GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

EE2823A
Quiz No. 2
March 1, 1999

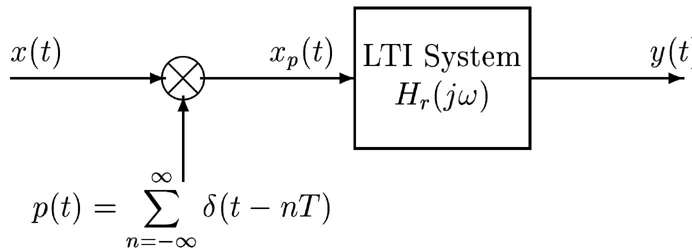
Name: ____

1. The exam is closed book. You may use one 8.5” by 11” sheet of notes (both sides). You are permitted to use a calculator. **I have given you a table of transform formulas as the last two pages. Tear then off and use them!**
2. Do all work in the space provided. If you need more room, use the *back* of the *previous* page and indicate that you have done so.
3. Indicate your answer clearly by circling it or drawing a box around it.
4. Think carefully about the problem before you begin to write. Move on to the next problem if you cannot come up with a plan for the solution.
5. If you want to receive partial credit, you should clearly indicate your reasoning and method of attack on the problem.

Problem	**Points**	**Score**
1	25	
2	25	
3	20	
4	20	
5	10	
TOTAL	100	

Problem W99.Q2.1 (25 %)

Consider the following impulse train sampling and reconstruction system:



(a) If $x(t) = 10 + 20\cos(160\pi t - \pi/4)$, plot the Fourier transform $X(j\omega)$.

(b) If $2\pi/T = 200\pi$, plot the Fourier transform $X_p(j\omega)$.

(c) For the conditions of part (b), determine the output $y(t)$ if the frequency response of the LTI system is

$$H_r(j\omega) = \begin{cases} T & |\omega| \le \pi/T \\ 0 & |\omega| > \pi/T \end{cases}$$

Problem W99.Q2.2: (25 %)

A signal $x(t) = 10 + 20\cos(200\pi t)$ is multiplied by a window function to obtain:

$$v(t) = w(t)x(t) = w(t)[10 + 20\cos(200\pi t)]$$

(a) Give an equation for $V(j\omega)$ in terms of $W(j\omega)$.

(b) Now suppose that $w(t)$ is the "rectangular window" defined by

$$w(t) = \begin{cases} 1 & |t| \leq T_1 \\ 0 & |t| > T_1 \end{cases}$$

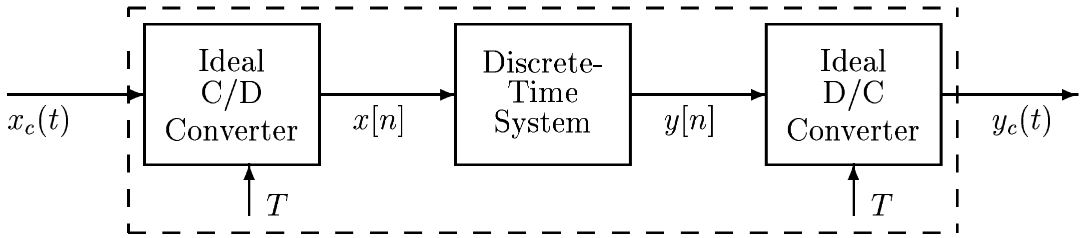
Determine the Fourier transform $W(j\omega)$.

(c) Using the results of parts (a) and (b), plot $V(j\omega)$ for the case $T_1 = 0.1$ sec.

(d) What is the *minimum* value for T_1 such that the "main lobes" of the copies of $W(j\omega)$ do not overlap?

Problem W99.Q2.3 (20 %)

All parts of this problem are concerned with the following system.



$$x[n] = x_c(nT) \qquad\qquad y_c(t) = \sum_{n=-\infty}^{\infty} y[n]\frac{\sin\frac{\pi}{T}(t-nT)}{\frac{\pi}{T}(t-nT)}$$

Assume that $X_c(j\omega) = 0$ for $|\omega| \geq 1000\pi$.

(a) Suppose that the discrete-time system is defined by $y[n] = x[n]$. What is the *minimum* value of $2\pi/T$ such that $y_c(t) = x_c(t)$?

(b) The input/output relation for the discrete-time system is

$$y[n] = 0.5x[n] + 0.5x[n-2]$$

For the value of T chosen in part (a), the input and output Fourier transforms are related by an equation of the form $Y_c(j\omega) = H_{\text{eff}}(j\omega)X_c(j\omega)$. Find an equation for the overall effective frequency response $H_{\text{eff}}(\omega)$, and plot $|H_{\text{eff}}(j\omega)|$ and $\arg[H_{\text{eff}}(j\omega)]$ below.

Problem W99.Q2.4 (20 %)

(a) Determine the impulse response of a causal system whose system function is

$$H(s) = \frac{(s+1)}{s+3} e^{-s2}$$

(b) If the input to an integrator system [impulse response $h(t) = u(t)$] is

$$x(t) = \delta(t) + 20\cos(10t)u(t),$$

determine the Laplace transform $Y(s)$ of the output (be sure to include the region of convergence of $Y(s)$).

