EE-2025 Fall-99

Lecture 14

IIR Filters: Feedback

15-Oct-99

Info: Web-CT, Lab, HW

- Quiz #2 is Monday 25–Oct
    - Calculator, 1 page Handwritten Notes
  - Review: Sunday 24–Oct & Sat(?) 23–Oct
- Prob Set #7 is due today
  - On–Line HW #6 due Sat 23–Oct
- Help Sessions
  - Mon (6:30) & Wed (6:00) in VL–456
  - TAs have office hours in CoC–310

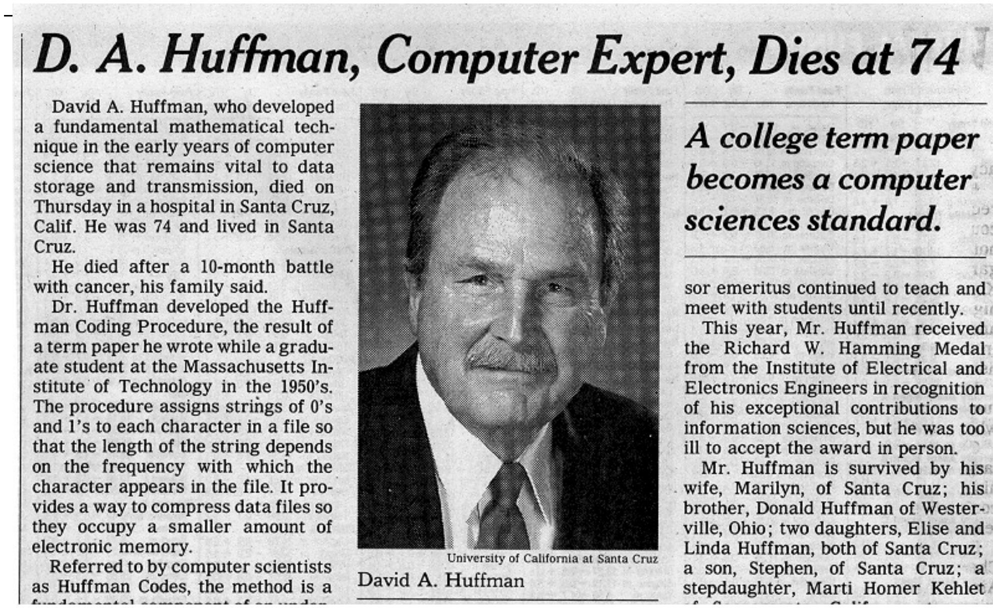
NEXT WEEK

- NO LAB
- GO TO A RECITATION
  - REGULARLY SCHEDULED
  - Or, at your LAB TIME on Wed & Thurs.
    - It will be a bit crowded

READING ASSIGNMENTS

- This Lecture:
  - Chapter 8, pp. 249–263
- Other Reading:
  - Recitation: Ch. 8, pp. 261–272
    - POLES & ZEROS
  - Next Lecture: Chapter 8, pp. 269–282

HUFFMAN CODES



D. A. Huffman, Computer Expert, Dies at 74

David A. Huffman, who developed a fundamental mathematical technique in the early years of computer science that remains vital to data storage and transmission, died on Thursday in a hospital in Santa Cruz, Calif. He was 74 and lived in Santa Cruz.

He died after a 10-month battle with cancer, his family said.

Dr. Huffman developed the Huffman Coding Procedure, the result of a term paper he wrote while a graduate student at the Massachusetts Institute of Technology in the 1950's. The procedure assigns strings of 0's and 1's to each character in a file so that the length of the string depends on the frequency with which the character appears in the file. It provides a way to compress data files so they occupy a smaller amount of electronic memory.

Referred to by computer scientists as Huffman Codes, the method is a

University of California at Santa Cruz

David A. Huffman

A college term paper becomes a computer sciences standard.

sor emeritus continued to teach and meet with students until recently.

This year, Mr. Huffman received the Richard W. Hamming Medal from the Institute of Electrical and Electronics Engineers in recognition of his exceptional contributions to information sciences, but he was too ill to accept the award in person.

