EE-2025 Fall-99

Lecture 11

Digital Filtering of Analog Signals

4-Oct-99

Info: Web-CT, Lab, HW

- Quiz #2 on 25-Oct (Monday)
  - Coverage: HW #4, #5, #6, #7 and #8
- MATLAB Help on Wednesdays
  - 6 PM, VL-456
- Prob Set #6 due FRIDAY
  - On-Line HW #5 also
- Lab #6: Filtering of IMAGES (Blurring)

READING ASSIGNMENTS

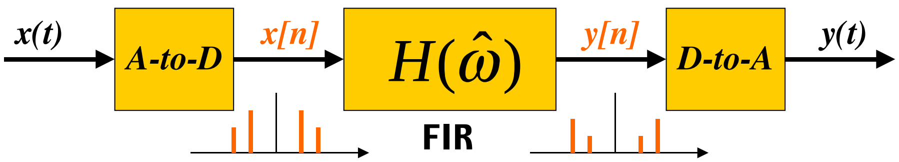
- This Lecture:
  - Chapter 6, pp. 188-194
- Other Reading:
  - Recitation: Ch. 6, pp. 176-188
    - FREQUENCY RESPONSE EXAMPLES
  - Next Lecture: Chapter 7, start

COURSE OBJECTIVE

- Students will be able to:
- Understand mathematical descriptions of signal processing algorithms and express those algorithms as computer implementations (MATLAB)

LECTURE OBJECTIVES

- Two Domains: Time & Frequency
- Track the spectrum of x[n] thru an FIR Filter.
- UNIFICATION: How does Frequency Response affect x(t) to produce y(t) ?



TIME & FREQ DOMAINS

- LTI: Linear & Time-Invariant
- COMPLETELY CHARACTERIZED by:
  - IMPULSE RESPONSE h[n] (time domain)
  - FREQUENCY RESPONSE

$$H(\hat{\omega}) = |H(\hat{\omega})| e^{j\varphi(\hat{\omega})}$$

- Two DOMAINS: time & frequency
  - Go back and forth

TIME & FREQUENCY

$$y[n] = \sum_{k=0}^{M} b_k x[n-k] = \sum_{k=0}^{M} h[k] x[n-k]$$

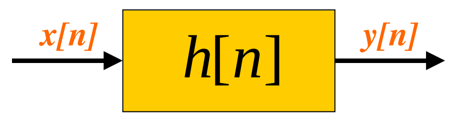
FIR DIFFERENCE EQUATION is the TIME-DOMAIN

$$H(\hat{\omega}) = \sum_{k=0}^{M} h[k] e^{-j\hat{\omega}k}$$

$$H(\hat{\omega}) = h[0]e^{-j0} + h[1]e^{-j\hat{\omega}} + h[2]e^{-j\hat{\omega}2} + h[3]e^{-j\hat{\omega}3} + \cdots$$