Problem W99.Q2.5 (10 %)

The system function of an LTI system is $H(s) = \dfrac{s^2+4}{(s+1)(s-1)}$.

(a) Plot the poles and zeros in the s-plane and shade the region of convergence of $H(s)$ if the system is causal.

(b) Plot the poles and zeros in the s-plane and shade the region of convergence of $H(s)$ if the system is stable.

FORMULAS FOR SECOND EXAM

Fourier Transform Pairs

$$x(t) = e^{-at}u(t) \quad \overset{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = \frac{1}{a + j\omega}$$

$$x(t) = \begin{cases} 1 & |t| < T_1 \\ 0 & |t| > T_1 \end{cases} \quad \overset{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = \frac{2\sin(\omega T_1)}{\omega}$$

$$x(t) = \frac{\sin \omega_b t}{\pi t} \quad \overset{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = \begin{cases} 1 & |\omega| < \omega_b \\ 0 & |\omega| > \omega_b \end{cases}$$

$$x(t) = 1 \quad \overset{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = 2\pi\delta(\omega)$$

$$x(t) = \delta(t) \quad \overset{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = 1$$

$$x(t) = u(t) \quad \overset{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = \pi\delta(\omega) + \frac{1}{j\omega}$$

$$x(t) = u_1(t) \quad \overset{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = j\omega$$

$$x(t) = \cos \omega_0 t \quad \overset{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = \pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)$$

$$x(t) = \sin \omega_0 t \quad \overset{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = -j\pi\delta(\omega - \omega_0) + j\pi\delta(\omega + \omega_0)$$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 kt} \quad \overset{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - k\omega_0)$$

$$a_k = \frac{1}{T}\int_0^T x(t)e^{-jk\omega_0 t}dt$$

Fourier Transform Theorems

$$ax_1(t) + bx_2(t) \quad \overset{\mathcal{F}}{\longleftrightarrow} \quad aX_1(j\omega) + bX_2(j\omega)$$

$$x(t - t_0) \quad \overset{\mathcal{F}}{\longleftrightarrow} \quad e^{-j\omega t_0}X(j\omega)$$

$$e^{j\omega_0 t}x(t) \quad \overset{\mathcal{F}}{\longleftrightarrow} \quad X(j(\omega - \omega_0))$$

$$\frac{dx(t)}{dt} \quad \overset{\mathcal{F}}{\longleftrightarrow} \quad j\omega X(j\omega)$$

$$-jtx(t) \quad \overset{\mathcal{F}}{\longleftrightarrow} \quad \frac{dX(j\omega)}{d\omega}$$

$$y(t) = x(t) * h(t) \quad \overset{\mathcal{F}}{\longleftrightarrow} \quad Y(j\omega) = H(j\omega)X(j\omega)$$

$$x(t)p(t) \quad \overset{\mathcal{F}}{\longleftrightarrow} \quad \frac{1}{2\pi}X(j\omega) * P(j\omega)$$

$$x(at) \quad \overset{\mathcal{F}}{\longleftrightarrow} \quad \frac{1}{|a|}X\left(\frac{j\omega}{a}\right)$$

$$X(jt) \quad \overset{\mathcal{F}}{\longleftrightarrow} \quad 2\pi x(-\omega)$$

Laplace Transform Pairs and Theorems

$$x(t) = e^{s_0 t}u(t) \quad \overset{\mathcal{L}}{\longleftrightarrow} \quad X(s) = \frac{1}{s - s_0} \qquad \Re e\{s\} > \Re e\{s_0\}$$

$$x(t) = -e^{s_0 t}u(-t) \quad \overset{\mathcal{L}}{\longleftrightarrow} \quad X(s) = \frac{1}{s - s_0} \qquad \Re e\{s\} < \Re e\{s_0\}$$

$$x(t) = \cos(\omega_0 t)u(t) \quad \overset{\mathcal{L}}{\longleftrightarrow} \quad X(s) = \frac{s}{s^2 + \omega_0^2} \qquad \Re e\{s\} > \Re e\{0\}$$

$$x(t) = \sin(\omega_0 t)u(t) \quad \overset{\mathcal{L}}{\longleftrightarrow} \quad X(s) = \frac{\omega_0}{s^2 + \omega_0^2} \qquad \Re e\{s\} > \Re e\{0\}$$

$$x(t) = u(t) \quad \overset{\mathcal{L}}{\longleftrightarrow} \quad X(s) = \frac{1}{s} \qquad \Re e\{s\} > 0$$

$$x(t) = \delta(t) \quad \overset{\mathcal{F}}{\longleftrightarrow} \quad X(s) = 1$$

$$ax_1(t) + bx_2(t) \quad \overset{\mathcal{L}}{\longleftrightarrow} \quad aX_1(s) + bX_2(s)$$

$$x(t - t_0) \quad \overset{\mathcal{L}}{\longleftrightarrow} \quad e^{-st_0}X(s)$$

$$\frac{dx(t)}{dt} \quad \overset{\mathcal{L}}{\longleftrightarrow} \quad sX(s)$$

$$y(t) = x(t) * h(t) \quad \overset{\mathcal{L}}{\longleftrightarrow} \quad Y(s) = H(s)X(s)$$