Mr. Huffman is survived by his wife, Marilyn, of Santa Cruz; his brother, Donald Huffman of Westerville, Ohio; two daughters, Elise and Linda Huffman, both of Santa Cruz; a son, Stephen, of Santa Cruz; a stepdaughter, Marti Homer Kehlet

POP QUIZ

- **Given:** $H(\hat{\omega}) = e^{-j\hat{\omega}} \cos(\hat{\omega})$
- **Plot the Magnitude and Phase**
- **Find the output, y[n]**
  - **When** $x[n] = \cos(0.25\pi n)$

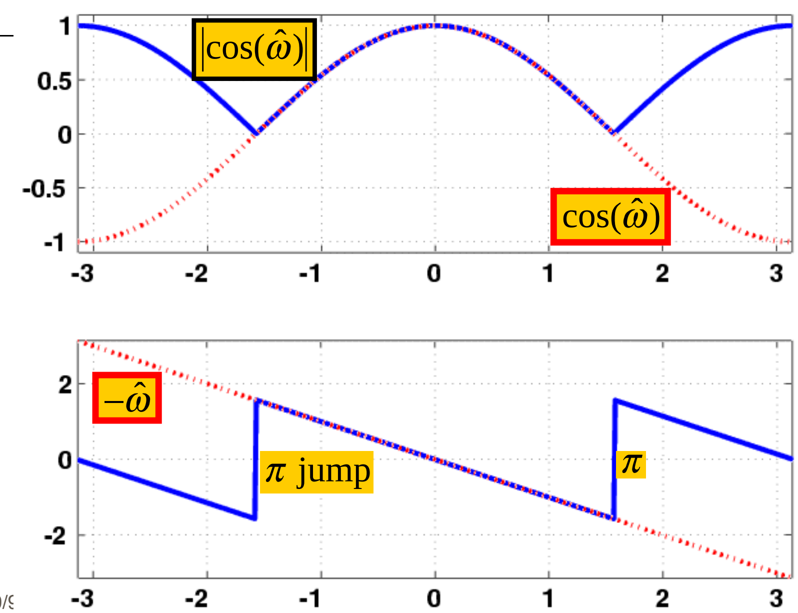
POP QUIZ: MAG & PHASE

- **Given:** $H(\hat{\omega}) = e^{-j\hat{\omega}} \cos(\hat{\omega})$
- **Derive Magnitude and Phase**

$$|H(\hat{\omega})| = |e^{-j\hat{\omega}}|\,|\cos(\hat{\omega})| = |\cos(\hat{\omega})|$$

$$\angle H(\hat{\omega}) = \begin{cases} -\hat{\omega} & \cos(\hat{\omega}) \geq 0 \\ -\hat{\omega} + \pi & \text{if } \cos(\hat{\omega}) < 0 \end{cases}$$

Ans: FREQ RESPONSE



L-pt RUNNING SUM H(z)

$$H(z)=\sum_{k=0}^{L-1}\tfrac{1}{L}z^{-k}=\frac{1-z^{-L}}{L(1-z^{-1})}=\frac{z^{L}-1}{Lz^{L-1}(z-1)}$$

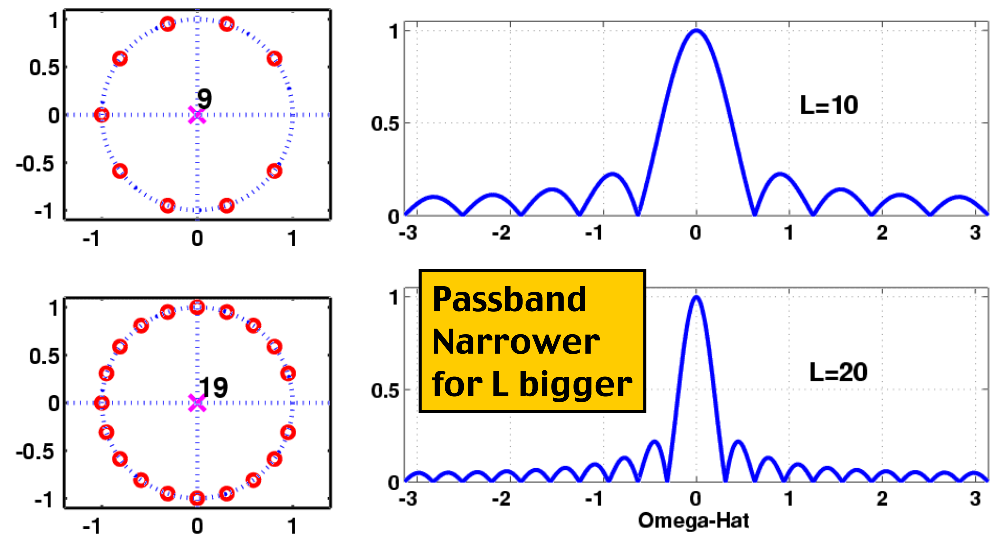
$$z^{L}-1=0 \quad \Rightarrow \quad z^{L}=1=e^{j2\pi k}$$

