GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

EE2201A
Problem Set No. 3

Date Assigned: April 16, 1999
Date Due: April 23, 1999

Reading Assignment: In Oppenheim and Willsky, read pp. 284-334, 231-244, and 583-601.

Homework Assignment: Turn in for grading only the starred problems: 3.2*, 3.3*, 3.4*, 3.5*, and 3.7*.

Problem 3.1

Consider the signal $x(t)$, whose Fourier transform is

$$X(j\omega) = \begin{cases} 10 & -2\pi < \omega < 2\pi \\ 0 & \text{otherwise} \end{cases}$$

This is the signal $x(t)$ that is referred to in all the problems below.

(a) $x(t)$ is the input to a linear time-invariant system whose impulse response is

$$h(t) = \frac{\sin[\pi(t-2)]}{\pi(t-2)}$$

Use Fourier transforms to determine an equation for the output $y(t) = x(t) * h(t)$ of the LTI system.

(b) Another signal has Fourier transform $Y(j\omega) = X(j(\omega - 10\pi)) + X(j(\omega + 10\pi))$. Plot the Fourier transform $Y(j\omega)$ of this signal. Use a theorem of Fourier transforms to determine the signal $y(t)$.

(c) Still another signal is $v(t) = (x(t))^2$. Determine and plot the Fourier transform $V(j\omega)$ of this signal.

(d) Give an equation for the Fourier transform of $w(t) = x(t-5)$.

(e) Plot the magnitude and phase of $R(j\omega)$, the Fourier transform of the signal $r(t) = x^{(1)}(t)$.

Try these first on your own and then see Problem 3.1 of EE2823, Winter 1999 for answers.

Problem 3.2*:

Use the tables of transforms and Fourier transform properties to determine either the time function or its transform for each of the following:

(a) $X(j\omega) = \dfrac{d}{dt}\left(\dfrac{10\sin(\omega_c t)}{\pi t}\right)$

(b) $x(t) = \dfrac{2\sin(400\pi t)}{\pi t}\cos(2000\pi t)$

(c) $x(t) = \delta(t) + e^{-t}u(t) - e^{-2t}u(t)$

(d) $x(t) = e^{-t}u(t) * e^{-2t}u(t)$

(e) $x(t) = \dfrac{10\sin(200\pi t)}{\pi t} * \dfrac{\sin(100\pi t)}{\pi t}$

(f) $X(j\omega) = \left(\dfrac{2\sin(\omega)}{\omega}\right)\sum_{k=-\infty}^{\infty}\left(\dfrac{2\pi}{10}\right)\delta(\omega - 2\pi k/10))$

Problem 3.3*:

Consider a causal LTI system whose frequency response is

$$H(j\omega) = \frac{1}{j\omega + 3}$$

For a particular input $x(t)$, the output of the system is

$$y(t) = e^{-3t}u(t) - e^{-4t}u(t)$$

Determine $x(t)$.

Problem 3.4*:

Consider the periodic signal $x(t)$, which is defined over one period by

$$x(t) = \begin{cases} 1 & 0 < t < 10 \\ 0 & 10 < t < 20 \end{cases}$$

The period of the signal is $T = 20$.

(a) Sketch the waveform of the signal $x(t)$. What is the fundamental frequency ω_0 for this signal?

(b) Determine the Fourier transform $X(j\omega)$ of the signal $x(t)$. Plot $X(j\omega)$ as a function of ω.

(c) The frequency response of a LTI highpass filter is

$$H(j\omega) = \begin{cases} 0 & |\omega| < \pi/20 \\ e^{-j\omega 5} & \pi/20 < |\omega| \end{cases}$$

Plot the magnitude of the frequency response, $|H(j\omega)|$ on the same graph as the plot of $X(j\omega)$. What is the effect of the factor $e^{-j\omega 5}$ on the output waveform?

(d) Determine the Fourier transform of the output of the system for the given input $x(t)$. Give the simplest possible equation for your answer.

(e) From your answer in part (d), determine a simple equation for the output $y(t)$ in terms of $x(t)$ that holds for this input $x(t)$.

Problem 3.5*:

(a) Consider an LTI system for which the input $x(t)$ and output $y(t)$ satisfy the differential equation

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = \frac{dx(t)}{dt} + 5x(t)$$

Determine the frequency response, $H(j\omega)$, of this system.

(b) If the frequency response of an LTI system is

$$H(j\omega) = \frac{(j\omega + 2)(j\omega + 1)}{(j\omega + 3)(j\omega + 4)}$$

what is the differential equation that is satisfied by the input $x(t)$ and the output $y(t)$?

Problem 3.6:

(a) Show that in general if a signal $x(t)$ is real, then its Fourier transform is "conjugate symmetric", i.e., $X(-j\omega) = X^*(j\omega)$, where * denotes complex conjugation.

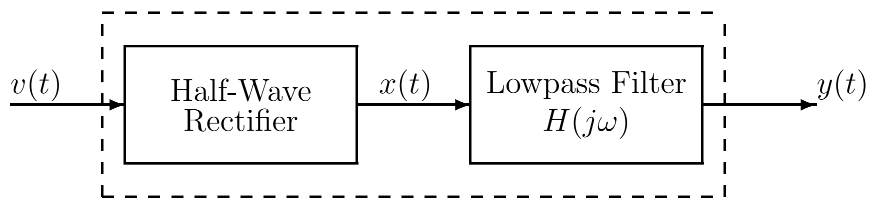
(b) Furthermore, show that conjugate symmetry implies that

$$\begin{aligned}
\mathcal{R}e\{X(-j\omega)\} &= \mathcal{R}e\{X(j\omega)\} && \text{even symmetry} \\
\mathcal{I}m\{X(-j\omega)\} &= -\mathcal{I}m\{X(j\omega)\} && \text{odd symmetry} \\
|X(-j\omega)| &= |X(-j\omega)| && \text{even symmetry} \\
\angle X(-j\omega) &= -\angle X(-j\omega) && \text{odd symmetry}
\end{aligned}$$

(c) Find the Fourier transform of the signal $x(t) = e^{-\alpha t}u(t)$ where α is real and $0 < \alpha$. Sketch the real and imaginary parts of $X(j\omega)$ as a function of ω and verify that the above symmetry conditions hold for this real signal.

Problem 3.7*

A DC power supply can be modeled as the following system:



The half-wave rectifier and the lowpass filter are described by the equations

$$x(t) = \begin{cases} v(t) & \text{if } v(t) > 0 \\ 0 & \text{if } v(t) \le 0 \end{cases} \qquad \text{and} \qquad H(j\omega) = \begin{cases} G & |\omega| < \omega_c \\ 0 & |\omega| > \omega_c \end{cases}$$

The input signal is $v(t) = 120\sqrt{2}\cos(120\pi t)$.

(a) Carefully sketch the signal $x(t)$ at the output of the half-wave rectifier. Label the time axis carefully. What frequencies will be present in the Fourier transform of $x(t)$?

(b) The objective of the lowpass filter is to remove all frequency components except the DC component. How should ω_c be chosen so that only the DC component remains?

(c) For the choice of ω_c in part (b) how should the gain G of the lowpass filter be chosen so that $y(t) = 10$ for all t?