GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

EE3230
Final Exam
June 9, 1998

Name: ______________________________

1. The exam is closed book. In addition to the table of Fourier and Laplace transforms given at the end of the test, you may use one page of notes.
2. Do all work in the space provided. If you need more room, use the *back* of the *previous* page.
3. Indicate your answer clearly by circling it or drawing a box around it.
4. Think carefully about the the problem before you begin to write. *None* of the problems should require extensive algebra or calculation. Move on to the next problem if you find yourself bogged down in algebra.

Problem	**Points**	**Score**
1	15	
2	15	
3	20	
4	15	
5	20	
6	15	
Extra Credit	5	
TOTAL	100 (105)	

Problem Sp98.F.1 (15 %)
A causal LTI system with input $x(t)$ and output $y(t)$ is described by the following system function:

$$H(s) = \frac{s+2}{(s+3)(s+4)}.$$

(a) Determine the differential equation that is satisfied by the input $x(t)$ and output $y(t)$.

(b) Is the system stable? Explain.

(c) Determine $Y(s)$, the two-sided Laplace transform of the output, if the input is

$$x(t) = u(-t) + e^{-2t}u(t)$$

Plot the poles and zeros of $Y(s)$ in the z-plane and indicate the region of convergence.

(d) Find the output time function $y(t)$ for $-\infty < t < \infty$ for the input given in part (b).

Problem Sp98.F.2 (15 %)

Consider the pulse signal

$$p(t) = \begin{cases} 1/\sqrt{T_1} & 0 < t < T_1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Plot $p(t)$ beside its equation in the above space.

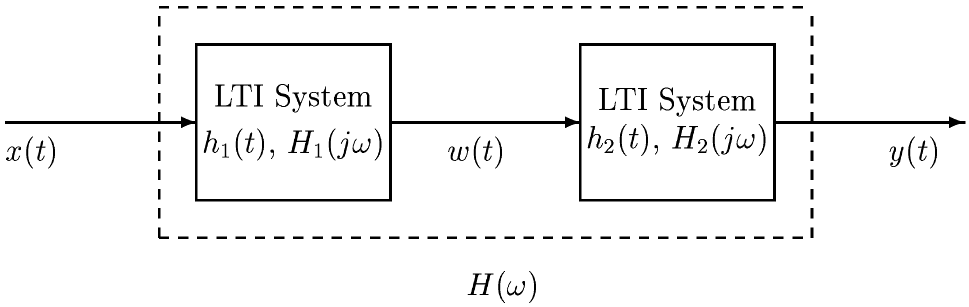
(b) A new signal is defined as $w(t) = p(t) * p(-t)$. Determine $w(t)$ and plot it in the space below.

(c) Find the Fourier transform of $w(t)$ and plot it in the space below.

(d) Plot the Fourier transform of $y(t) = w(t)\cos(\frac{10\pi}{T_1}t)$.

Problem Sp98.F.3 (20 %)

A cascade of two linear time-invariant systems is shown below.



The frequency response of the first LTI system is

$$H_1(j\omega) = \begin{cases} e^{-j\omega\tau_0} & |\omega| < 200\pi \\ 0 & |\omega| > 200\pi \end{cases}$$

where $\tau_0 = (1/400)$ seconds. The impulse response of the second system is

$$h_2(t) = \delta(t) + \delta(t - 2\tau_0)$$

(a) Find $H(j\omega)$, the frequency response of the overall system. Express your answer in the form $H(j\omega) = A(\omega)e^{j\theta(\omega)}$, where $A(\omega)$ and $\theta(\omega)$ are real functions of ω.

(b) What is the delay of the overall system?

Problem Sp98.F.3 (cont.) (20 %)

(c) Plot the magnitude and phase of $H(j\omega)$.

(d) Find $h(t)$, the impulse response of the overall system from input $x(t)$ to output $y(t)$.

(e) Find the output $y(t)$ if the input is $x(t) = \sum_{k=-\infty}^{\infty} e^{j100\pi kt}$.

(Using the results of the previous parts, you should be able to write down the answer with little or no additional analysis. If you were not successful in obtaining the answer to one or more of the previous parts, you should still be able to give a partially correct answer to this part.)

Problem Sp98.F.4 (15 %)

Consider a bandlimited signal $x(t)$ whose Fourier transform has the property $X(j\omega) = 0$ for $|\omega| \geq \omega_1$.

