GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

Final Exam

Date: June 9, 1999 Course: EE 2201B

Name: ______________________________
Last, First

- Closed book and closed notes. Three $8\frac{1}{2}'' \times 11''$ handwritten sheets and the tables from the previous quizzes are allowed. Two hour and fifty minute time limit.
- None of the problems require involved calculations. Reconsider your approach before doing something tedious.
- All work should be performed on the exam itself. If more space is needed, use the backs of the pages.
- This quiz will be conducted under the rules and guidelines of the Georgia Tech Honor Code and no cheating will be tolerated.

Problem	*Score*
1	
2	
3	
4	
5	
6	
Total	

Problem 1: (15 points)

Find and sketch the convolution of $p_{\frac{1}{2}}(t)$ with each of the following signals, where

$$p_{\frac{1}{2}}(t) = \begin{cases} 1, & |t| < \frac{1}{2} \\ 0, & |t| > \frac{1}{2} \end{cases}$$

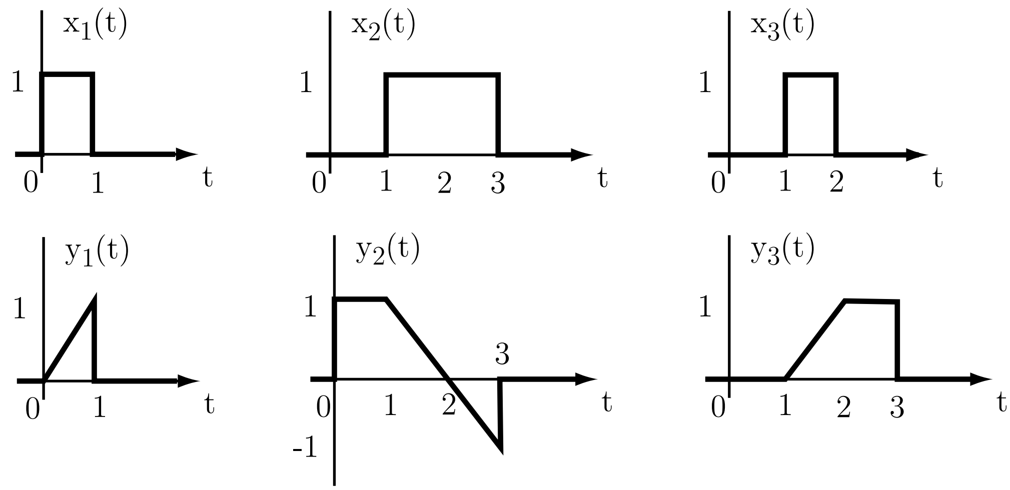
(a) $\delta(t-7) - \delta(t+1)$

(b) $\sin(10\pi t)$

(c) $\cos^2(10\pi t)$

Problem 2: (20 points)

A linear continuous-time system is observed to have the following input-output pairs of signals (each input $x_i(t)$ has corresponding output $y_i(t)$, $i = 1, 2, 3$).

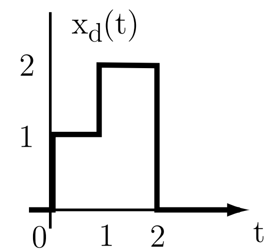


(a) Is this system causal? Why?

(b) Is this system time-invariant? Why?

(c) Is this system memoryless? Why?

(d) Find the output for this system when the input is

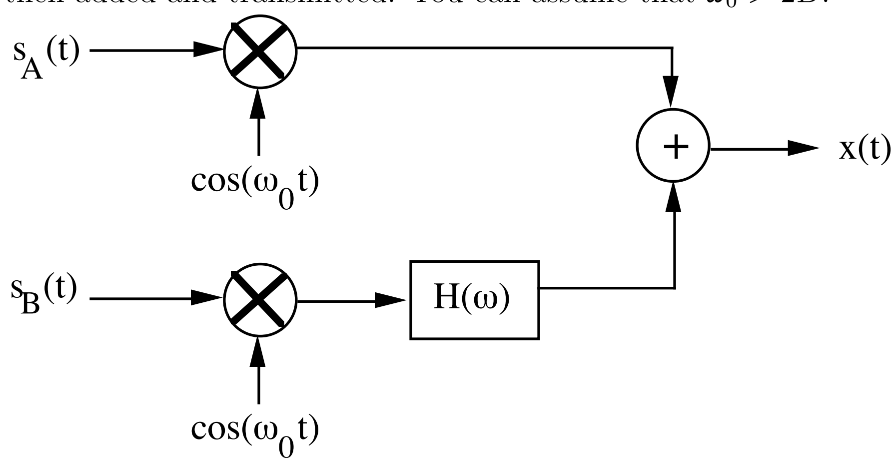


Problem 3: (25 points)

