GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

EE3230
Quiz No. 2
May 14, 1998

Name: ____

1. The exam is closed book. You may use one 8.5" by 11" sheet of notes (both sides). You are permitted to use a calculator. **I have given you a sheet of Fourier transform formulas as the last page. Tear it off and use it!**
2. Do all work in the space provided. If you need more room, use the *back* of the *previous* page and indicate that you have done so.
3. Indicate your answer clearly by circling it or drawing a box around it.
4. Think carefully about the problem before you begin to write. Move on to the next problem if you cannot come up with a plan for the solution.
5. If you want to receive partial credit, you should clearly indicate your reasoning and method of attack on the problem.

Problem	**Points**	**Score**
1	25	
2	25	
3	25	
4	25	
TOTAL	100	

Problem Sp98.Q2.1 (25 %)

Consider the signal $x(t)$, whose Fourier transform is

$$X(j\omega) = \begin{cases} 10 & -2\pi < \omega < 2\pi \\ 0 & \text{otherwise} \end{cases}$$

which is going to be used in the following problems:

(a) $x(t)$ is the input to a linear time-invariant system whose impulse response is

$$h(t) = \frac{\sin[\pi(t-2)]}{\pi(t-2)}$$

Use Fourier transforms to determine an equation for the output $y(t) = x(t) * h(t)$ of the LTI system.

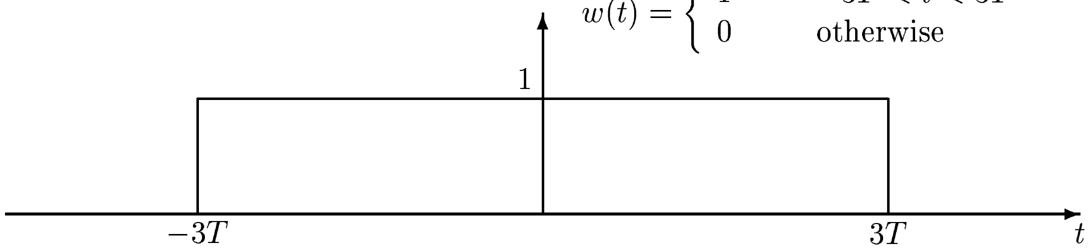
(b) Another signal is formed by modulation: $y(t) = x(t)\cos(10\pi t)$. Plot the Fourier transform $Y(j\omega)$ of this signal.

(c) Still another signal is $v(t) = (x(t))^2$. Plot the Fourier transform $V(j\omega)$ of this signal.

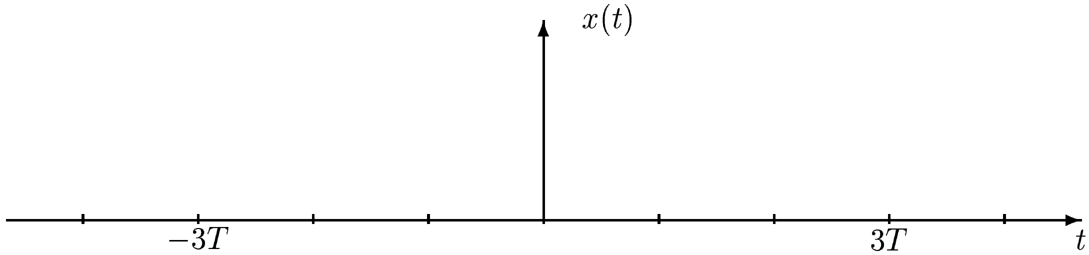
Problem Sp98.Q2.2 (25 %)

Consider the signal $x(t) = w(t)\cos(\omega_c t)$, where $\omega_c = 2\pi/T$ and

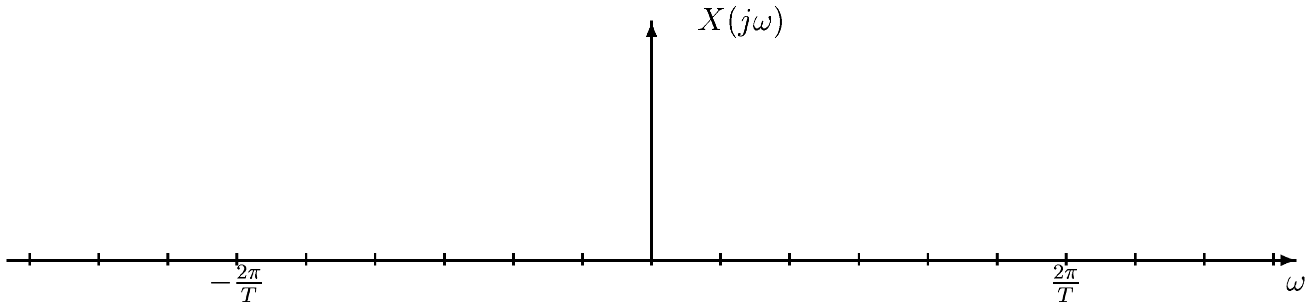
$$w(t) = \begin{cases} 1 & -3T < t < 3T \\ 0 & \text{otherwise} \end{cases}$$



(c) Carefully sketch $x(t)$ below.