$$z=e^{j(2\pi/L)k} \quad \text{for } k=1,2,\ldots L\text{-}1$$

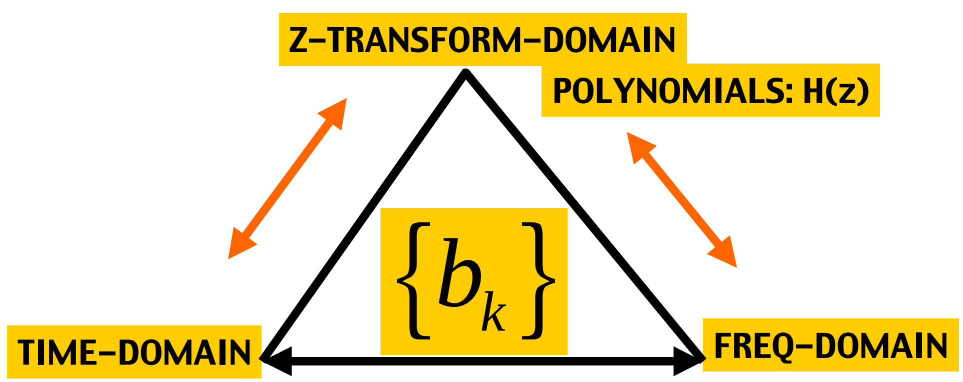
ZEROS on UNIT CIRCLE

(z–1) in denominator cancels k=0 term

FILTER DESIGN: CHANGE L



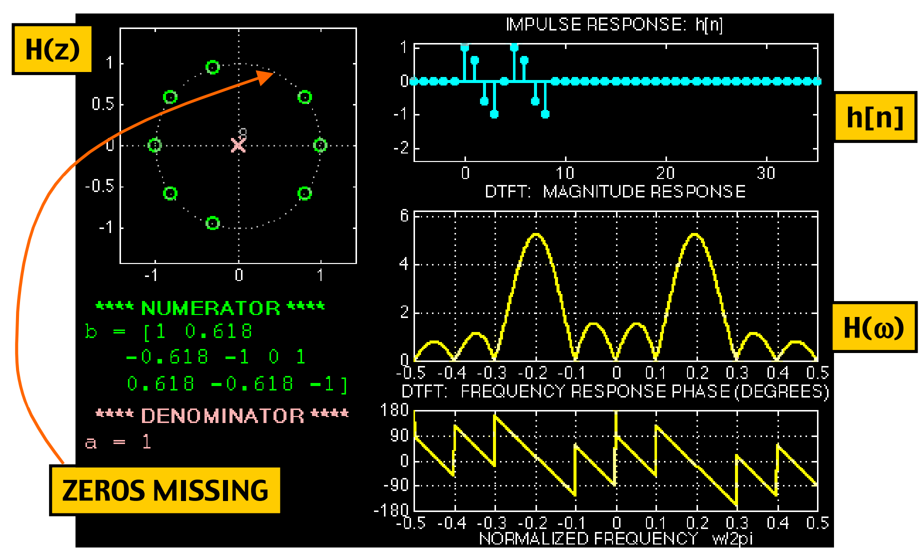
THREE DOMAINS



$$y[n]=\sum_{k=0}^{M}b_k x[n-k]$$

$$H(\hat{\omega})=\sum_{k=0}^{M}b_k e^{-j\hat{\omega}k}$$

3 DOMAINS MOVIE: FIR



LECTURE OBJECTIVES

- **INFINITE IMPULSE RESPONSE FILTERS**
  - **Define IIR Filters**
  - **Have FEEDBACK: PREVIOUS OUTPUTS**

$$y[n] = \sum_{\ell=1}^{N} a_\ell y[n-\ell] + \sum_{k=0}^{M} b_k x[n-k]$$

- **Show how to compute the output y[n]**
  - **FIRST-ORDER CASE (N=1)**
  - **Impulse Response h[n] <--> H(z)**

