GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ECE
EE 3230
SOLUTIONS to PROBLEM SET # 2

2.1 (a)

For $t<0$: $y(t)=0$

For $t>0$: $y(t)=\int_0^t e^{-a\tau}e^{-b(t-\tau)}\,d\tau = e^{-bt}\int_0^t e^{(b-a)\tau}\,d\tau$

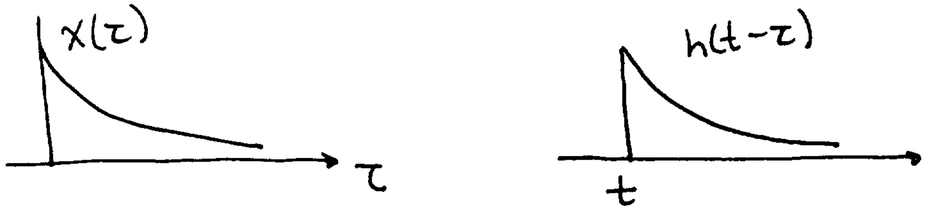
$$= \frac{e^{-bt}}{b-a}\left[e^{(b-a)t}-1\right]$$

$$y(t)=\frac{1}{b-a}\left[e^{-at}-e^{-bt}\right]$$

$$\Rightarrow y(t)=\frac{1}{b-a}\left[e^{-at}-e^{-bt}\right]u(t)$$

(assuming $b>a$)

(b)



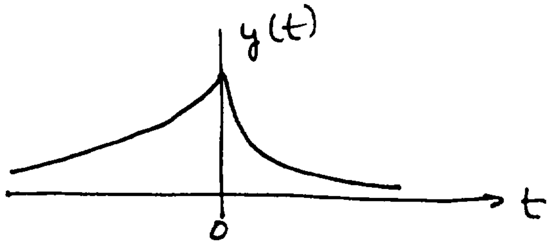
For $t<0$: $y(t)=\int_0^\infty e^{-a\tau}e^{+a(t-\tau)}\,d\tau = e^{+at}\int_0^\infty e^{-2a\tau}\,d\tau$

$$= \frac{e^{+at}}{-2a}\left[-1\right] = \frac{1}{2a}e^{+at}$$

For $t>0$: $y(t)=\int_t^\infty e^{-a\tau}e^{+a(t-\tau)}\,d\tau = e^{+at}\int_t^\infty e^{-2a\tau}\,d\tau$

$$= \frac{e^{-at}}{-2a}\left[-e^{-2at}\right] = \frac{1}{2a}e^{-3at}$$

$$\Rightarrow y(t) = \frac{1}{2a}\left[e^{at}u(-t) + e^{-3at}u(t)\right]$$



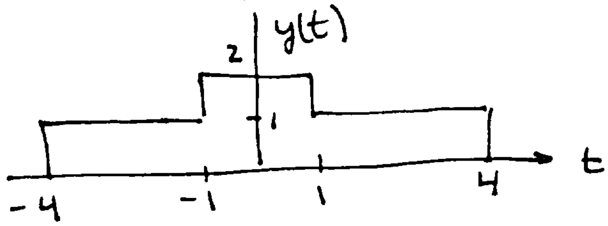
(f) $x(t) = u(t+2) - u(t-3)$, $h(t) = \delta(t+2) + \delta(t-1)$.

$\Rightarrow$ Just convolving with impulses! Use superposition:

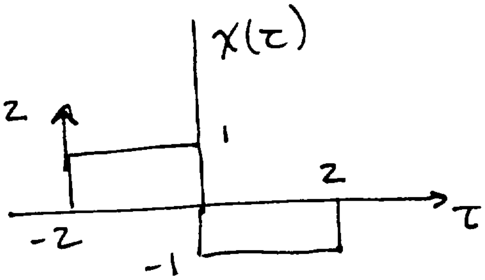
$$x(t) * \delta(t+2) = x(t+2) = u(t+4) - u(t-1)$$

