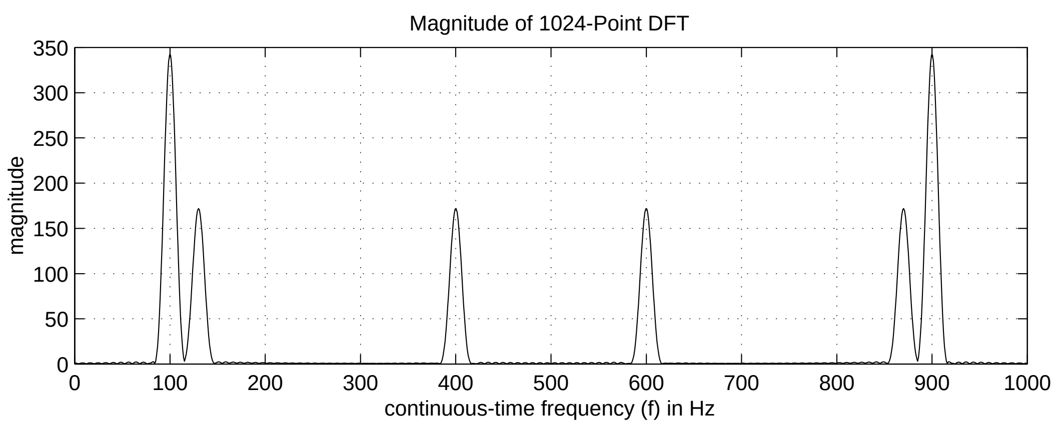
Now if we switch to the Hamming window, which has twice as wide a main lobe, we will need twice the window length to obtain the same resolution as the rectangular window. The following plot shows the spectrum for the Hamming window with $M = 128$ and $= 1024$.



Problem 10: Modify Program B to use a 128-point Hamming window with the three component signal. Use $N = 1024$ to compute the magnitude of the DFT of the windowed signal. Where are the peaks in the spectrum estimate? Does the peak due to the component `B*cos(2*pi*f2*n*T)` occur where you expect it to occur? Explain.

```
B=5;f2=600;
x=A*cos(2*pi*f0*n*T+phi)+0.5*A*cos(2*pi*f1*n*T)+B*cos(2*pi*f2*n*T);
```

This will add a third sinusoidal component to the input signal to be sampled. Since its frequency is 600 Hz and our sampling rate remains at 1000 samples/second, we will have aliasing. Therefore, in the interval $0 \leq f < 1000$ Hz, we should see the component at 600 and its negative frequency alias at 1000-600=400 Hz. This is evident in the following plot.



