GEORGIA INSTITUTE OF TECHNOLOGY

School of Electrical and Computer Engineering

Quiz #2

Date: May 29, 1997

Course: EE 3230

Name: ______________________________
Last, First

- Closed book, closed notes, two $8\frac{1}{2}'' \times 11''$ handwritten sheets are allowed. Eighty minute time limit.
- None of the problems require involved calculations. Reconsider your approach before doing something tedious.
- All work should be performed on the quiz itself. If more space is needed, use the backs of the pages.

Problem	*Score*
1	
2	
3	
4	
Total	

Problem 1:

Consider the following time-domain signal attributes:

(i) $x(0) = 0$

(ii) $Ev\{x(t)\} = 0$

(iii) $x(t)$ is real

(iv) $x(t)$ is periodic

(v) $\int_{-\infty}^{\infty} tx(t)dt = 0$

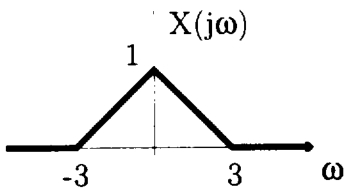
Determine which of these attributes are possessed by the signals having the following Fourier transforms:

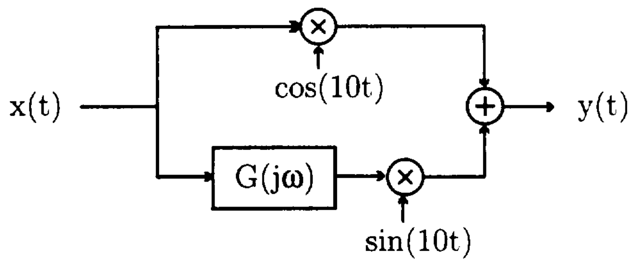
(a) $X(j\omega) = e^{-4\omega^2}$

(b) $X(j\omega) = 2\pi\delta(\omega + 2) - 2\pi\delta(\omega - 2)$

Problem 2:

Consider the following modulation scheme with its associated input:





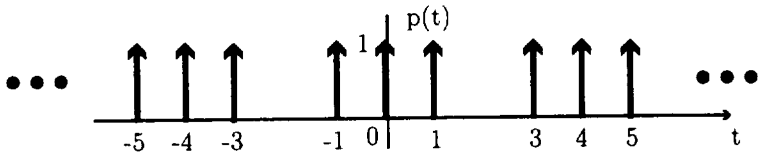
where $G(j\omega) = \begin{cases} -j, & \omega > 0 \\ j, & \omega < 0 \end{cases}.$

(a) Find $Y(j\omega)$, the Fourier transform of the output $y(t)$.

(b) Give a simple system for recovering $x(t)$ from $y(t)$.

Problem 3:

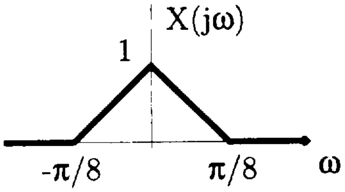
Consider the following periodic signal:



(a) Find and sketch $P(j\omega)$, the Fourier transform of $p(t)$.

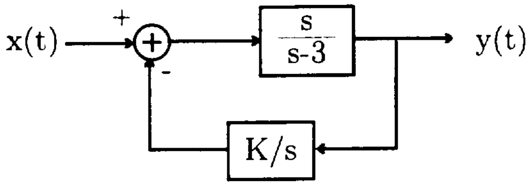
(b) Give the Fourier series representation of $p(t)$.

(c) Sketch $Y(j\omega)$, the Fourier transform of $y(t) = x(t)p(t)$ if $x(t)$ has the Fourier transform:



Problem 4:

Consider the following feedback system:



where K is a constant to be determined. Assume that all systems are causal and that $H(s)$ is the transfer function for the overall system.

(a) Find $h(t)$, the impulse response for the overall system, in terms of K.

(b) For what values of K is the overall system stable?

(c) How many inverse systems are there for $H(s)$? Explain your reasoning.

TABLE 4.1 Fourier Transform Properties

Property	Time domain	Frequency domain
Linearity	$ax_1(t) + bx_2(t)$	$aX_1(j\omega) + bX_2(j\omega)$
Time shift	$x(t - t_0)$	$e^{-j\omega t_0}X(j\omega)$
Modulation	$e^{j\omega_0 t}x(t)$	$X[j(\omega - \omega_0)]$
Axis scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{j\omega}{a}\right)$
Axis reversal	$x(-t)$	$X(-j\omega)$
Duality	$X(jt)$	$2\pi x(-\omega)$
	$\frac{1}{2\pi}X(-jt)$	$x(\omega)$
Differentiation	$\frac{dx(t)}{dt}$	$j\omega X(j\omega)$
	$-jtx(t)$	$\frac{dX(j\omega)}{d\omega}$
Integration	$\int_{-\infty}^{t} x(\tau)\,d\tau$	$\frac{1}{j\omega}X(j\omega) + \pi X(0)\,\delta(\omega)$
	$-\frac{1}{jt}x(t) + \pi x(0)\,\delta(t)$	$\int_{-\infty}^{\omega} X(j\lambda)\,d\lambda$
Convolution	$h(t) * x(t)$	$H(j\omega)X(j\omega)$
Multiplication	$x(t)p(t)$	$\frac{1}{2\pi}X(j\omega) * P(j\omega)$
Conjugation	$x^*(t)$	$X^*(-j\omega)$
Symmetries	$x(t)$ real	$\mathrm{Re}\{X(j\omega)\}$ even
		$\mathrm{Im}\{X(j\omega)\}$ odd
		$\lvert X(j\omega)\rvert$ even
		$\angle X(j\omega)$ odd
	$\mathrm{Ev}\{x(t)\}$	$\mathrm{Re}\{X(j\omega)\}$
	$\mathrm{Od}\{x(t)\}$	$j\,\mathrm{Im}\{X(j\omega)\}$
Parseval's relation finite energy	$\int_{-\infty}^{\infty} \lvert x(t)\rvert^2\,dt$	$\frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(j\omega)\rvert^2\,d\omega$
periodic signal	$\frac{1}{T}\int_{T} \lvert x(t)\rvert^2\,dt$	$\sum_{k=-\infty}^{\infty} \lvert a_k\rvert^2$