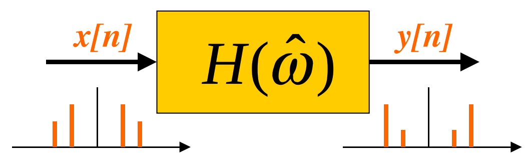
Ex: DELAY by 2 SYSTEM

Find $h[n]$ and $H(\hat{\omega})$ for $y[n] = x[n-2]$



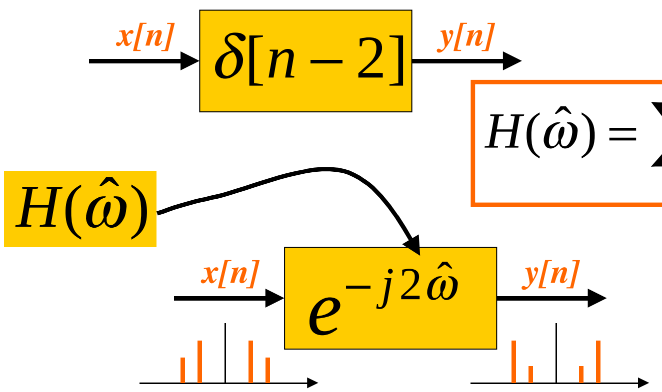
$$b_k = \{0, 0, 1\}$$

$$h[n] = \delta[n-2]$$



DELAY by 2 SYSTEM

Find $h[n]$ and $H(\hat{\omega})$ for $y[n] = x[n-2]$



$$H(\hat{\omega}) = \sum_k \delta[k-2]e^{-j\hat{\omega}k}$$

k=2 ONLY

GENERAL DELAY PROPERTY

Find $h[n]$ and $H(\hat{\omega})$ for $y[n] = x[n-n_d]$

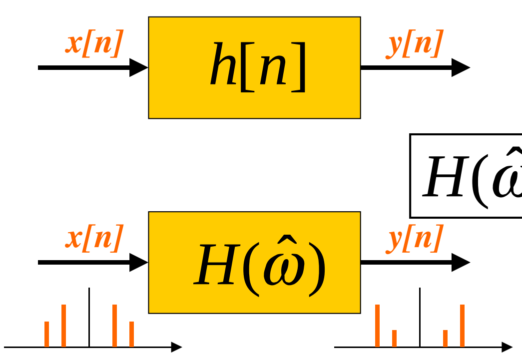
$$h[n] = \delta[n-n_d]$$

$$H(\hat{\omega}) = \sum_k \delta[k-n_d]e^{-j\hat{\omega}k} = e^{-j\hat{\omega}n_d}$$

ONLY ONE non-ZERO TERM for k

FREQ DOMAIN --> TIME ??

- **START with** $H(\hat{\omega})$ and find $h[n]$ or b_k



$$h[n] = ?$$

$$H(\hat{\omega}) = 7e^{-j2\hat{\omega}}\cos(\hat{\omega})$$

FREQ DOMAIN --> TIME

$$H(\hat{\omega}) = 7e^{-j2\hat{\omega}}\cos(\hat{\omega})$$

EULER's Formula

$$= 7e^{-j2\hat{\omega}}(0.5e^{j\hat{\omega}} + 0.5e^{-j\hat{\omega}})$$

$$= (3.5e^{-j\hat{\omega}} + 3.5e^{-j3\hat{\omega}})$$

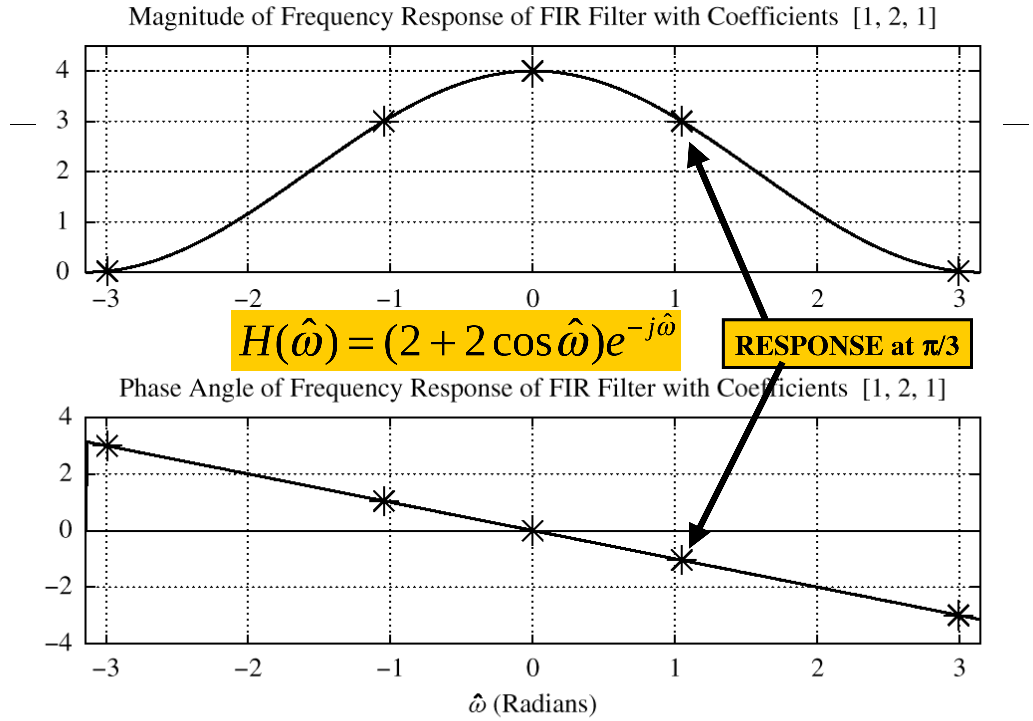
$$h[n] = 3.5\delta[n-1] + 3.5\delta[n-3]$$

$$b_k = \{0, 3.5, 0, 3.5\}$$

FREQ. RESPONSE PLOTS

- DENSE GRID **(ww)** from −π to +π
- **`yy = freqz(bb,1,ww)`**
  - VECTOR **bb** contains Filter Coefficients
  - DSP-First: `yy = freekz(bb,1,ww)`

