GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

Quiz #2

Date: November 16, 1995 Course: EE 3230

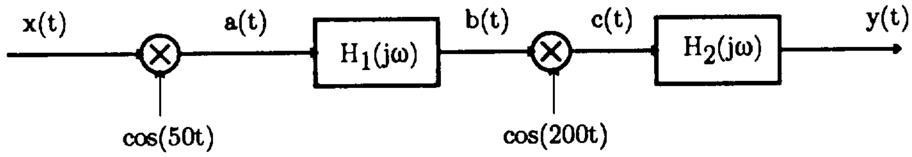
Name: ______________________________
Last, First

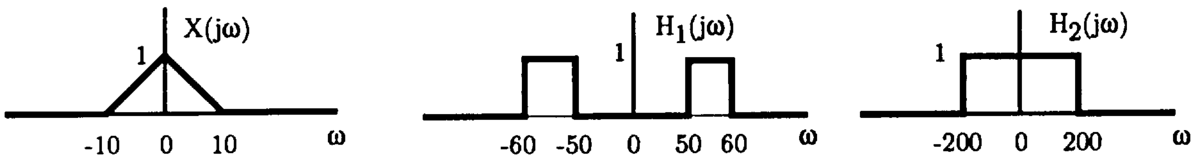
- Closed book, closed notes, two $8\frac{1}{2}'' \times 11''$ handwritten sheets are allowed. Eighty minute time limit.
- None of the problems require involved calculations. Reconsider your approach before doing something tedious.
- All work should be performed on the quiz itself. If more space is needed, use the backs of the pages.

Problem	*Score*
1	
2	
3	
4	
Total	

Problem 1:

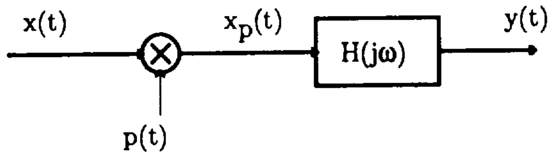
For the system shown below, accurately sketch $A(j\omega)$, $B(j\omega)$, $C(j\omega)$, and $Y(j\omega)$.





Problem 2:

Consider the system below with $p(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT)$



(a) Give the constraints on $x(t)$ and T such that $x(t)$ can be reconstructed from $x_p(t)$.

(b) Give the frequency response $H(j\omega)$ such that $y(t) = x(t)$, provided that $x(t)$ and T satisfy the constraints in part (a).

(c) Consider the signal $x_p(t)$ for $x(t) = \cos(2\pi(50)t)$ and $T = 0.004s$. Find a value of f_x such that a *different* input $x(t) = \cos(2\pi f_x t)$ will result in exactly the same $x_p(t)$.

Problem 3:

(a) Find the Laplace transform of $x(t) = e^{5t}[u(t+1) - u(t-2)]$. Be careful with the region of convergence since this is a finite duration signal.

(b) Find the Laplace transform of $y(t) = e^{5t}[u(2-t) - u(-1-t)]$. This is also a finite duration signal.

(c) By comparing $X(s)$ and $Y(s)$, determine the relationship between the signals $x(t)$ and $y(t)$.

Problem 4:

Consider the system transfer function $H(s) = \frac{2s+6}{s^2+5s+4}$.

(a) Find a differential equation in $x(t)$ and $y(t)$ that represents a system with transfer function $H(s)$.

(b) If the system is causal, find its impulse response $h(t)$.

(c) Find the inverse system's transfer function $H_I(s)$.

(d) Is it possible for the inverse system to be both causal and stable? Why?

TABLE 5.1 Laplace Transform Properties

Property	*Time domain*	*Transform*	*ROC*
Linearity	$ax_1(t) + bx_2(t)$	$aX_1(s) + bX_2(s)$	$R' \supset R_1 \cap R_2$
Time shift	$x(t - t_0)$	$e^{-st_0}X(s)$	$R' = R$
Modulation	$e^{s_0t}x(t)$	$X(s - s_0)$	$R' = R + \text{Re}\{s_0\}$
Axis scaling	$x(at)$	$\frac{1}{\lvert a \rvert}X\left(\frac{s}{a}\right)$	$R' = aR$
Axis reversal	$x(-t)$	$X(-s)$	$R' = -R$
Differentiation	$\frac{dx(t)}{dt}$	$sX(s)$	$R' \supset R$
	$-tx(t)$	$\frac{dX(s)}{ds}$	$R' = R$
Integration	$\int_{-\infty}^{t} x(\tau)\, d\tau$	$\frac{1}{s}X(s)$	$R' \supset R \cap \text{Re}\{s\} > 0$
Convolution	$x_1(t) * x_2(t)$	$X_1(s)X_2(s)$	$R' \supset R_1 \cap R_2$

TABLE 5.2 Common Laplace Transforms

Signal	*Time domain*	*Transform*	*ROC*
Impulse	$\delta(t)$	1	All s
Unit step	$u(t)$	$\frac{1}{s}$	$\text{Re}\{s\} > 0$
	$-u(-t)$	$\frac{1}{s}$	$\text{Re}\{s\} < 0$
Exponential	$e^{-at}u(t)$	$\frac{1}{s + a}$	$\text{Re}\{s\} > -a$
	$-e^{-at}u(-t)$	$\frac{1}{s + a}$	$\text{Re}\{s\} < -a$
Weighted exponential	$te^{-at}u(t)$	$\frac{1}{(s + a)^2}$	$\text{Re}\{s\} > -a$
Causal sine	$[\sin \omega_0 t]u(t)$	$\frac{\omega_0}{s^2 + \omega_0^2}$	$\text{Re}\{s\} > 0$
Causal cosine	$[\cos \omega_0 t]u(t)$	$\frac{s}{s^2 + \omega_0^2}$	$\text{Re}\{s\} > 0$
Damped sine	$e^{-at}[\sin \omega_0 t]u(t)$	$\frac{\omega_0}{(s + a)^2 + \omega_0^2}$	$\text{Re}\{s\} > -a$
Damped cosine	$e^{-at}[\cos \omega_0 t]u(t)$	$\frac{s + a}{(s + a)^2 + \omega_0^2}$	$\text{Re}\{s\} > -a$