GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

EE2201A
Problem Set No. 1

Date Assigned: April 2, 1999
Date Due: April 9, 1999

Reading Assignment: In Oppenheim and Willsky, read pp. 30-56 and 90-136.

Homework Assignment: In all problems, write some explanation of your approach to the solution, i.e., give more than the answer. Turn in for grading only the starred problems: 1.1*, 1.2*, 1.3*, 1.6*, and 1.8*.

Optional Problems:

Look at the problems in Problem Set #1 for Winter, Spring, and Fall, 1998 in EE3230 and Problem Set #1 for EE2823 for Winter 1999. These problems all relate to what we will study during the next week.

Problem 1.1*:

For each of the following systems, determine whether or not the system is (1) Time-invariant, (2) Linear, (3) Causal, and (4) Stable.

(a) $y(t) = Ax(t+5) + B$ where A and B are constants.

(b) $y(t) = \int_{t-3}^{t} x(\tau)d\tau$

(c) $y(t) = x(t)\cos(4000\pi t)$

(d) $y(t) = e^{x(t)}$.

Problem 1.2*:

The impulse response of an LTI continuous-time system is such that $h(t) = 0$ for $t \leq T_1$ and for $t \geq T_2$. By drawing appropriate figures as recommended for evaluating convolution integrals, show that if $x(t) = 0$ for $t \leq T_3$ and for $t \geq T_4$ then $y(t) = x(t) * h(t) = 0$ for $t \leq T_5$ and for $t \geq T_6$. In the process of proving this result you should obtain expressions for T_5 and T_6 in terms of T_1, T_2, T_3, and T_4.

Problem 1.3*:

A linear time-invariant system has impulse response:

$$h(t) = \begin{cases} e^t & -2 < t < 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) Plot $h(t)$ and plot $h(t-\tau)$ as a function of τ for $t = 3$.

(b) Is the system stable? Justify your answer.

(c) Is the system causal? Justify your answer.

(d) Find the output $y(t)$ when the input is $x(t) = \delta(t-1)$

(e) Find the output $y(t)$ when the input is $x(t) = u_1(t) = \dfrac{d\delta(t)}{dt}$.

(f) Find the output $y(t)$ when the input is $x(t) = \begin{cases} 1 & -1 < t < 1 \\ 0 & \text{otherwise} \end{cases}$.

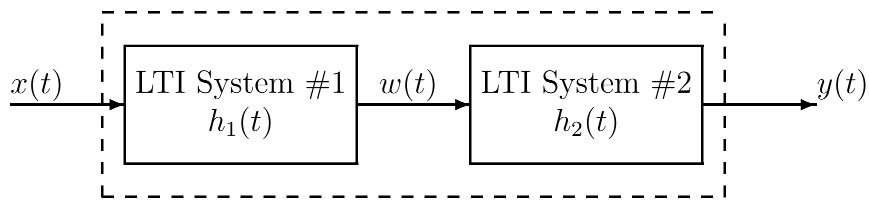
Problem 1.4:

A linear time-invariant system has impulse response

$$h(t) = \begin{cases} e^t & 2 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$

The input to this system is $x(t) = u(t-1)$. Find and plot the output $y(t)$ for $-\infty < t < \infty$.

Problem 1.5:



The first system is described by the input/output relation

$$w(t) = \frac{dx(t)}{dt}$$

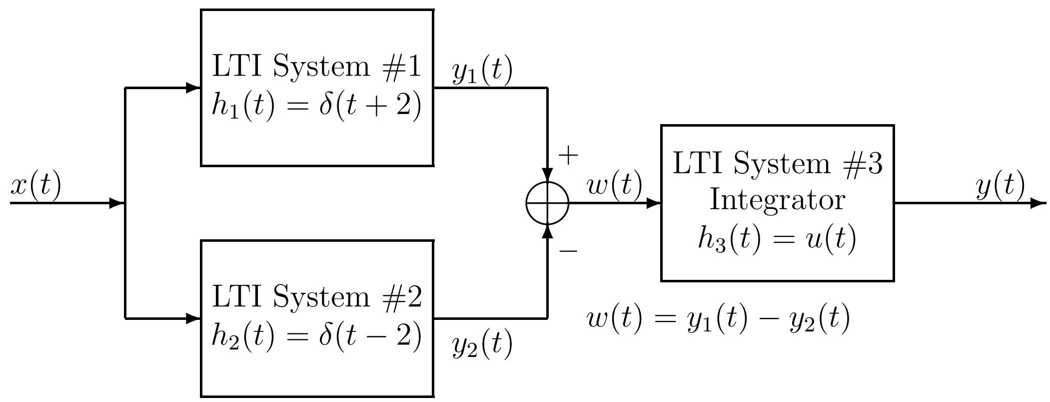
and the second system has impulse response

$$h_2(t) = u(t-5) - u(t-10)$$

(a) Find the impulse response of the overall system; i.e., find the output $y(t) = h(t)$ when the input is $x(t) = \delta(t)$.

(b) Give a general expression for $y(t)$ in terms of $x(t)$ that holds for any input signal.

Problem 1.6*



(a) What is the impulse response of the overall LTI system (i.e., from $x(t)$ to $y(t)$)? Give your answer both as an equation and as a carefully labeled sketch.

(b) Is the overall system a causal system? (Explain to receive credit.) Is it a stable system? (Explain to receive credit.)

Problem 1.7

An LTI system has impulse response

$$h(t) = e^{a(t+t_0)}u(t+t_0)$$

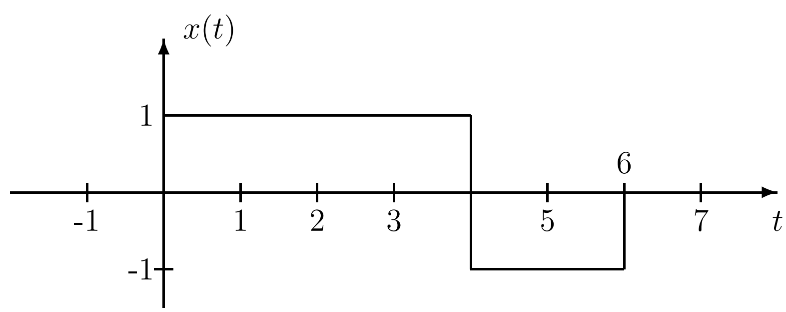
where a is a real number.

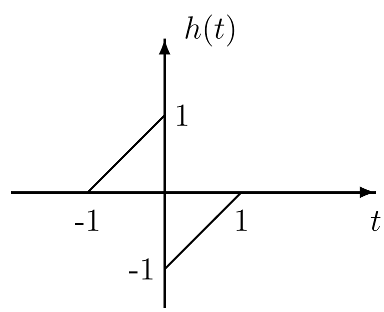
(a) Under what conditions on a and t_0 will the system be *causal*? Justify your answer to receive full credit.

(b) Under what conditions on a and t_0 will the system be *stable*? Justify your answer to receive full credit.

Problem 1.8*

If the input $x(t)$ and the impulse response $h(t)$ of an LTI system are the following:





(a) Determine $y(0)$, the value of the output at $t = 0$.

(b) Determine the complete set of values of t such that the output $y(t) = 0$. *You do not need to find $y(t)$ at any other values of t.*