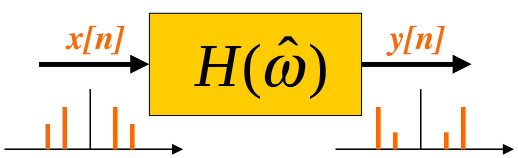
$$\mathcal{H}(\hat{\omega}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k}$$



EXAMPLE 6.2

Find $y[n]$ when $H(\hat{\omega})$ is known

& $x[n] = 2e^{j\pi/4}e^{j(\pi/3)n}$



$$H(\hat{\omega}) = (2+2\cos\hat{\omega})e^{-j\hat{\omega}}$$

EXAMPLE 6.2 (answer)

Find $y[n]$ when $x[n] = 2e^{j\pi/4}e^{j(\pi/3)n}$

Answer: Eval $H(\hat{\omega})$ at $\hat{\omega} = \pi/3$.

$$H(\hat{\omega}) = (2+2\cos\hat{\omega})e^{-j\hat{\omega}}$$

$$H(\hat{\omega}) = 3e^{-j\pi/3} \quad @\ \hat{\omega} = \pi/3$$

$$y[n] = \left(3e^{-j\pi/3}\right) \times 2e^{j\pi/4}e^{j(\pi/3)n} = 6e^{-j\pi/12}e^{j(\pi/3)n}$$

SINUSOID thru FIR

$$x[n] = X_0 + \sum_{k=1}^{N} \left(\frac{X_k}{2} e^{j\hat{\omega}_k n} + \frac{X_k^*}{2} e^{-j\hat{\omega}_k n} \right)$$

$$= X_0 + \sum_{k=1}^{N} |X_k| \cos(\hat{\omega}_k n + \angle X_k)$$

if $\mathcal{H}(-\hat{\omega}) = \mathcal{H}^*(\hat{\omega})$, the corresponding output is

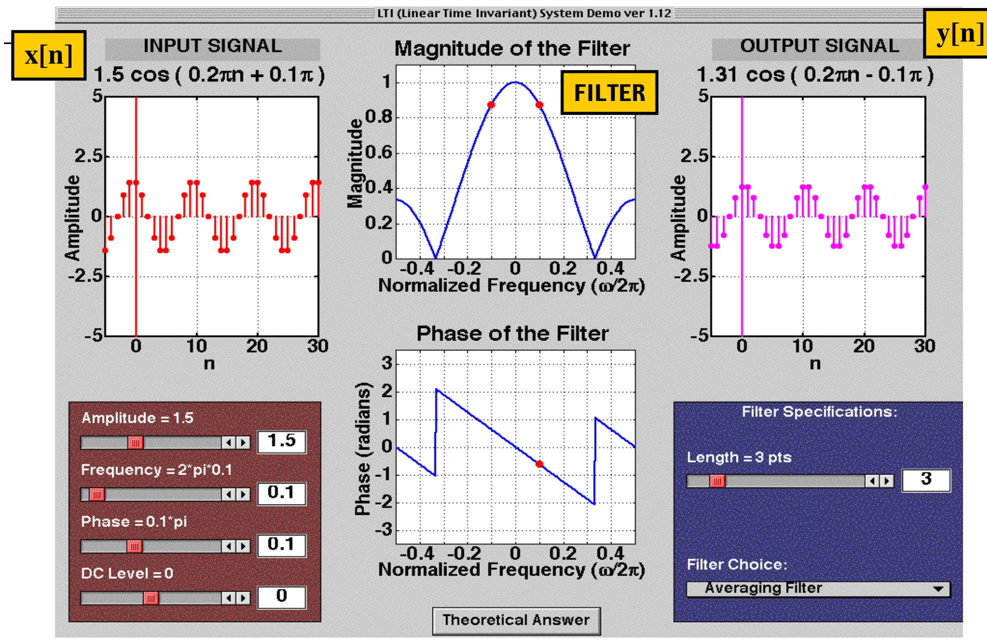
$$y[n] = \mathcal{H}(0) X_0 + \sum_{k=1}^{N} \left(\mathcal{H}(\hat{\omega}_k) \frac{X_k}{2} e^{j\hat{\omega}_k n} + \mathcal{H}(-\hat{\omega}_k) \frac{X_k^*}{2} e^{-j\hat{\omega}_k n} \right)$$