TABLE 4.2 Common Fourier Transforms

Signal	Time domain	Frequency domain
Rectangular pulse	$x(t) = \begin{cases} 1, & \lvert t\rvert < T_p/2 \\ 0, & \lvert t\rvert > T_p/2 \end{cases}$	$T_p \operatorname{sinc} \frac{\omega T_p}{2}$
Sinc pulse	$\frac{\omega_b}{\pi} \operatorname{sinc} \omega_b t$	$X(j\omega) = \begin{cases} 1, & \lvert\omega\rvert < \omega_b \\ 0, & \lvert\omega\rvert > \omega_b \end{cases}$
Impulse	$\delta(t)$	1, for all ω
Unit step	$u(t)$	$\frac{1}{j\omega} + \pi\,\delta(\omega)$
Delayed impulse	$\delta(t - t_0)$	$e^{-j\omega t_0}$
Causal exponential	$e^{-at}u(t),\ a > 0$	$\frac{1}{a + j\omega}$
Weighted exponential	$te^{-at}u(t),\ a > 0$	$\frac{1}{(a + j\omega)^2}$
dc signal	a_0, for all t	$2\pi a_0\,\delta(\omega)$
Complex sinusoid	$e^{j\omega_0 t}$	$2\pi\,\delta(\omega - \omega_0)$
Sine wave	$\sin \omega_0 t$	$(\pi/j)[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$
Cosine wave	$\cos \omega_0 t$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$
Square wave	Figure 4.1 (odd symmetry)	$\sum_{k=-\infty,\ k\ \mathrm{odd}}^{\infty} \frac{4}{jk}\,\delta(\omega - k\omega_0)$
Impulse train	$\sum_{k=-\infty}^{\infty} \delta(t - kT)$	$\omega_0 \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_0)$
Rectangular pulse train	Figure 4.2 (even symmetry)	$\sum_{k=-\infty}^{\infty} \frac{2}{k} \sin(k\omega_0 T_p/2)\,\delta(\omega - k\omega_0)$

TABLE 5.1 Laplace Transform Properties

Property	*Time domain*	*Transform*	*ROC*
Linearity	$ax_1(t) + bx_2(t)$	$aX_1(s) + bX_2(s)$	$R' \supset R_1 \cap R_2$
Time shift	$x(t - t_0)$	$e^{-st_0}X(s)$	$R' = R$
Modulation	$e^{s_0 t}x(t)$	$X(s - s_0)$	$R' = R + \mathrm{Re}\{s_0\}$
Axis scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{s}{a}\right)$	$R' = aR$
Axis reversal	$x(-t)$	$X(-s)$	$R' = -R$
Differentiation	$\frac{dx(t)}{dt}$	$sX(s)$	$R' \supset R$
	$-tx(t)$	$\frac{dX(s)}{ds}$	$R' = R$
Integration	$\int_{-\infty}^{t} x(\tau)\,d\tau$	$\frac{1}{s}X(s)$	$R' \supset R \cap \mathrm{Re}\{s\} > 0$
Convolution	$x_1(t) * x_2(t)$	$X_1(s)X_2(s)$	$R' \supset R_1 \cap R_2$

TABLE 5.2 Common Laplace Transforms

Signal	*Time domain*	*Transform*	*ROC*
Impulse	$\delta(t)$	1	All s
Unit step	$u(t)$	$\frac{1}{s}$	$\mathrm{Re}\{s\} > 0$
	$-u(-t)$	$\frac{1}{s}$	$\mathrm{Re}\{s\} < 0$
Exponential	$e^{-at}u(t)$	$\frac{1}{s+a}$	$\mathrm{Re}\{s\} > -a$
	$-e^{-at}u(-t)$	$\frac{1}{s+a}$	$\mathrm{Re}\{s\} < -a$
Weighted exponential	$te^{-at}u(t)$	$\frac{1}{(s+a)^2}$	$\mathrm{Re}\{s\} > -a$
Causal sine	$[\sin \omega_0 t]u(t)$	$\frac{\omega_0}{s^2 + \omega_0^2}$	$\mathrm{Re}\{s\} > 0$
Causal cosine	$[\cos \omega_0 t]u(t)$	$\frac{s}{s^2 + \omega_0^2}$	$\mathrm{Re}\{s\} > 0$
Damped sine	$e^{-at}[\sin \omega_0 t]u(t)$	$\frac{\omega_0}{(s+a)^2 + \omega_0^2}$	$\mathrm{Re}\{s\} > -a$
Damped cosine	$e^{-at}[\cos \omega_0 t]u(t)$	$\frac{s+a}{(s+a)^2 + \omega_0^2}$	$\mathrm{Re}\{s\} > -a$