GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

EE3230
Quiz No. 3
May 28,1998

Name: ____

1. The exam is closed book. You may use one 8.5" by 11" sheet of notes (both sides). You are permitted to use a calculator. **I have given you two sheets of Fourier and Laplace transform formulas as the last two pages. Tear them off and use them!**
2. Do all work in the space provided. If you need more room, use the *back* of the *previous* page and indicate that you have done so.
3. Indicate your answer clearly by circling it or drawing a box around it.
4. Think carefully about the problem before you begin to write. Move on to the next problem if you cannot come up with a plan for the solution.
5. If you want to receive partial credit, you should clearly indicate your reasoning and method of attack on the problem.

Problem	**Points**	**Score**
1	30	
2	25	
3	25	
4	20	
TOTAL	100	

Problem Sp98.Q3.1: (30 %)

A causal linear time-invariant system has system function

$$H(s) = \frac{(s+j20)(s-j20)}{(s+1+j20)(s+1-j20)} = \frac{(s+j20)(s-j20)}{(s+1)^2+400}$$

(a) Determine the differential equation that is satisfied by the input $x(t)$ and the output $y(t)$.

(b) Plot the poles and zeros in the s-plane and shade the region of convergence of $H(s)$.

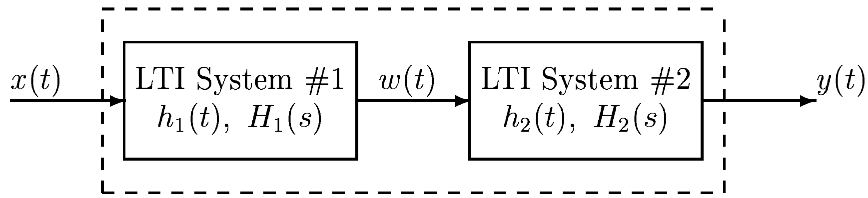
(c) Is the system stable? How do you know?

(d) Determine the frequency response of the system.

(e) If the input is $x(t) = e^{j\omega_0 t}$ for $-\infty < t < \infty$, for what values of ω_0 will it be true that the output will be $y(t) = 0$ for $-\infty < t < \infty$.

(f) Sketch the magnitude of the frequency response as a function of ω. Make your plot as accurate as possible by considering what happens at $\omega = 0$, $\omega = \infty$, and other key frequencies.

Problem Sp98.Q3.2: (25 %)



The first system is a causal system with system function

$$H_1(s) = \frac{s-2}{s+2}$$

(a) Determine the system function, $H_2(s)$ of the second system if it is desired that for *any* input $x(t)$, the output of the overall cascade system is $y(t) = x(t)$.

(b) There are two different possible impulse responses that could correspond to $H_2(s)$. Plot the poles and zeros of $H_2(s)$ in the s-plane and indicate the two possible regions of convergence.

(c) Determine the impulse response $h_2(t)$ of a *causal* system for the second system.

(d) Determine the impulse response $h_2(t)$ of a *stable* system for the second system.

Problem Sp98.Q3.3: (25 %)

Consider a real signal $x(t)$. This signal is convolved with the signal $x(-t)$ to give

$$c(t) = x(t) * x(-t) = \int_{-\infty}^{\infty} x(t-\tau)x(-\tau)d\tau = \int_{-\infty}^{\infty} x(t+\tau)x(\tau)d\tau$$

This new function is called the *autocorrelation function* for $x(t)$.

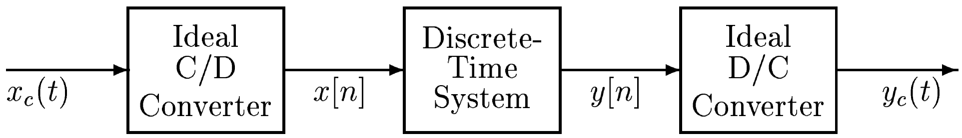
(a) From the definition, show that the Laplace transform of $c(t)$ is in general equal to $C(s) = X(s)X(-s)$. *Hint: What is the Laplace transform of* $x(-t)$*?*

(b) If $x(t) = e^{-2t}u(t)$, use the result stated in part (a) to determine the Laplace transform $C(s)$ of the corresponding $c(t)$. Be sure to give the region of convergence.

(c) Make a partial fraction expansion of $C(s)$ and from it determine $c(t)$ for the given $x(t)$. Verify for your answer that $c(-t) = c(t)$.