$$= \mathcal{H}(0) X_0 + \sum_{k=1}^{N} |\mathcal{H}(\hat{\omega}_k)| |X_k| \cos\left(\hat{\omega}_k n + \angle X_k + \angle \mathcal{H}(\hat{\omega}_k)\right)$$

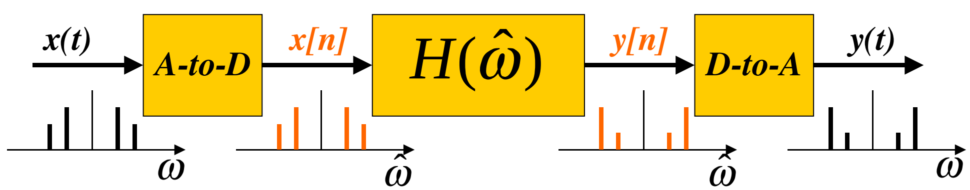
MULTIPLY MAGS

ADD PHASES

LTI Demo with Sinusoids

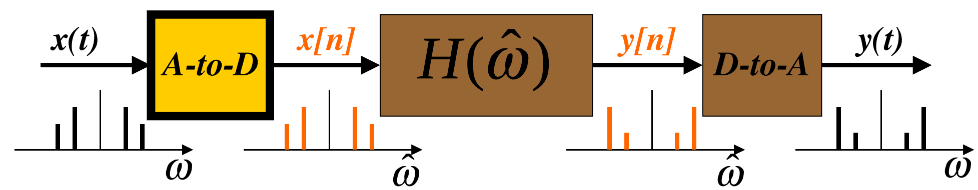


DIGITAL "FILTERING"



- ω SPECTRUM of x(t) (SUM of SINUSOIDS)
- SPECTRUM of x[n]
  - Is ALIASING a PROBLEM ?
- $\hat{\omega}$ SPECTRUM y[n] (FIR Gain or Nulls)
- ω Then, OUTPUT y(t) = SUM of SINUSOIDS

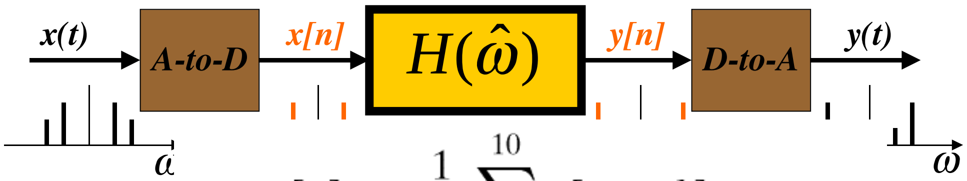
FREQUENCY SCALING



- TIME SAMPLING: $t = nT_s$
  - IF NO ALIASING:
  - FREQUENCY SCALING

$$\hat{\omega} = \omega T_s = \frac{\omega}{f_s}$$

11-pt AVERAGER Example



$$y[n] = \frac{1}{11}\sum_{k=0}^{10} x[n-k]$$

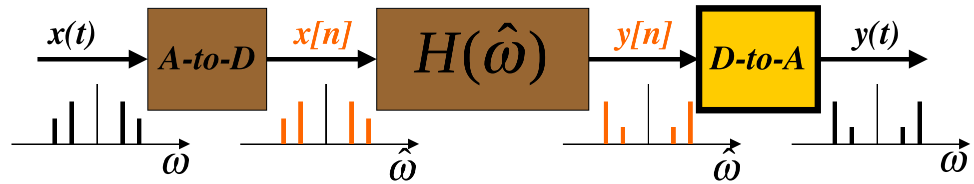
- 250 Hz
- 25 Hz

$$\mathcal{H}(\hat{\omega}) = \frac{\sin(\hat{\omega}11/2)}{11\sin(\hat{\omega}/2)}e^{-j\hat{\omega}5}$$