(b) Determine an equation for $X(j\omega)$ in terms of $W(j\omega)$, and then determine $W(j\omega)$ and substitute it into your equation. Plot $X(j\omega)$ carefully marking all significant amplitudes and frequencies. *Note: The axis below is marked off conveniently for your plot.*

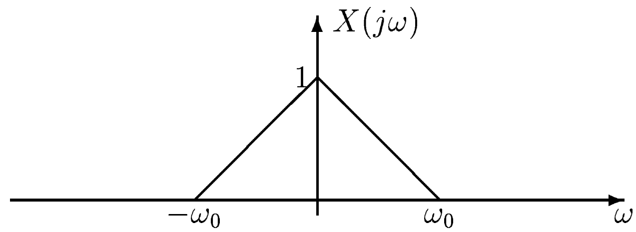


Problem Sp98.Q2.3: (30 %)

A signal $y(t) = x(t)p(t)$ is formed by modulating a periodic squarewave $p(t)$ given by

$$p(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \quad \text{with} \quad a_k = \begin{cases} \dfrac{\sin(\pi k/2)}{\pi k} & k \neq 0 \\ 0 & k = 0 \end{cases}$$

where $\omega_0 = 2\pi/T$. Assume that the input signal $x(t)$ has a Fourier transform as depicted below



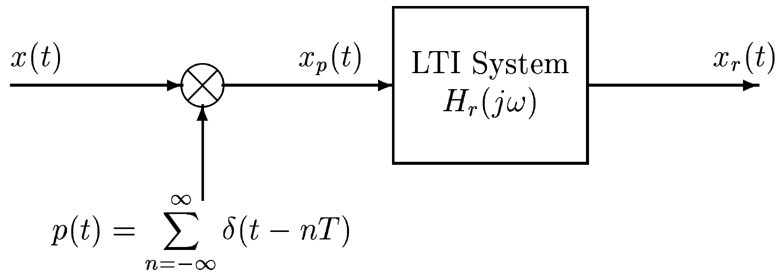
(a) Give an equation for $Y(j\omega)$ in terms of $X(j\omega)$.

$Y(j\omega) =$

(b) For the Fourier transform $X(j\omega)$ given in the graph above, plot $Y(j\omega)$ for frequencies $-6\omega_0 < \omega < 6\omega_0$.

(c) From your plot in (b), you should be able to see how to recover the original signal $x(t)$ by using a combination of ideal filters and modulators. Draw a block diagram of the demodulator system showing all filter frequency responses (cutoff frequencies, gains), and modulator frequencies.

Problem Sp98.Q2.4 (25 %)



The input signal for the above sampling/reconstruction system is

$$x(t) = 10\cos(30\pi t + \pi/4) + 20\cos(100\pi t - \pi/2) \qquad -\infty < t < \infty$$

and the frequency response of the lowpass reconstruction filter is

$$H_r(j\omega) = \begin{cases} T & |\omega| < \pi/T \\ 0 & |\omega| > \pi/T \end{cases}$$

where T is the sampling period.

(a) Determine the Fourier transform of $x(t)$ and plot it below.

(b) How should the sampling frequency $2\pi/T$ be chosen so that $x_r(t) = x(t)$?

(c) Now assume that $2\pi/T = 150\pi$. Plot the Fourier transform $X_p(j\omega)$ for frequencies $-150\pi < \omega < 150\pi$ and give an equation for the reconstructed output $x_r(t)$.

FORMULAS FOR SECOND EXAM

Fourier Transform Pairs

$$x(t) = e^{-at}u(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = \frac{1}{a + j\omega}$$

$$x(t) = \begin{cases} 1 & |t| < T_1 \\ 0 & |t| > T_1 \end{cases} \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = \frac{2\sin(\omega T_1)}{\omega}$$

$$x(t) = \frac{\sin \omega_b t}{\pi t} \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = \begin{cases} 1 & |\omega| < \omega_b \\ 0 & |\omega| > \omega_b \end{cases}$$

$$x(t) = \delta(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = 1$$

$$x(t) = u(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = \pi\delta(\omega) + \frac{1}{j\omega}$$

$$x(t) = u_1(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = j\omega$$

$$x(t) = 1 \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = 2\pi\delta(\omega)$$

$$x(t) = e^{j\omega_0 t} \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = 2\pi\delta(\omega - \omega_0)$$

$$x(t) = \cos \omega_0 t \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = \pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)$$

$$x(t) = \sin \omega_0 t \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = -j\pi\delta(\omega - \omega_0) + j\pi\delta(\omega + \omega_0)$$

$$\left.\begin{aligned} x(t) &= \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 k t} \\ a_k &= \frac{1}{T}\int_0^T x(t) e^{-jk\omega_0 t} dt \end{aligned}\right\} \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - k\omega_0)$$

Fourier Transform Theorems

$$a x_1(t) + b x_2(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad a X_1(j\omega) + b X_2(j\omega)$$

$$x(t - t_0) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad e^{-j\omega t_0} X(j\omega)$$

$$e^{j\omega_0 t} x(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j(\omega - \omega_0))$$

$$\frac{dx(t)}{dt} \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad j\omega X(j\omega)$$

$$-jtx(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad \frac{dX(j\omega)}{d\omega}$$

$$y(t) = x(t) * h(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad Y(j\omega) = H(j\omega)X(j\omega)$$

$$x(t)p(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad \frac{1}{2\pi} X(j\omega) * P(j\omega)$$

$$x(at) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad \frac{1}{|a|} X\left(\frac{j\omega}{a}\right)$$

$$X(jt) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad 2\pi x(-\omega)$$