The South Carolina Amplitude Modulation company proposes the use of a new A.M. stereo transmission scheme. Signals $s_A(t)$ and $s_B(t)$ are each bandlimited to a bandwidth B, i.e., $S_A(j\omega) = S_B(j\omega) = 0$ for $|\omega| > B$. In the new scheme, the signal $s_A(t)$ is amplitude modulated as shown below. Signal $s_B(t)$ is modulated with the same carrier and then passed through a filter with a transfer function

$$H(j\omega) = \begin{cases} +1 & , \quad |\omega| < \omega_0 \\ -1 & , \quad |\omega| > \omega_0 \end{cases}$$

the result is then added and transmitted. You can assume that $\omega_0 > 2B$.



(a) What is the bandwidth of $x(t)$? That is, what is the smallest value of W such that $X(j\omega) = 0$ for all $\omega > W$?

(b) If we sample $x(t)$ to form $x[n] = x(nT)$, $n = 0, \pm 1, \pm 2, \ldots$, for what range of values of T are we able to recover $x(t)$ through simple lowpass filtering?

(c) With $s_B(t) \equiv 0$ and $s_A(t) = \frac{\sin \pi t}{\pi t}$, sketch $X(j\omega)$, the transform of $x(t)$.

(d) With $s_B(t) \equiv 0$, find a demodulator for recovering $s_A(t)$.

(e) Find a stereo demodulator to recover *both* $s_A(t)$ and $s_B(t)$.

Problem 4: (20 points)

Given that $e^{-t^2/2}$ has the Fourier transform $\sqrt{2\pi}e^{-\omega^2/2}$, use the properties of the Fourier transform to fill in the blanks in the following table:

$\boldsymbol{x(t)}$	$\boldsymbol{X(j\omega)}$
$e^{-\frac{1}{2}(t^2-4t+4)}$	
$e^{-\frac{t^2}{2}+j2t}$	
	$2\pi e^{-2\omega^2}$
	$\sqrt{2\pi}\ \omega e^{-\omega^2/2}$

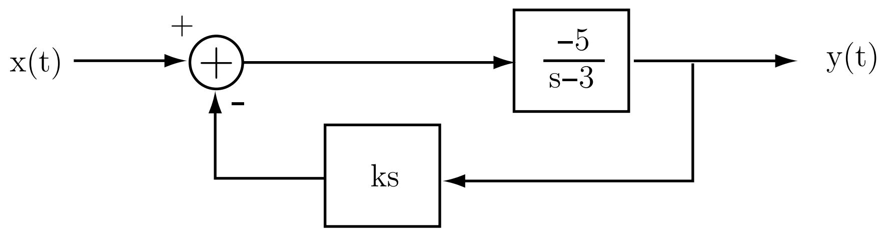
Problem 5:

We know that $x[n] = a^n u[n]$ has the discrete-time Fourier transform $X(e^{j\Omega}) = \frac{1}{1-ae^{-j\Omega}}$ for $|a| < 1$. Using the properties of the discrete-time Fourier transform, match the following signals with their transforms.

Signals	**Transforms**
$a^n u[n+n_0]$	$\frac{1}{1-ae^{-j\Omega}e^{j\Omega_0}}$
$\mathcal{O}d\{a^n u[n]\}$	$\frac{a^{n_0}e^{-j\Omega n_0}}{1-ae^{-j\Omega}}$
$r[n] = u[n] * a^n u[n]$	$a^{-n_0}e^{j\Omega n_0}\frac{1}{1-ae^{-j\Omega}}$
$e^{j\Omega_o n}a^n u[n]$	$\frac{aj}{(1-ae^{-j\Omega})^2}$
$na^n u[n]$	$\frac{1}{1-(a+1)e^{-j\Omega}+ae^{-2j\Omega}} + \frac{\pi}{1-a}\sum_{k=-\infty}^{\infty}\delta(\Omega - 2\pi k)$
	$\frac{-aj\sin\Omega}{1+a^2-2a\cos\Omega}$
	$\frac{ae^{-j\Omega}}{(1-ae^{-j\Omega})^2}$
	None of the above.

Problem 6: (20 points)

Consider the following causal feedback system:



where k is a real-valued constant.

(a) If the input is $x(t) = e^{-3t/2}u\left(t - \frac{3}{2}\right)$, find $X(s)$, the Laplace transform of $x(t)$.

(b) For $k = 0$ find $y(t)$ when the input is $x(t) = 3e^{-4t}u(t)$.

(c) Find $H(s)$, the transfer function of the entire feedback system, as a function of k. Give a pole/zero plot for $H(s)$ and indicate the region of convergence (when plotting, assume that $k = 1$).

(d) For what values of k is this feedback system stable?