?

$$x(t) = \cos(2\pi(25)t) + \sin(2\pi(250)t)$$

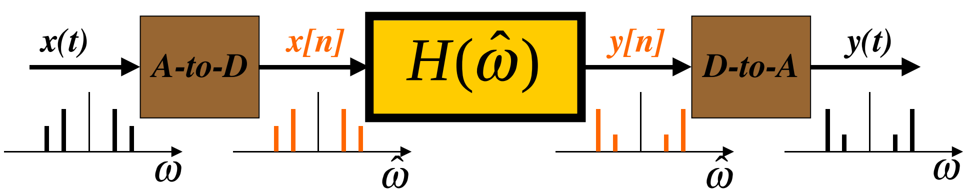
D-A FREQUENCY SCALING



- **TIME SAMPLING:** $t = nT_s \Rightarrow n \leftarrow t\, f_s$
- **RECONSTRUCT up to $0.5f_s$**
  - **FREQUENCY SCALING**

$$\omega = \hat{\omega}\, f_s$$

TRACK the FREQUENCIES

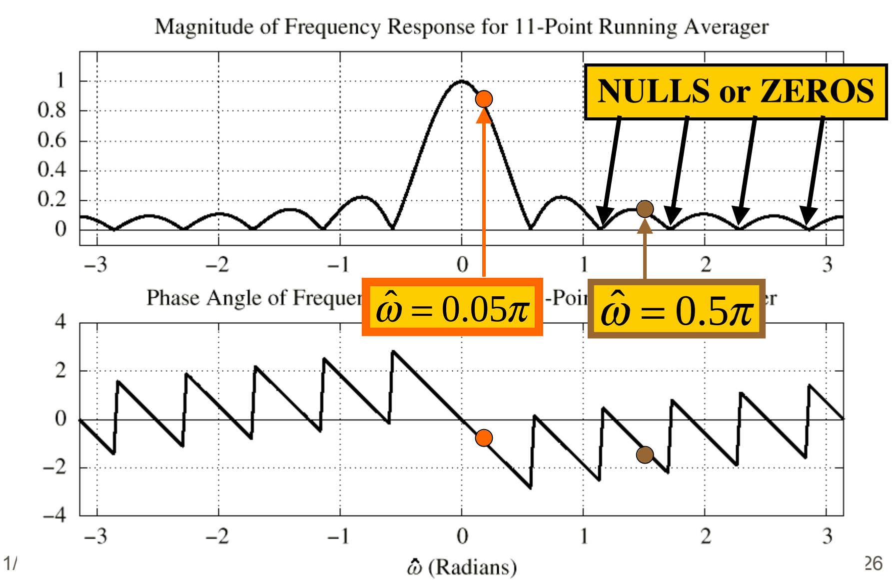


250 Hz	0.5π	$H(0.5\pi)$	0.5π	250 Hz
25 Hz	$.05\pi$	$H(0.05\pi)$	$.05\pi$	25 Hz

Fs = 1000 Hz

NO new freqs

11-pt AVERAGER



EVALUATE Freq. Response

$$H(\hat{\omega}) = \frac{\sin(\hat{\omega}11/2)}{11\sin(\hat{\omega}/2)} e^{-j\hat{\omega}5}$$

At $\hat{\omega} = 0.5\pi$

$$H(\hat{\omega}) = \frac{\sin((0.5\pi)11/2)}{11\sin(0.5\pi/2)} e^{-j(0.5\pi)5}$$

$$= \frac{\sin(2.75\pi)}{11\sin(0.25\pi)} e^{-j2.5\pi}$$

$$= 0.0909 e^{-j0.5\pi}$$

EVALUATE Freq. Response

$$x(t) = \cos(2\pi(25)t) + \sin(2\pi(250)t)$$

evaluating at 25 and 250 Hz.