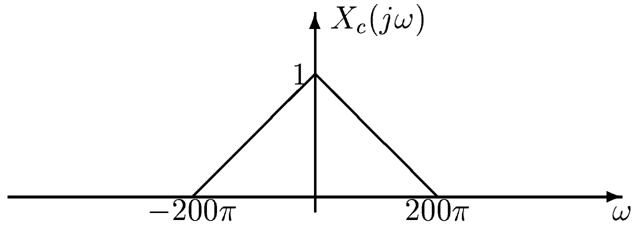
Problem Sp98.Q3.4: (20 %)

All parts of this problem are concerned with the following system.



$$x[n] = x_c(nT) \qquad\qquad y_c(t) = \sum_{n=-\infty}^{\infty} y[n] \frac{\sin \frac{\pi}{T}(t-nT)}{\frac{\pi}{T}(t-nT)}$$

Assume that the input signal $x_c(t)$ has a bandlimited Fourier transform as depicted below.



(a) Suppose that the discrete-time system is defined by $y[n] = x[n]$. What is the **maximum** value of T such that $y_c(t) = x_c(t)$?

(b) Now suppose that the discrete-time system is a highpass filter defined for one period of its frequency response by

$$H(e^{j\hat{\omega}}) = \begin{cases} 0 & |\hat{\omega}| < \pi/2 \\ 10 & \pi/2 < |\hat{\omega}| < \pi \end{cases}$$

Plot $Y_c(j\omega)$ below for the case $2\pi/T = 400\pi$ and the input is the given input.

(c) Determine $2\pi/T$ the sampling rate (or rates) such that $y_c(t) = 0$ when the discrete-time system is $H(e^{j\hat{\omega}})$ as in part (b) and the input is the given input.

FORMULAS FOR THIRD EXAM

Fourier Transform Pairs

$$x(t) = e^{-at}u(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = \frac{1}{a + j\omega}$$

$$x(t) = \begin{cases} 1 & |t| < T_1 \\ 0 & |t| > T_1 \end{cases} \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = \frac{2\sin(\omega T_1)}{\omega}$$

$$x(t) = \frac{\sin \omega_b t}{\pi t} \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = \begin{cases} 1 & |\omega| < \omega_b \\ 0 & |\omega| > \omega_b \end{cases}$$

$$x(t) = 1 \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = 2\pi\delta(\omega)$$

$$x(t) = \delta(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = 1$$

$$x(t) = u(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = \pi\delta(\omega) + \frac{1}{j\omega}$$

$$x(t) = u_1(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = j\omega$$

$$x(t) = \cos \omega_0 t \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = \pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)$$

$$x(t) = \sin \omega_0 t \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = -j\pi\delta(\omega - \omega_0) + j\pi\delta(\omega + \omega_0)$$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 kt} \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - k\omega_0)$$

$$a_k = \frac{1}{T}\int_0^T x(t)e^{-jk\omega_0 t}dt$$

Fourier Transform Theorems

$$ax_1(t) + bx_2(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad aX_1(j\omega) + bX_2(j\omega)$$

$$x(t - t_0) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad e^{-j\omega t_0}X(j\omega)$$

$$e^{j\omega_0 t}x(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j(\omega - \omega_0))$$

$$\frac{dx(t)}{dt} \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad j\omega X(j\omega)$$

$$-jtx(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad \frac{dX(j\omega)}{d\omega}$$

$$y(t) = x(t) * h(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad Y(j\omega) = H(j\omega)X(j\omega)$$

$$x(t)p(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad \frac{1}{2\pi}X(j\omega) * P(j\omega)$$

$$x(at) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad \frac{1}{|a|}X\left(\frac{j\omega}{a}\right)$$

$$X(jt) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad 2\pi x(-\omega)$$

Laplace Transform Pairs and Theorems

$$x(t) = e^{s_0 t}u(t) \quad \stackrel{\mathcal{L}}{\longleftrightarrow} \quad X(s) = \frac{1}{s - s_0} \qquad \Re e\{s\} > \Re e\{s_0\}$$

$$x(t) = -e^{s_0 t}u(-t) \quad \stackrel{\mathcal{L}}{\longleftrightarrow} \quad X(s) = \frac{1}{s - s_0} \qquad \Re e\{s\} < \Re e\{s_0\}$$

$$x(t) = u(t) \quad \stackrel{\mathcal{L}}{\longleftrightarrow} \quad X(s) = \frac{1}{s} \qquad \Re e\{s\} > 0$$

$$x(t) = \delta(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(s) = 1$$

$$ax_1(t) + bx_2(t) \quad \stackrel{\mathcal{L}}{\longleftrightarrow} \quad aX_1(s) + bX_2(s)$$

$$x(t - t_0) \quad \stackrel{\mathcal{L}}{\longleftrightarrow} \quad e^{-st_0}X(s)$$

$$\frac{dx(t)}{dt} \quad \stackrel{\mathcal{L}}{\longleftrightarrow} \quad sX(s)$$

$$y(t) = x(t) * h(t) \quad \stackrel{\mathcal{L}}{\longleftrightarrow} \quad Y(s) = H(s)X(s)$$