

GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

EE2201A
Problem Set No. 6

Date Assigned: May 7, 1999
Date Due: May 14, 1999

Reading Assignment: In Oppenheim and Willsky read pp. 5143-545.

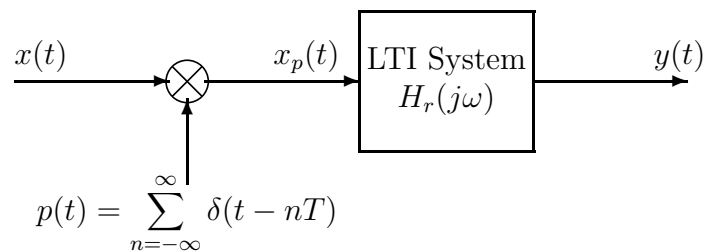
Homework Assignment: Turn in for grading only the starred problems: 6.1*, 6.2*, and 6.3*.

Practice Problems:

Look at Problems on Problem Sets 5 and 6 of EE3230, Winter, Spring, Fall of 1998 and Problem Set 6 of EE2823A Winter 1999.

Problem 6.1*

Consider the following impulse train sampling and reconstruction system:



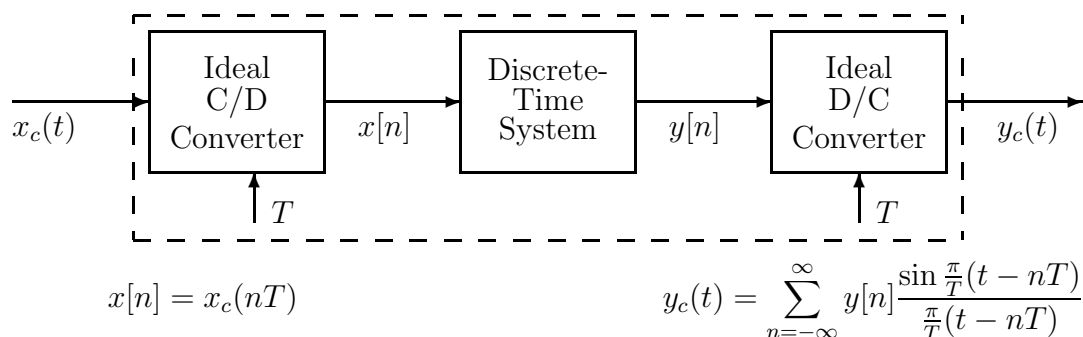
- (a) If $x(t) = 10 + 20 \cos(160\pi t - \pi/4)$, plot the Fourier transform $X(j\omega)$.
- (b) If $2\pi/T = 200\pi$, plot the Fourier transform $X_p(j\omega)$.
- (c) For the conditions of part (b), determine the output $y(t)$ if the frequency response of the LTI system is

$$H_r(j\omega) = \begin{cases} T & |\omega| \leq \pi/T \\ 0 & |\omega| > \pi/T \end{cases}$$

- (d) For the input in part (a), how should we choose the sampling rate $2\pi/T$ so that $y(t) = x(t)$?

Problem 6.2*

All parts of this problem are concerned with the following system.



Assume that $X_c(j\omega) = 0$ for $|\omega| \geq 400\pi$.

- Suppose that the discrete-time system is defined by $y[n] = x[n]$. What is the *minimum* value of $2\pi/T$ such that $y_c(t) = x_c(t)$?
- If the input is $x_c(t) = 10 + 20 \cos(160\pi t - \pi/4)$, the sampling frequency is $2\pi/T = 200\pi$, and $y[n] = x[n]$, plot $Y_c(j\omega)$ and give an equation for $y_c(t)$.
- The input/output relation for the discrete-time system is

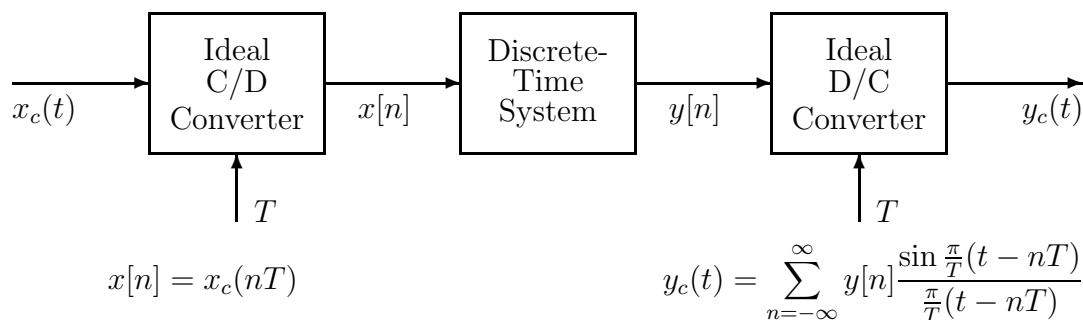
$$y[n] = 0.25x[n] + 0.5x[n - 2] + 0.25x[n - 4]$$

If $2\pi/T = 400\pi$, the input and output Fourier transforms are related by an equation of the form $Y_c(j\omega) = H_{\text{eff}}(j\omega)X_c(j\omega)$. Find an equation for the overall effective frequency response $H_{\text{eff}}(\omega)$, and plot $|H_{\text{eff}}(j\omega)|$ and $\arg[H_{\text{eff}}(j\omega)]$.