$f_s = 1000$

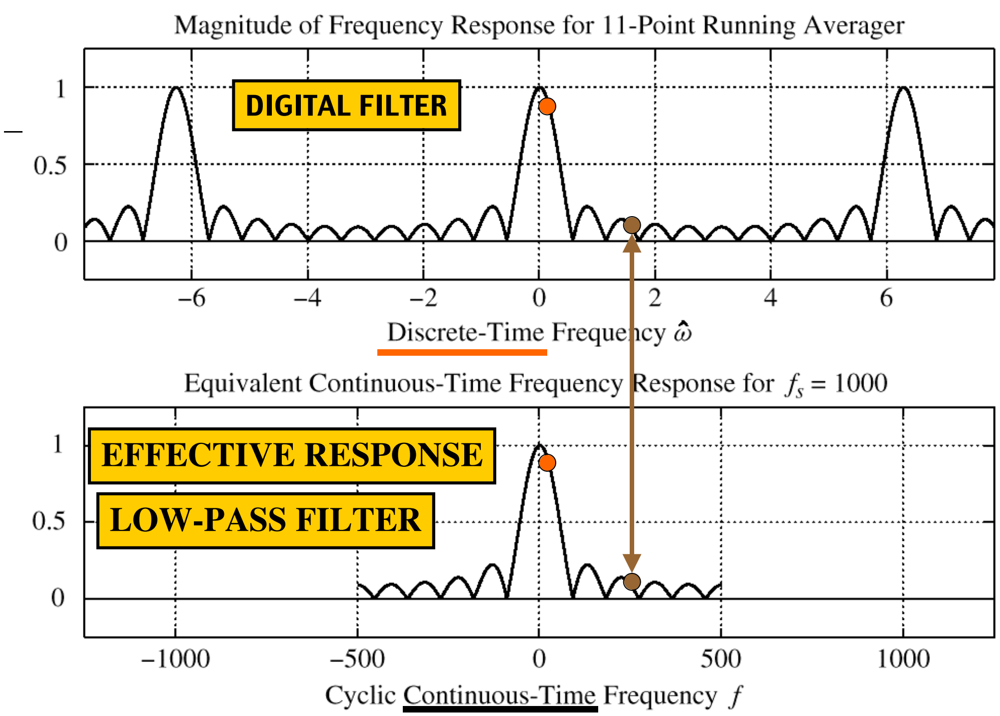
$$\mathcal{H}(2\pi(25)/1000) = \frac{\sin(\pi(25)(11)/1000)}{11\sin(\pi(25)/1000)} e^{-j2\pi(25)(5)/1000} = 0.8811 e^{-j\pi/4}$$

MAG SCALE

PHASE CHANGE

$$\mathcal{H}(2\pi(250)/1000) = \frac{\sin(\pi(250)(11)/1000)}{11\sin(\pi(250)/1000)} e^{-j2\pi(250)(5)/1000} = 0.0909 e^{-j\pi/2}$$

$$y(t) = 0.8811\cos(2\pi(25)t - \pi/4) + 0.0909\sin(2\pi(250)t - \pi/2)$$

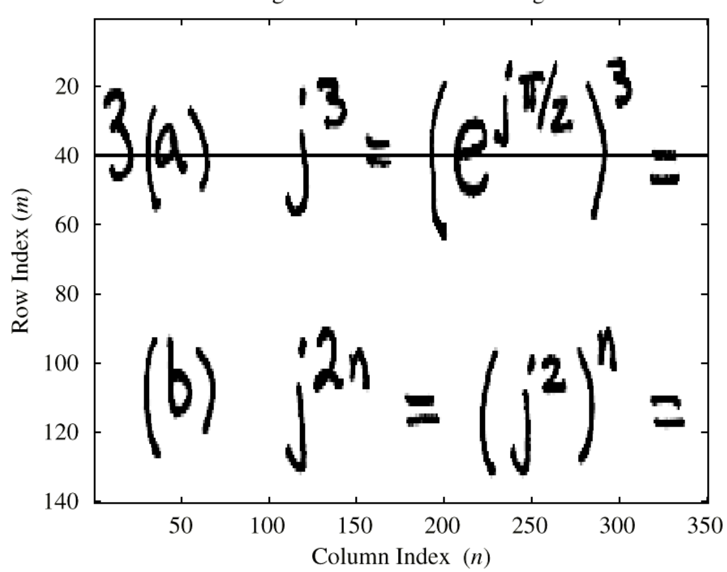


FILTER TYPES

- LOW–PASS FILTER (LPF)
  - BLURRING
  - ATTENUATES HIGH FREQUENCIES
- HIGH–PASS FILTER (HPF)
  - SHARPENING for IMAGES
  - BOOSTS THE HIGHS
  - REMOVES DC
- BAND–PASS FILTER (BPF)

