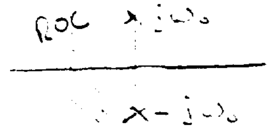
Solutions PS # 5 EE 3230 5/5/94

5.1) a) $X(s) = \int_{-\infty}^{\infty} \sin(\omega_0 t) u(-t) e^{-st}\, dt = \int_{-\infty}^{0} \sin(\omega_0 t) e^{-st}\, dt$

$$= \frac{1}{2j}\int_{-\infty}^{0} e^{-(s-j\omega_0)t}\, dt - \frac{1}{2j}\int_{-\infty}^{0} e^{-(s+j\omega_0)t}\, dt$$

$$X(s) = -\frac{\frac{1}{2j}}{s-j\omega_0} + \frac{\frac{1}{2j}}{s+j\omega_0} \qquad \mathrm{Re}\{s\} < 0$$

$$= \frac{\frac{1}{2j}[-s-j\omega_0+s-j\omega_0]}{(s-j\omega_0)(s+j\omega_0)} = \frac{-\omega_0}{s^2+\omega_0^2}$$



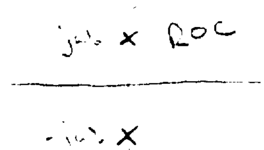
FT exists but cannot be found by $s=j\omega$

b) $X(s) = \int_{-\infty}^{\infty} \sin(\omega_0 t + \pi/4) u(t) e^{-st}\, dt = \int_{0}^{\infty} \sin(\omega_0 t + \pi/4) e^{-st}\, dt$

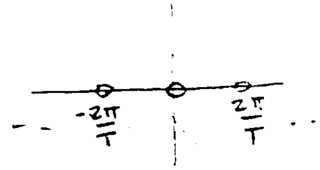
$$= \frac{e^{j\pi/4}}{2j}\int_{0}^{\infty} e^{-(s-j\omega_0)t}\, dt - \frac{e^{-j\pi/4}}{2j}\int_{0}^{\infty} e^{-(s+j\omega_0)t}\, dt$$

$$X(s) = \frac{\frac{e^{j\pi/4}}{2j}}{s-j\omega_0} - \frac{\frac{e^{-j\pi/4}}{2j}}{s+j\omega_0} = \frac{\frac{1}{2j}\left[e^{j\pi/4}(s+j\omega_0) - e^{-j\pi/4}(s-j\omega_0)\right]}{s^2+\omega_0^2}$$

FT exists but cannot be found by $s=j\omega$



c) $X(s) = \int_{-\infty}^{\infty} [\delta(t+T) - \delta(t-T)] e^{-st}\, dt = e^{sT} - e^{-sT}$



ROC is entire s-plane

FT exists & can be found by $s = j\omega$

f) $X(s) = \int_{-\infty}^{\infty} e^{-a|t|} \cos(\omega_0 t) e^{-st}\, dt \qquad a > 0$

$$= \int_{0}^{\infty} e^{-at} \cos(\omega_0 t) e^{-st}\, dt + \int_{-\infty}^{0} e^{at} \cos(\omega_0 t) e^{-st}\, dt$$

$$= \frac{1}{2}\int_{0}^{\infty} e^{-(a+s-j\omega_0)t}\, dt + \frac{1}{2}\int_{0}^{\infty} e^{-(a+s-j\omega_0)t}\, dt$$

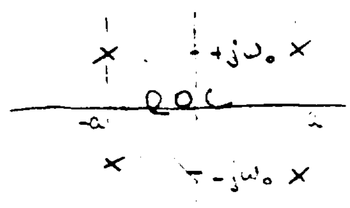
$$+ \frac{1}{2}\int_{-\infty}^{0} e^{-(s-a-j\omega_0 t)}\, dt + \frac{1}{2}\int_{-\infty}^{0} e^{-(s-a+j\omega_0)t}\, dt$$

$$= \frac{\frac{1}{2}}{a+s-j\omega_0} + \frac{\frac{1}{2}}{a+s+j\omega_0} \qquad s > -a$$

$$\frac{\frac{1}{2}}{a-s+j\omega_0} + \frac{\frac{1}{2}}{a-s-j\omega_0} \qquad s < a$$

$$= \frac{\frac{1}{2}(a+s+j\omega_0) + \frac{1}{2}(a+s-j\omega_0) + \frac{1}{2}(a-s+j\omega_0) + \frac{1}{2}(a-s-j\omega_0)}{[(a+s)^2+\omega_0^2][(a-s)^2+\omega_0^2]}$$

$$= \frac{2a}{[(a+s)^2+\omega_0^2][(a-s)^2+\omega_0^2]}$$



FT exists [illegible]

5.2) [illegible]

a) $$\frac{3s}{(s-2)(s+1)} = \frac{1}{s+1} + \frac{2}{s-2} \quad \text{by PFE}$$

for $\mathrm{Re}\{s\} > 2$ $\quad x(t) = \left[e^{-t} + 2e^{2t}\right] u(t)$

b) for $-1 < \mathrm{Re}\{s\} < 2$

$$x(t) = e^{-t} u(t) - 2e^{2t} u(-t)$$

c) for $\mathrm{Re}\{s\} < -1$

$$x(t) = \left[-e^{-t} - 2e^{2t}\right] u(-t)$$

5.3) [illegible]

a) $X(s) = \frac{1}{(s+a)^2}$

since $t\,x(t) \leftrightarrow -\frac{dX(s)}{ds}$ $\Rightarrow$ $x(t) = t e^{-at} u(t)$

since $\mathrm{Re}\{s\} > -a$

if $\mathrm{Re}\{s\} < -a \Rightarrow x(t) = -t e^{-at} u(-t)$

b) for $\mathrm{Re}\{s\} > -a$ $\quad x(t) = \frac{1}{(N-1)!} t^{N-1} e^{-at} u(t)$

for $\mathrm{Re}\{s\} < -a$ $\quad x(t) = -\frac{1}{(N-1)!} t^{N-1} e^{-at} u(-t)$

5.4)

a) $e^{-at}u(t-t_0) = e^{-at_0}e^{-a(t-t_0)}u(t-t_0)$

$$X(s) = e^{-at_0}\frac{e^{-t_0 s}}{s+a} = \frac{e^{-(s+a)t_0}}{s+a} \qquad Re\{s\} > -a$$

c) $e^{j\omega_0 t}u(t) \Rightarrow X(s) = \frac{1}{s-j\omega_0} \qquad Re\{s\} > 0$

h) $X(s) = s\left[\int_{-\infty}^{0} e^{at} e^{-st}\,dt + \int_{0}^{\infty} e^{-at} e^{-st}\,dt\right]$

$$= -\frac{s}{s-a} + \frac{s}{s+a} = \frac{-2sa}{s^2-a^2} \qquad -a < Re\{s\} < a$$

g) $X(s) = \frac{2}{(s+a)^3} \qquad Re\{s\} > -a$