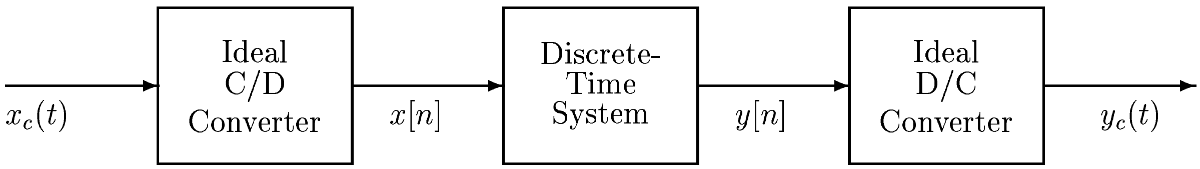
(a) First consider the signal $g(t) = [x(t)]^2 = x(t) \cdot x(t)$. Sketch a "typical" $X(j\omega)$ and then use properties of Fourier transforms to show that $G(\omega) = 0$ for $|\omega| \geq \omega_2$. **Find ω_2 in terms of ω_1.**

(b) Now consider the signal $v(t) = [x(t) + \cos \omega_c t]^2$. Sketch the Fourier transform $V(j\omega)$ assuming a "typical" $X(j\omega)$. *You may use the result of part (a) even if you were unable to find ω_2, and remember that* $\cos^2 \theta = \frac{1}{2}[1 + \cos 2\theta]$.

(c) Determine the condition on ω_c in terms of ω_1 and ω_2 such that it is possible to find a LTI system whose output will be $y(t) = x(t) \cos \omega_c t$ if the input is $v(t)$. Carefully sketch the frequency response of the LTI system.

Problem Sp98.F.5 (20 %)

All parts of this problem are concerned with the following system.



$$x[n] = x_c(nT) \qquad\qquad y_c(t) = \sum_{n=-\infty}^{\infty} y[n] \frac{\sin \frac{\pi}{T}(t - nT)}{\frac{\pi}{T}(t - nT)}$$

(a) Assume that $X_c(j\omega) = 0$ for $|\omega| \geq 100\pi$, and suppose that the discrete-time system is defined by $y[n] = x[n]$. What is the *minimum* value of $2\pi/T$ such that $y_c(t) = x_c(t)$?

(b) If the input is $x_c(t) = \cos(30\pi t + \pi/3)$, the sampling frequency is $2\pi/T = 40\pi$, plot the discrete-time Fourier transform $X(e^{j\omega T})$ as a function of ω for $-40\pi < \omega < 40\pi$.

(c) If $y[n] = x[n]$, and the signal $x_c(t) = \cos(30\pi t + \pi/3)$ is sampled with sampling frequency $2\pi/T = 40\pi$ as in part (b), what is the reconstructed signal $y_c(t)$?

Problem Sp98.F.5 (cont) (20%)

(d) Now suppose that the input/output relation for the discrete-time system is

$$y[n] = 0.25(x[n] + x[n-1] + x[n-2] + x[n-3])$$

For the signal and sampling rate conditions assumed in part (a), the input and output Fourier transforms are related by an equation of the form $Y_c(j\omega) = H_{eff}(j\omega)X_c(j\omega)$. Find an equation for the overall effective frequency response $H_{eff}(j\omega)$.

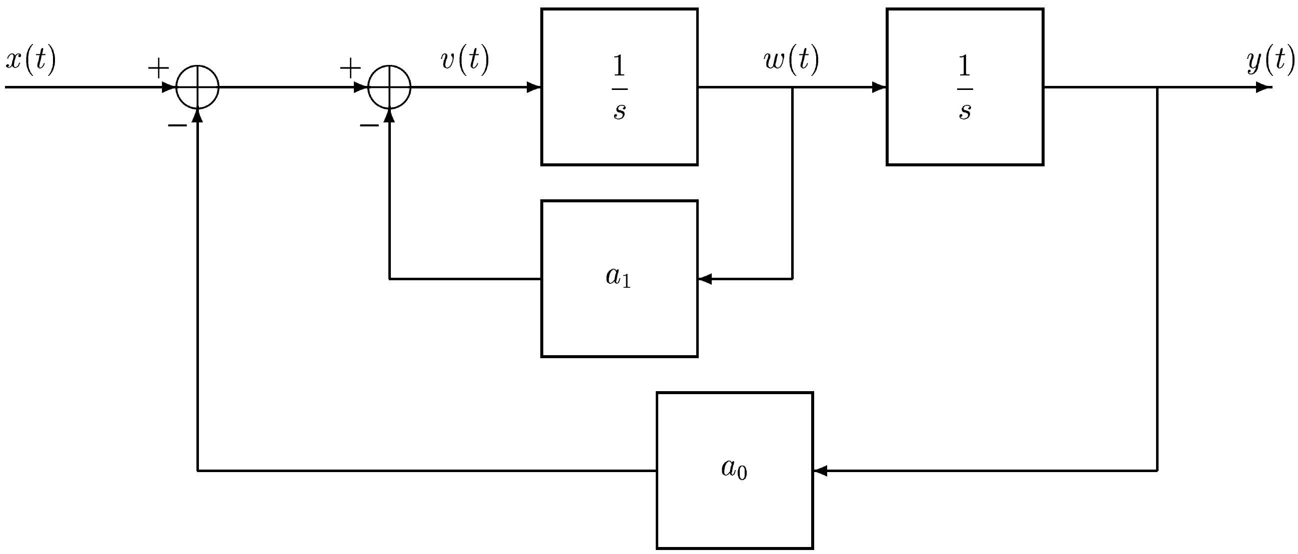
(e) Suppose that the discrete-time system is LTI with frequency response $H(e^{j\hat{\omega}})$ and assume that $X_c(j\omega) = 0$ for $|\omega| \geq 100\pi$. Determine an appropriate value for T, and sketch the corresponding frequency response $H(e^{j\hat{\omega}})$ (as a function of $\hat{\omega}$) or give an equation for $H(e^{j\hat{\omega}})$ such that

$$Y_c(j\omega) = \begin{cases} 2X_c(j\omega) & |\omega| < 50\pi \\ 0 & |\omega| > 50\pi \end{cases}$$