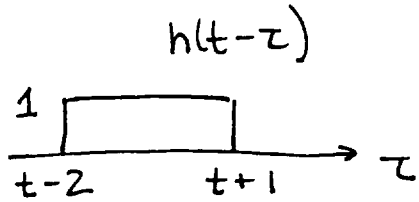
$$x(t) * \delta(t-1) = x(t-1) = u(t+1) - u(t-4)$$

$$\Rightarrow y(t) = u(t+4) + u(t+1) - u(t-1) - u(t-4)$$



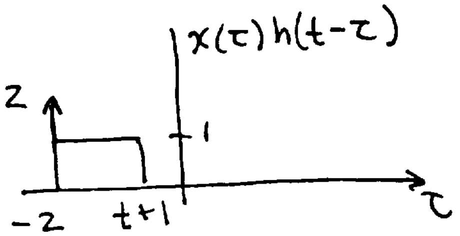
(h)





For $(t+1) < -2 \Rightarrow t < -3$: $y(t) = 0$

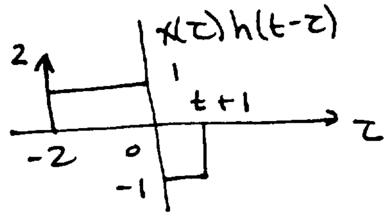
For $-2 < (t+1) < 0 \Rightarrow -3 < t < -1$:



$$y(t) = 2 + [(t+1) - (-2)]$$

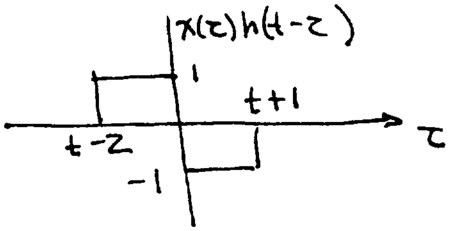
$$= 2 + t + 1 + 2 = 5 + t$$

For $(t+1)>0$ & $(t-2)<-2 \Rightarrow -1<t<0$:



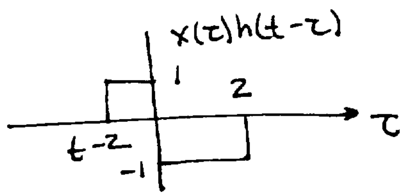
$$y(t) = 2+2-[t+1]$$
$$= 3-t$$

For $(t-2)>-2$ & $(t+1)<2 \Rightarrow 0<t<1$:



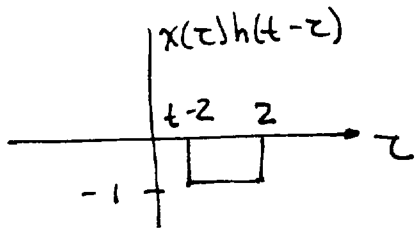
$$y(t) = -(t-2)-[t+1]$$
$$= -t+2-t-1 = 1-2t$$

For $(t+1)>2$ & $(t-2)<0 \Rightarrow 1<t<2$:



$$y(t) = -(t-2)-2$$
$$= -t$$

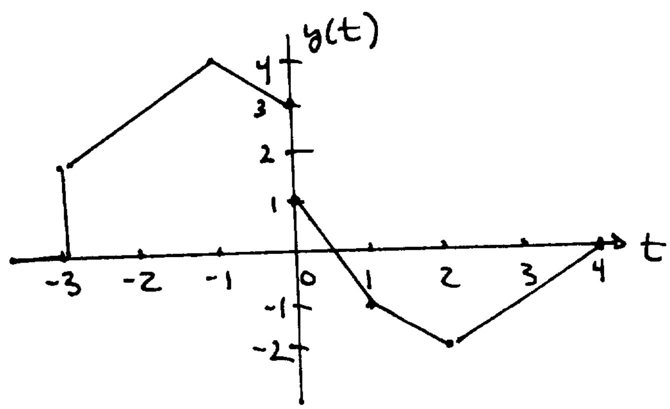
For $(t-2)>0$ & $(t-2)<2 \Rightarrow 2<t<4$:



$$y(t) = -[2-(t-2)]$$
$$= t-4$$

For $(t-2)>2 \Rightarrow t>4$: $y(t)=0$

$$\Rightarrow y(t) = \begin{cases} 0, & t<-3 \\ 5+t, & -3<t<-1 \\ 3-t, & -1<t<0 \\ 1-2t, & 0<t<1 \\ -t, & 1<t<2 \\ t-4, & 2<t<4 \\ 0, & t>4 \end{cases}$$



2.2 (a) Show that $\phi_{xw}(t) = \int_{-\infty}^{\infty} x(t+\tau) w^*(\tau) d\tau = x(t) * w^*(-t)$

Know: $x(t) * h(t) = \int_{-\infty}^{\infty} x(t-\tau) h(\tau) d\tau$

Let $h(t) = w^*(-t) \Rightarrow x(t) * w^*(-t) = \int_{-\infty}^{\infty} x(t-\tau) w^*(-\tau) d\tau$

Let $T = -\tau \Rightarrow x(t) * w^*(-t) = -\int_{\infty}^{-\infty} x(t+T) w^*(T) dT$

$$= \int_{-\infty}^{\infty} x(t+T) w^*(T) dT \quad \checkmark$$

Now $\phi_{xx}(t) = x(t) * x^*(-t)$ is true by letting $w(t) = x(t)$.

(b) $\phi_{wx}(t) = \int_{-\infty}^{\infty} w(t+T) x^*(T) dT = w(t) * x^*(-t)$

$$\phi_{wx}^*(-t) = w^*(-t) * x(t) = x(t) * w^*(-t) = \phi_{xw}(t).$$

Similarly, $\phi_{xx}(t) = x(t) * x^*(-t)$

$\phi_{xx}(-t) = x(-t) * x^*(t) \Rightarrow \phi_{xx}(t) = \phi_{xx}^*(-t)$.

(d) $\phi_{yw}(t) = x(t-t_0) * w^*(-t) = \int_{-\infty}^{\infty} x(t-t_0+\tau) w^*(\tau) d\tau$

$$= \phi_{xw}(t-t_0)$$

$$\phi_{yx}(t) = \int_{-\infty}^{\infty} x(t-t_0+\tau) x^*(\tau) d\tau = \phi_{xx}(t-t_0)$$

$$\phi_{yy}(t) = \int_{-\infty}^{\infty} x(t-t_0+\tau) x^*(\tau-t_0) d\tau = \int_{-\infty}^{\infty} x(t+T) x^*(T) dT$$

(for $T = \tau - t_0$)

$\Rightarrow \phi_{yy}(t) = \phi_{xx}(t)$.

2.3 (a) M, C, TI

(b) C, L, TI, I ; $x(t)=\int_{-\infty}^{t} y(\tau)\,d\tau$

(c) L, S

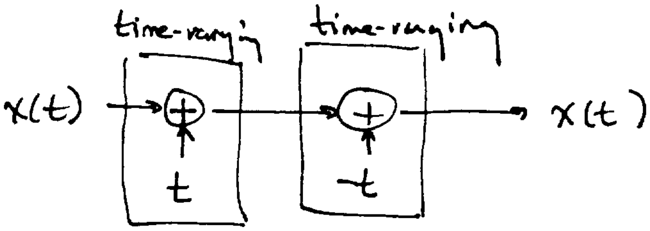
(d) L, I ; $x(t)=\frac{d}{d(2t)}\,y(t)$

(e) M, C, L, S

(f) $y(t)=\int_{-\infty}^{\infty} x(\tau)\,\delta(\tau-t-1)\,d\tau = x(t+1)$

L, TI, S, I, $x(t)=y(t-1)$

2.4 (b) False



(c) True

(f) True

(h) False