GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

EE2201A
Homework Assignment No. 8

Date Assigned: May 28, 1999
Date Due: June 4, 1998

Reading Assignment: In Oppenheim and Willsky, study pp. 654-702 and 816-836.

Homework Assignment: Turn in for grading only Problems 8.4*, 8.5*, and 8.6*.

NOTICE: The Final Exam will be given on Wednesday, June 9, 1999 at 8:00am. I will give you two sheets of formulas on Fourier and Laplace transforms. The exam will cover the entire course.

Problem 8.1

(a) Determine the impulse response of a causal system whose system function is

$$H(s) = \frac{(s+1)}{s+3}e^{-s2}$$

(b) If the input to an integrator system [impulse response $h(t) = u(t)$] is

$$x(t) = \delta(t) + 20\cos(10t)u(t),$$

determine the Laplace transform $Y(s)$ of the output (be sure to include the region of convergence of $Y(s)$). Use partial fraction expansion to determine the output $y(t)$.

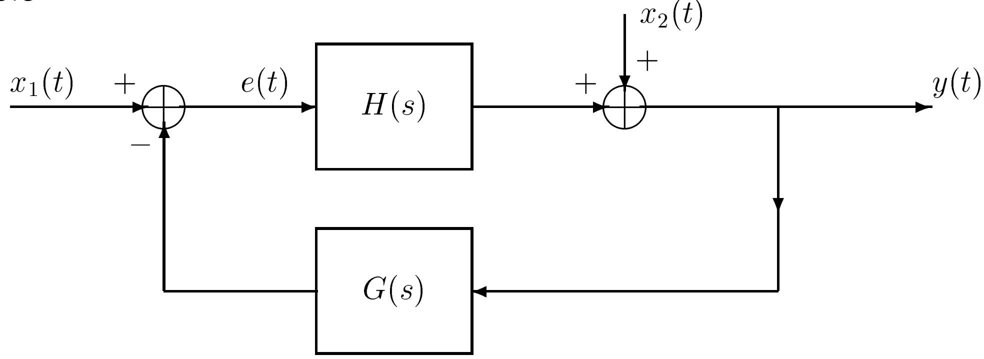
Problem 8.2

The system function of an LTI system is $H(s) = \dfrac{s^2+4}{(s+1)(s-1)}$.

(a) Plot the poles and zeros in the s-plane and shade the region of convergence of $H(s)$ if the system is causal. Determine $h(t)$.

(b) Plot the poles and zeros in the s-plane and shade the region of convergence of $H(s)$ if the system is stable. Determine $h(t)$.

Problem 8.3



(a) Use superposition to **show** that it is possible to express the Laplace transform of the output, $Y(s)$ as $Y(s) = Q(s)X_1(s) + R(s)X_2(s)$, and in the process, determine the transfer function $R(s)$ between the input $X_2(s)$ and the output $Y(s)$. *As a check, note that when $x_2(t) = 0$, the system reduces to the one that we discussed in class.*

(b) In the above system, suppose that $H(s) = 1/s$ and $G(s) = K/s$. Is it possible to find a value of K so that the overall system is stable? If so, determine the range of values for stability. If it is not possible, explain why not.

(c) For $H(s) = 1/s$ and $G(s) = 100/s$ and inputs $x_1(t) = u(t)$ and $x_2(t) = 0$, determine the output $y(t)$.

Problem 8.4*

A causal linear time-invariant system has system function

$$H(s) = \frac{(s + j2)(s - j2)}{(s - 2)(s + 3)}.$$

(a) Determine the differential equation that is satisfied by the input $x(t)$ and the output $y(t)$.

(b) Plot the poles and zeros in the s-plane and shade the region of convergence of $H(s)$.

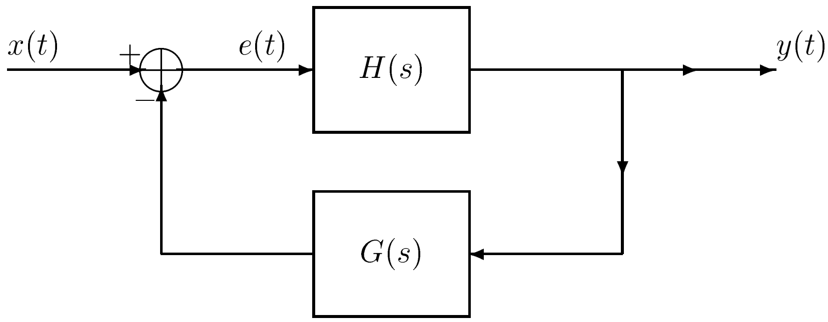
(c) Is the system stable? How do you know?

(d) Determine the response of the system to a unit step input $x(t) = u(t)$.

(e) Does the frequency response of this system exist? If so, what is it? If not, why not?

Problem 8.5*

Consider the following causal feedback system:



where the system functions of the causal systems are

$$H(s) = \frac{s}{(s+4)(s+9)} \qquad \text{and} \qquad G(s) = K.$$

(a) For what set of values of K would this system be a stable system?

(b) Find the value of K such that the system has poles on the $j\omega$ axis. Where are these poles?

(c) For the value of K found in part (b), what is the impulse response of the system?

Problem 8.6*:

In each of the following cases, $H(s)$ is the system function of a causal LTI system. Use the technique of drawing vectors from the poles and zeros to the $j\omega$ axis to sketch the magnitude of the frequency response $|H(j\omega)|$ for each of the following cases. In each case, determine the poles and zeros and plot them in the s-plane. Plot $|H(j\omega)|$ beside your pole-zero plot.

(a) $H(s) = \dfrac{s^2+25}{s+1}$

(b) $H(s) = \dfrac{26}{(s+1)^2+25}$

(c) $H(s) = 10\dfrac{s}{s+100}$

(d) $H(s) = 100\dfrac{(s-2)(s-5)}{(s+2)(s+5)}$.

(e) $H(s) = \dfrac{s^2+25}{(s+1)^2+25}$

Additional Review Problems:

See Problem Sets 7 and 8 of EE3230, Fall 1998.