GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

EE2201A
Problem Set No. 2

Date Assigned: April 9, 1999
Date Due: April 16, 1999

Reading Assignment: In Oppenheim and Willsky, read pp.177-195, 226-231, and 284-330.

Homework Assignment: In all problems, write some explanation of your approach to the solution, i.e., give more than the answer. Turn in for grading only the starred problems: 2.2*, 2.3* 2.4*, and 2.5*.

Problems with answers:

Take a look on the Web at Problem Set 2 from Winter, Spring, and Fall of 1998 and EE2823A from Winter 1999. These problems and solutions cover the same material as this problem set.

Problem 2.1:

Try your hand at expressing each of the following in a simpler form:

(a) $[e^{-t}\cos(2\pi t)][\delta(t)-\delta(t+3)] =$

(b) $\int_{-\infty}^{\infty} \sin(2\pi\tau+\pi/3)\delta(\tau-2)d\tau =$

(c) $\int_{-\infty}^{\infty} \sin(2\pi\tau+\pi/3)\delta(t-\tau)d\tau =$

(d) $u_1(t-3) * \sin(2\pi t+\pi/3) =$

(e) $[\delta(t)-u(t)] * [\delta(t-1)+\delta(t-2)] =$

(f) $u(t) * \delta(t-3) * u_1(t) =$

Answers available under EE3230, Fall 1998.

Problem 2.2*

An LTI system has impulse response $h(t) = e^{-\alpha t}u(t)$.

(a) Find the frequency response $H(j\omega)$ of this system. Under what conditions on α does the frequency response exist?

(b) Using convolution, find the output of this system for the input $x(t) = e^{j\omega_0 t}u(t)$ for all t.

(c) Under what conditions on α will the output obtained in part (b) approach $H(j\omega_0)e^{j\omega_0 t}$ as $t \longrightarrow \infty$?

Problem 2.3*:

A linear time-invariant system has impulse response

$$h(t) = \frac{d}{dt}\left(e^{-4t}u(t)\right)$$

(a) Determine the frequency response $H(j\omega)$ of the system.

(b) Use superposition to find the output due to the input

$$x(t) = 10 + 10\cos(4t) + \delta(t-3).$$

Hint: Use the easiest method to find the output due to each component of the input.

Problem 2.4*:

An LTI system is defined by the following input/output relation:

$$y(t) = 0.5x(t) + x(t-4) + 0.5x(t-8). \qquad (1)$$

(a) Determine the impulse response $h(t)$ of the overall system; i.e., determine the output when the input is an impulse.

(b) Substitute your answer for $h(t)$ into the the integral formula

$$H(j\omega) = \int_{-\infty}^{\infty} h(t)e^{-j\omega t}dt$$

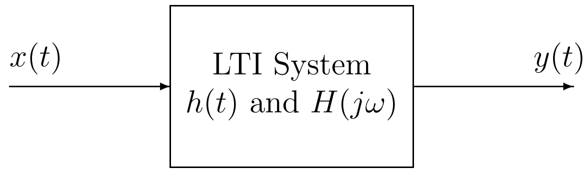
to obtain the frequency response.

(c) Apply the system definition given in Eq. (**??**) directly to the input $x(t) = e^{j\omega t}$ for $-\infty < t < \infty$ to determine the frequency response $H(j\omega)$ of the overall system. You should, of course, obtain the same answer as in part (b).

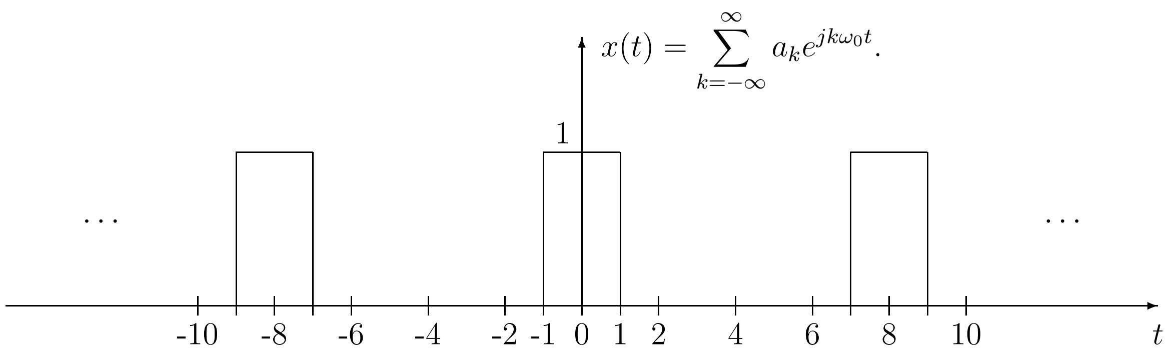
(d) Sketch the magnitude and phase of $H(j\omega)$ as a function of ω.

Problem 2.5*:

Consider the LTI system below:



The input to this system is the periodic pulse wave $x(t)$ depicted below:



(a) Determine ω_0 and the coefficients a_k in the Fourier series representation of $x(t)$.

(b) Plot the spectrum of the signal $x(t)$; i.e., make a plot showing the a_ks plotted at the frequencies $k\omega_0$ for $-4\omega_0 \leq \omega \leq 4\omega_0$.

(c) If the frequency response of the system is the ideal lowpass filter

$$H(j\omega) = \begin{cases} 1 & |\omega| < 0.5\pi \\ 0 & |\omega| > 0.5\pi \end{cases}$$

what is the output of the system when the input is $x(t)$ on the previous page? Give an equation for $y(t)$. *Use the plot of part (b) and a plot of the frequency response to help you solve this problem.*

(d) If the frequency response is $H(j\omega) = (j\omega)e^{-j\omega 3}$, plot the output of the system $y(t)$ when the input is $x(t)$ as plotted above.