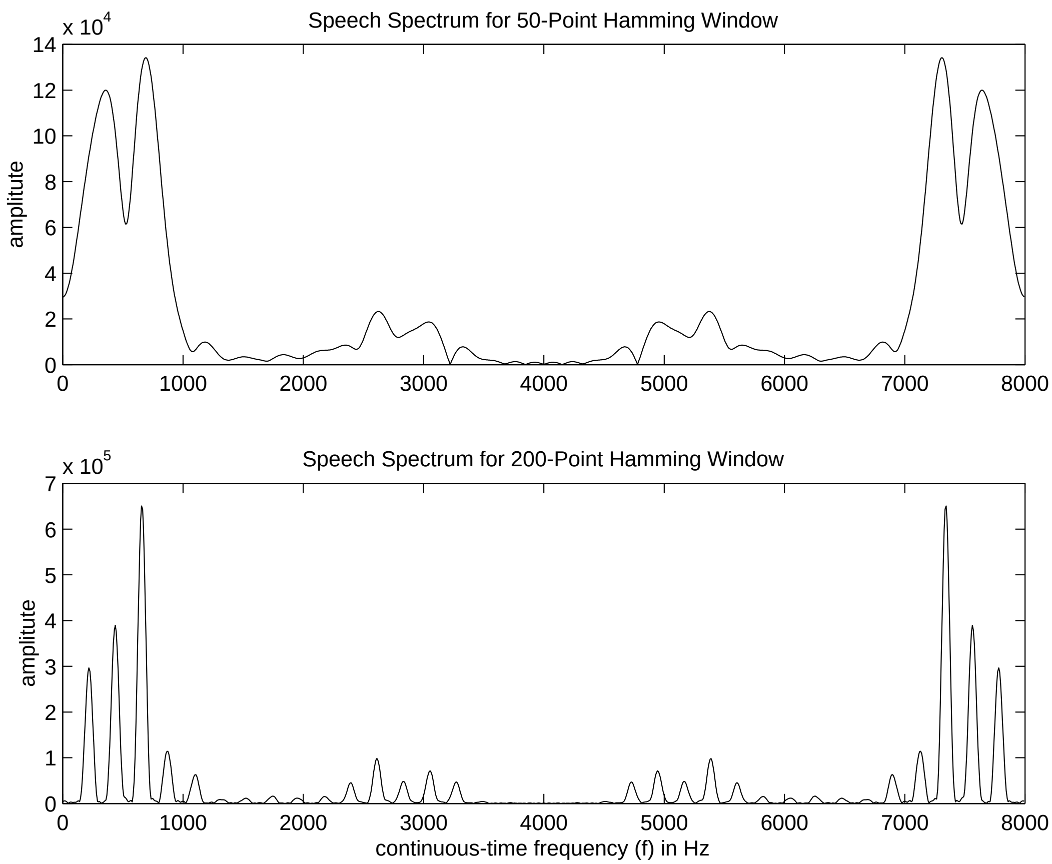
3 Analysis of a Speech Signal

Problem 11: Look at the waveform in Figure 3. It looks almost periodic. From the figure, estimate the period and fundamental frequency of the original continuous-time signal, $x_c(t)$ during this time interval.

Counting samples, we see that the "period" of the signal is approximately 37 samples. Since the sampling rate is given as 8000 samples/sec, the sample spacing is $T = 1/8000$ msec. Therefore the period of the signal is approximately $T_0 = 37/8000 = 4.625$ msec and the fundamental frequency is approximately $f_0 = 1/T_0 = 8000/37 \approx 216$ Hz

Problem 12: Use the techniques that we have explored in this project to compute and plot samples of the magnitude spectrum of the speech signal. Use Hamming windows of length $M = 50$ and $M = 200$ and $N = 1024$ for the DFT. Start your segments of the speech signal at sample number 6200 as in the waveform display example above. Display the magnitude of the DFTs as a function of cyclic continuous-time frequency in a two part plot. Hand in a hard copy of the plot with your report. Explain why the two graphs look different. From the $M = 200$ graph you should be able to estimate the fundamental frequency of the signal. Compare it to your time-domain estimate in Problem 11.

The following figure shows the magnitude specta for $M = 50$ (upper curve) and $M = 200$ (lower curve). (This plot was not made with the function `plotdftc( )` since I wanted to compare the two cases side-by-side.) As should be expected, the individual harmonics of the signal are better resolved for the longer window. If you look carefully at the lower graph, you will see that there are 4 equally spaced peaks between $0 \leq f \leq 850$ Hz. Therefore, the fundamental spacing is approximately $850/4 \approx 212$ Hz, which agrees quite well with our time-domain estimate. Note that while the upper curve does not show the periodic structure, it nevertheless has the same general shape as the more refined spectrum. This is simply because the shorter window causes more smoothing in the frequency domain.



4 Modified MATLAB Function to Plot the DFT

Here is the modified function now called `plotdftc( )`. It labels the frequency axis in terms of continuous-time cyclic frequency.

```
%         Modified plotdft function
function        V=plotdftc(v,N,T)
%       USAGE:  V=plotdftc(v,N,T)
%                    v=input sequence
%                    N=dft length
%                    V=dft of v
%                    T=sampling period
V=fft(v,N);
k=(0:N-1)/(N*T);       %  <-- here is the main modification
figure(1)              %   Plot the real and imaginary parts
subplot(211)
if(N>=256), plot(k,real(V)), else, jstem(k,real(V),10), end;
grid; ax=axis;axis([0,1/T,ax(3),ax(4)]);
ylabel('real part');
title(['Real Part of ',num2str(N),'-Point DFT'])
subplot(212)
if(N>=256), plot(k,imag(V)), else, jstem(k,imag(V),10), end;
grid; ax=axis;axis([0,1/T,ax(3),ax(4)]);
title(['Imaginary Part of ',num2str(N),'-Point DFT'])
ylabel('imaginary part')
xlabel('continuous-time frequency (f) in Hz')
%
figure(2)              %   Plot the magnitude and phase
subplot(211)
if(N>=256), plot(k,abs(V)), else, jstem(k,abs(V),10), end;
grid; ax=axis;axis([0,1/T,ax(3),ax(4)]);
ylabel('magnitude');
title(['Magnitude of ',num2str(N),'-Point DFT'])
subplot(212)
if(N>=256), plot(k,angle(V)), else, jstem(k,angle(V),10), end;
grid; ax=axis;axis([0,1/T,ax(3),ax(4)]);
ylabel('phase in radians')
xlabel('continuous-time frequency (f) in Hz')
title(['Phase of ',num2str(N),'-Point DFT'])
```