LOGICAL THREAD

- **FIND the IMPULSE RESPONSE, h[n]**
  - **INFINITELY LONG**
  - **IIR Filters**

$$H(z) = \sum_{n=0}^{\infty} h[n] z^{-n}$$

- **EXPLOIT THREE DOMAINS:**
  - **Show Relationship for IIR:**

$$h[n] \leftrightarrow H(z) \leftrightarrow H(e^{j\hat{\omega}})$$

ONE FEEDBACK TERM

- **ADD PREVIOUS OUTPUTS**

$$y[n] = a_1 y[n-1] + b_0 x[n] + b_1 x[n-1]$$

PREVIOUS

FEEDBACK

FIR PART of the FILTER

FEED-FORWARD

- **CAUSALITY**
  - **NOT USING FUTURE OUTPUTS or INPUTS**

FILTER COEFFICIENTS

- **ADD PREVIOUS OUTPUTS**

$$y[n] = 0.8 y[n-1] + 3x[n] - 2x[n-1]$$

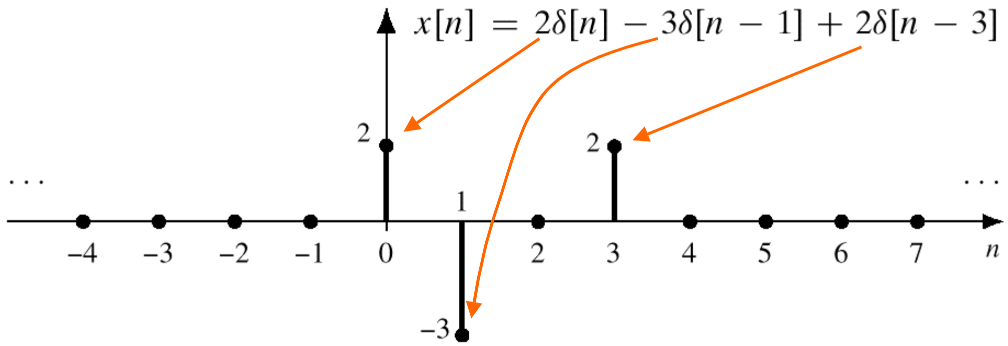
FEEDBACK COEFFICIENT

SIGN CHANGE

- **MATLAB**
  - `yy = filter([3,-2],[1,-0.8],xx)`

COMPUTE OUTPUT

$$y[n] = 0.8y[n-1] + 5x[n]$$



COMPUTE y[n]

- **FEEDBACK DIFFERENCE EQUATION:**

$$y[n] = 0.8y[n-1] + 5x[n]$$

- **NEED y[−1] to get started**

$$y[0] = 0.8y[-1] + 5x[0]$$

AT REST CONDITION

- **y[n] = 0, for n<0**
- **BECAUSE x[n] = 0, for n<0**

INITIAL REST CONDITIONS

1. The input must be assumed to be zero prior to some starting time n_0, i.e., $x[n] = 0$ for $n < n_0$. We say that such inputs are *suddenly applied*.
2. The output is likewise assumed to be zero prior to the starting time of the signal, i.e., $y[n] = 0$ for $n < n_0$. We say that the system is *initially at rest* if its output is zero prior to the application of a suddenly applied input.

