GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

Final Exam

Date: August 31, 1994 | Course: EE 3230

Name: ______________________________
Last, First

- Closed book, closed notes, three $8\frac{1}{2}'' \times 11''$ handwritten sheets are allowed. Two hour and fifty minute time limit.
- None of the problems require involved calculations. Reconsider your approach before doing something tedious.
- All work should be performed on the quiz itself. If more space is needed, use the backs of the pages.

Problem	*Score*
1	
2	
3	
4	
5	
6	
7	
Total	

Problem 1:

A continuous-time *sliding window averager* is a system whose input $x(t)$ and output $y(t)$ are continous-time signals related by

$$y(t) = \frac{1}{T_1 + T_2} \int_{t-T_1}^{t+T_2} x(\tau) d\tau$$

with $T_1 \geq 0$ and $T_2 \geq 0$.

(a) Show that this relation can be expressed as a convolution $y(t) = h(t) * x(t)$ and determine the impulse response $h(t)$.

(b) Suppose that the input is $x(t) = u(t)$. Compute the output $y(t)$.

(c) Under what conditions on T_1 and T_2 is the system causal?

Problem 2:

A phase modulator is described by the input/output relation

$$y(t) = A\cos[\omega_c + kx(t)]$$

where $x(t)$ is the input and $y(t)$ is the output.

Is the system:

(a) linear?

(b) time-invariant?

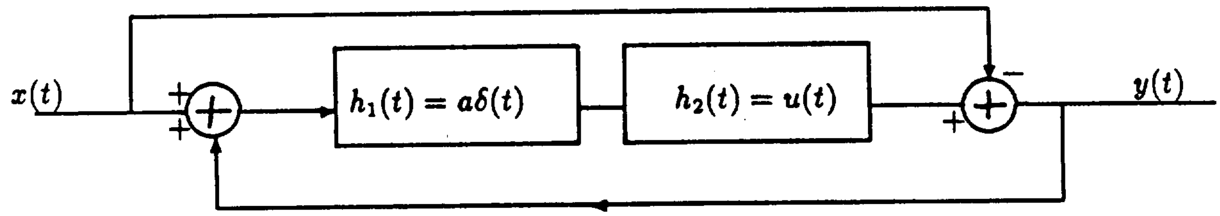
(c) causal?

(d) stable?

(e) memoryless?

Problem 3:

Consider the following system:



(a) Find $H(s)$, the transfer function for this system.

(b) What is $h(t)$, the impulse response for this system?

(c) If this system is causal, is it stable?

(d) If $x(t) = e^{2t}u(t)$, find $y(t)$.

Problem 4:

A causal linear time-invariant discrete-time system is described by the transfer function

$$H(z) = \frac{z}{z^2 - \frac{3}{2}z - 1}$$

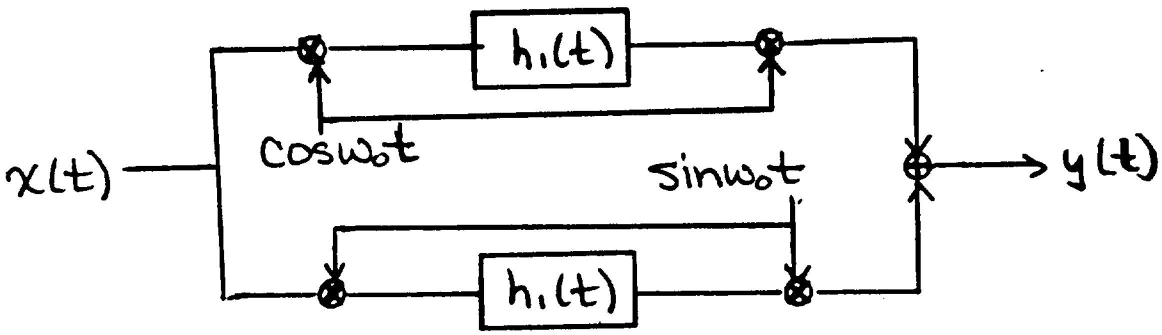
(a) Find $h[n]$, the impulse response for this system.

(b) You should have found this to be an unstable system. Find a stable impulse response (non-causal) that has the same transfer function $H(z)$.

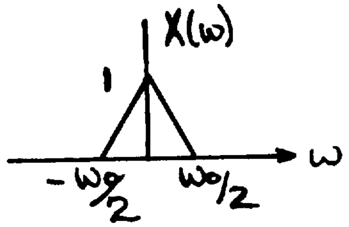
(c) Find a linear constant-coefficient difference equation that describes a system with this transfer function $H(z)$.

Problem 5:

The system shown below contains four AM modulators



(a) If $h_1(t) = \frac{2\sin\omega_o t}{\pi t}$ and $x(t)$ has the spectrum



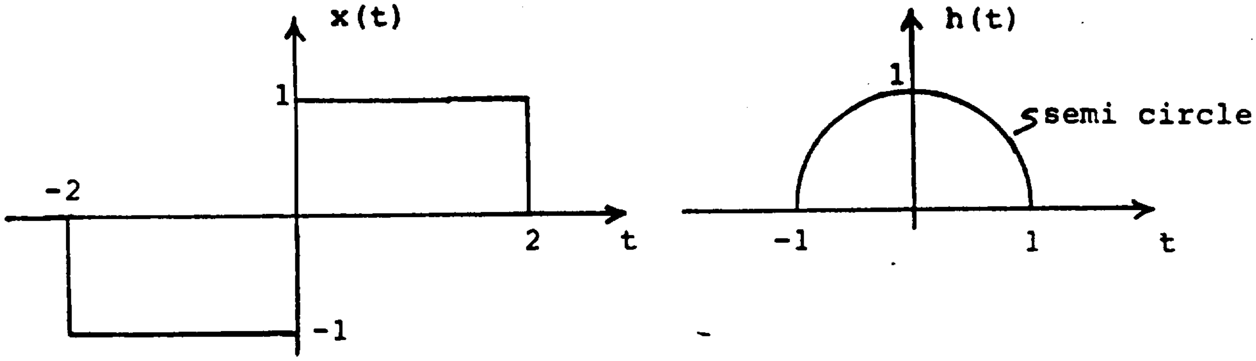
sketch $Y(j\omega)$.

(b) Next, $y(t)$ is to be sampled. What is the upper bound on the sampling period, T, such that the signal can be reconstructed from its samples?

(c) Consider again the amplitude modulation system given at the beginning of this problem, but with an arbitrary $h_1(t)$. Find the overall transfer function $H(j\omega)$ for this system as a function of $H_1(j\omega)$,

Problem 6:

In the following figures, $x(t)$ is the input and $h(t)$ is the impulse response of a linear time-invariant system whose output is $y(t)$.



(a) What is the *complete* set of values of t for which $y(t) = 0$?

(b) For what value of t does $y(t)$ have its largest positive value? What is $y(t)$ at that time?

(c) Carefully sketch below the derivative, $y^{(1)}(t) = \frac{dy(t)}{dt}$, of the output when the input and impulse response are given as in the above figure. Be careful, there is an easy way to do this.

Problem 7:

Consider an LTI system with input $x(t)$, output $y(t)$, and impulse response $h(t)$. We are given the following information:

$$X(s) = \frac{s+2}{s-2}$$

$$x(t) = 0\ , \quad t > 0$$

$$y(t) = -\frac{2}{3}e^{2t}u(-t) + \frac{1}{3}e^{-t}u(t)$$

(a) Determine $H(s)$ and its region of convergence.

(b) Determine $h(t)$.