- For the sampling rate and filter of part (c) and the input signal of part (b), determine the output signal $y_c(t)$.

Problem 6.3*

All parts of this problem are concerned with the following system.



Assume that $X_c(\omega) = 0$ for $|\omega| \geq 1000\pi$.

- Suppose that the discrete-time system is defined by $y[n] = x[n]$. What is the *minimum* value of $2\pi/T$ such that $y_c(t) = x_c(t)$?
- Determine the relationship between $y[n]$ and $x[n]$ so that if the sampling rate satisfies the condition of (a), then $y_c(t) = x_c(t - 50T)$.

(c) The input/output relation for the discrete-time system is

$$y[n] = \frac{1}{3} (x[n] + x[n-1] + x[n-2])$$

For the value of T chosen in part (a), the input and output Fourier transforms are related by an equation of the form $Y_c(j\omega) = H_{eff}(j\omega)X_c(j\omega)$. Find an equation for the overall effective frequency response $H_{eff}(j\omega)$. Plot the magnitude and phase of $H_{eff}(j\omega)$. Use MATLAB to do this or sketch it by hand.

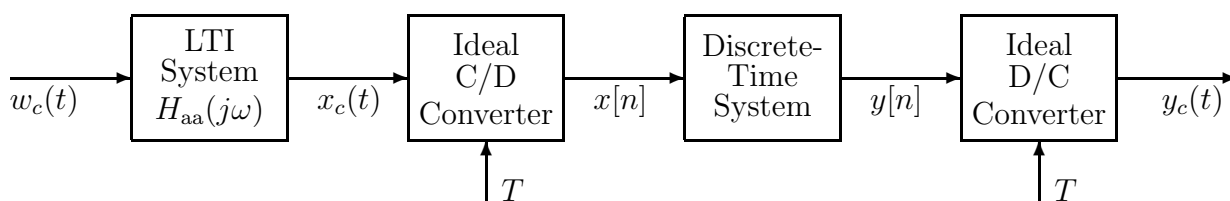
(d) Suppose that no aliasing distortion occurs in sampling and that the frequency response of the discrete-time system is defined over one period ($-\pi \leq \hat{\omega} \leq \pi$) by

$$H(e^{j\hat{\omega}}) = \begin{cases} 0 & |\hat{\omega}| < \pi/2 \\ e^{-j\hat{\omega}10} & \pi/2 < |\hat{\omega}| \leq \pi \end{cases}$$

where $\hat{\omega} = \omega T$. Plot the magnitude and phase of $H_{eff}(j\omega) = Y_c(j\omega)/X_c(j\omega)$.

Problem 6.4

All parts of this problem are concerned with the following system.



$$x[n] = x_c(nT)$$

$$y_c(t) = \sum_{n=-\infty}^{\infty} y[n] \frac{\sin \frac{\pi}{T}(t - nT)}{\frac{\pi}{T}(t - nT)}$$

- (a) The first LTI system has frequency response such that $H_{aa}(j\omega) = 0$ for $|\omega| \geq 1000\pi$. This system is a lowpass filter, but not necessarily an *ideal* lowpass filter; i.e., its passband may not be perfectly flat. A filter used like this in a sampling system is often called an *anti-aliasing filter*. Why?
- (b) How should T be chosen if we want $y_c(t) = x_c(t)$ when the discrete-time system is defined by the difference equation $y[n] = x[n]$?
- (c) If the filter $H_{aa}(j\omega)$ satisfies the condition given in part (a) and the sampling rate is chosen to avoid aliasing as in part (b), and the discrete-time system is defined by $y[n] = x[n]$, give an expression for $y_c(t)$ in terms of $w_c(t)$ and the impulse response $h_{aa}(t)$ of the anti-aliasing filter.
- (d) If the filter $H_{aa}(j\omega)$ satisfies the condition given in part (a) and the sampling rate is chosen to avoid aliasing as in part (b), and the discrete-time system now is defined by $y[n] = x[n] + x[n-1]$, what is the effective frequency response $H_{eff}(j\omega)$ of the overall system from $w_c(t)$ to $y_c(t)$; i.e., what is $H_{eff}(j\omega)$ in the equation

$$Y_c(j\omega) = H_{eff}(j\omega)W_c(j\omega)$$

Be as specific as you can.