B & W IMAGE

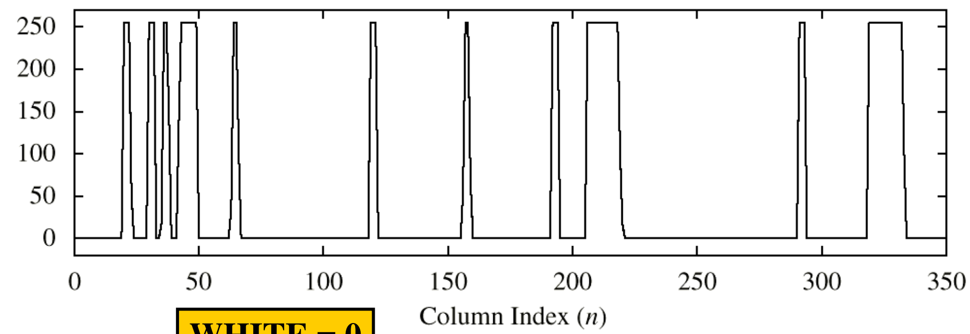
Original Black and White Image



ROW of B&W IMAGE

BLACK = 255

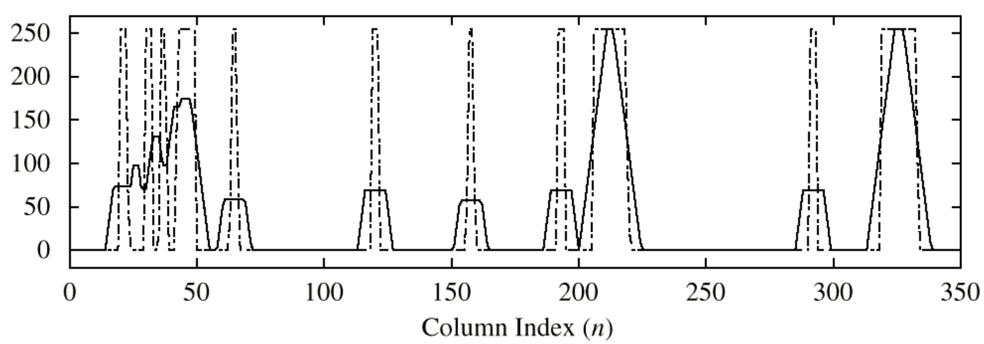
Row 40 of the Image



WHITE = 0

FILTERED ROW of IMAGE

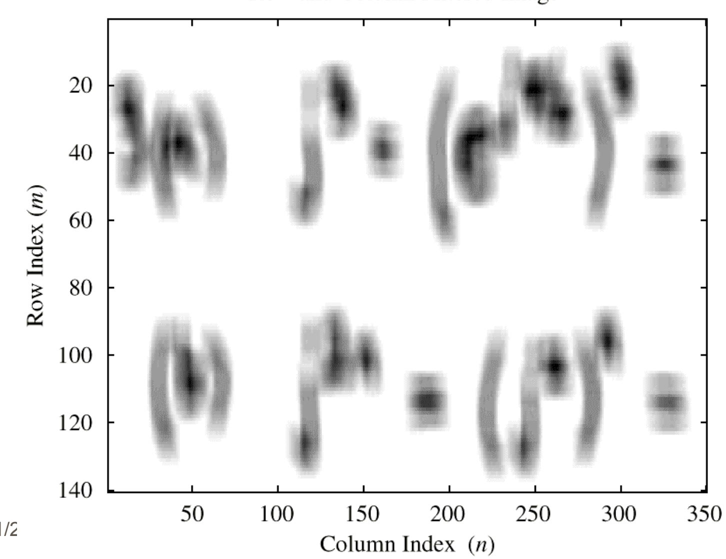
11-Point Averaging: 5-Sample Delay Equalization



ADJUSTED DELAY by 5 samples

FILTERED B&W IMAGE

Row and Column Filtered Image



LPF: BLUR

B&W IMAGE with COSINE

FILTERED: 11-pt AVG