You must give both T and a plot of $H(e^{j\hat{\omega}})$ for the complete solution. *There are many correct solutions for this problem depending on the choice of T. I will give you 2 points extra credit if you give me the solution that uses the absolute* **maximum** *value of T that will work.*

Problem Sp98.F.6 (15 %)

In the feedback system below, the quantities in the boxes are the system functions of the blocks. (a_0 and a_1 are constant gains.)



(a) From the above diagram, give an equations for $v(t)$ and $w(t)$ in terms of the output $y(t)$.

(b) Simplify the above block diagram using Laplace transform equations and obtain an expression for the overall system function $H_{oa}(s) = Y(s)/X(s)$.

(c) Determine the differential equation that is satisfied by the input $x(t)$ and output $y(t)$.

EXTRA CREDIT (3 %)

Draw a block diagram similar to the one on the previous page for which the input output differential equation is

$$\frac{d^3y(t)}{dt^3} + a_2\frac{d^2y(t)}{dt^2} + a_1\frac{dy(t)}{dt} + a_0y(t) = b_0x(t)$$

FORMULAS FOR FINAL EXAM

Fourier Transform Pairs

$$x(t) = e^{-at}u(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = \frac{1}{a + j\omega}$$

$$x(t) = \begin{cases} 1 & |t| < T_1 \\ 0 & |t| > T_1 \end{cases} \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = \frac{2\sin(\omega T_1)}{\omega}$$

$$x(t) = \frac{\sin \omega_b t}{\pi t} \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = \begin{cases} 1 & |\omega| < \omega_b \\ 0 & |\omega| > \omega_b \end{cases}$$

$$x(t) = 1 \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = 2\pi\delta(\omega)$$

$$x(t) = \delta(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = 1$$

$$x(t) = u(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = \pi\delta(\omega) + \frac{1}{j\omega}$$

$$x(t) = u_1(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = j\omega$$

$$x(t) = \cos \omega_0 t \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = \pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)$$

$$x(t) = \sin \omega_0 t \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = -j\pi\delta(\omega - \omega_0) + j\pi\delta(\omega + \omega_0)$$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 k t} \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - k\omega_0)$$

$$a_k = \frac{1}{T}\int_0^T x(t) e^{-jk\omega_0 t} dt$$

Fourier Transform Theorems

$$a x_1(t) + b x_2(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad a X_1(j\omega) + b X_2(j\omega)$$

$$x(t - t_0) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad e^{-j\omega t_0} X(j\omega)$$

$$e^{j\omega_0 t} x(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j(\omega - \omega_0))$$

$$\frac{dx(t)}{dt} \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad j\omega X(j\omega)$$

$$-jt x(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad \frac{dX(j\omega)}{d\omega}$$

$$y(t) = x(t) * h(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad Y(j\omega) = H(j\omega)X(j\omega)$$

$$x(t)p(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad \frac{1}{2\pi} X(j\omega) * P(j\omega)$$

$$x(at) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad \frac{1}{|a|} X\left(\frac{j\omega}{a}\right)$$

$$X(jt) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad 2\pi x(-\omega)$$

Laplace Transform Pairs and Theorems

$$x(t) = e^{s_0 t}u(t) \quad \stackrel{\mathcal{L}}{\longleftrightarrow} \quad X(s) = \frac{1}{s - s_0} \qquad \Re e\{s\} > \Re e\{s_0\}$$

$$x(t) = -e^{s_0 t}u(-t) \quad \stackrel{\mathcal{L}}{\longleftrightarrow} \quad X(s) = \frac{1}{s - s_0} \qquad \Re e\{s\} < \Re e\{s_0\}$$

$$x(t) = u(t) \quad \stackrel{\mathcal{L}}{\longleftrightarrow} \quad X(s) = \frac{1}{s} \qquad \Re e\{s\} > 0$$

$$x(t) = \delta(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(s) = 1$$

$$ax_1(t) + bx_2(t) \quad \stackrel{\mathcal{L}}{\longleftrightarrow} \quad aX_1(s) + bX_2(s)$$

$$x(t - t_0) \quad \stackrel{\mathcal{L}}{\longleftrightarrow} \quad e^{-st_0}X(s)$$

$$\frac{dx(t)}{dt} \quad \stackrel{\mathcal{L}}{\longleftrightarrow} \quad sX(s)$$

$$y(t) = x(t) * h(t) \quad \stackrel{\mathcal{L}}{\longleftrightarrow} \quad Y(s) = H(s)X(s)$$