COMPUTE y[0]

- **THIS STARTS THE RECURSION:**

With the initial rest assumption, $y[n] = 0$ for $n < 0$,

$$y[0] = 0.8y[-1] + 5(2) = 0.8(0) + 5(2) = 10$$

- **APPLIES TO ALL FEEDBACK TERMS**

$$y[n] = a_1 y[n-1] + a_2 y[n-2] + \sum_{k=0}^{2} b_k x[n-k]$$

COMPUTE MORE y[n]

- **CONTINUE THE RECURSION:**

$$y[1] = 0.8y[0] + 5x[1] = 0.8(10) + 5(-3) = -7$$

$$y[2] = 0.8y[1] + 5x[2] = 0.8(-7) + 5(0) = -5.6$$

$$y[3] = 0.8y[2] + 5x[3] = 0.8(-5.6) + 5(2) = 5.52$$

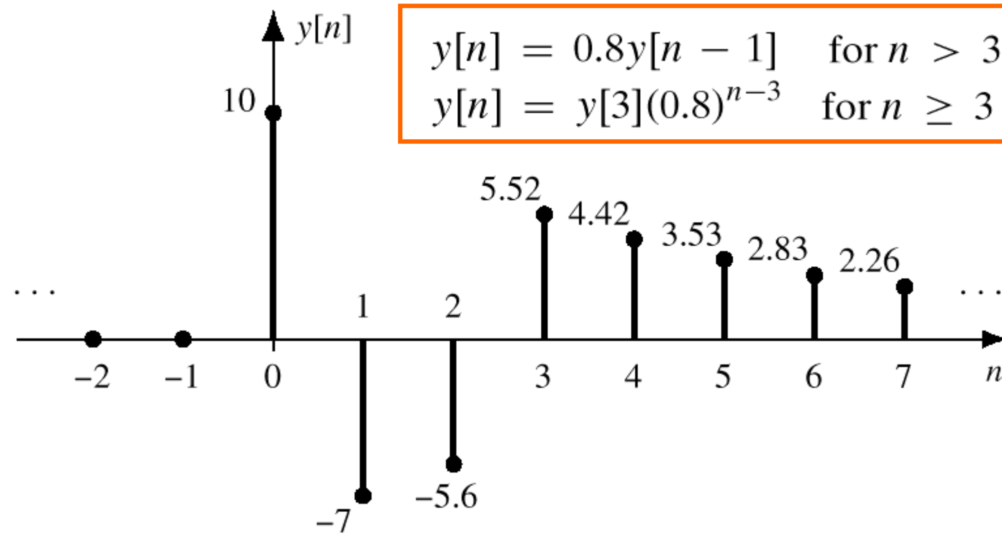
$$y[4] = 0.8y[3] + 5x[4] = 0.8(5.52) + 5(0) = 4.416$$

$$y[5] = 0.8y[4] + 5x[5] = 0.8(4.416) + 5(0) = 3.5328$$

$$y[6] = 0.8y[5] + 5x[6] = 0.8(3.5328) + 5(0) = 2.8262$$

PLOT y[n]

$$y[n] = 0.8y[n-1] \quad \text{for } n > 3$$
$$y[n] = y[3](0.8)^{n-3} \quad \text{for } n \geq 3$$



IMPULSE RESPONSE

$$h[n] = a_1 h[n-1] + b_0 \delta[n]$$

n	$n<0$	0	1	2	3	4
$\delta[n]$	0	1	0	0	0	0
$h[n-1]$	0	0	b_0	$b_0(a_1)$	$b_0(a_1)^2$	$b_0(a_1)^3$
$h[n]$	0	b_0	$b_0(a_1)$	$b_0(a_1)^2$	$b_0(a_1)^3$	$b_0(a_1)^4$

From this table it is obvious that the general formula is

$$h[n] = \begin{cases} b_0(a_1)^n & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

$$h[n] = b_0(a_1)^n u[n]$$

$$u[n] = 1, \quad \text{for } n \geq 0$$

IMPULSE RESPONSE

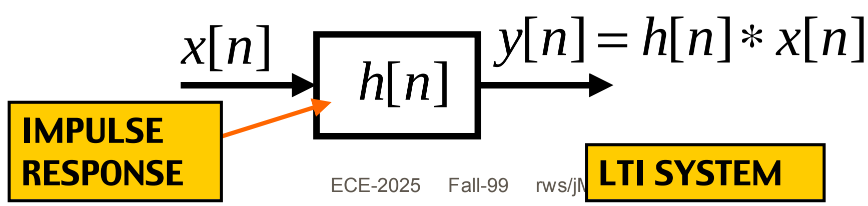
- **DIFFERENCE EQUATION:**

$$y[n] = 0.8y[n-1] + 3x[n]$$

- **Find h[n]**

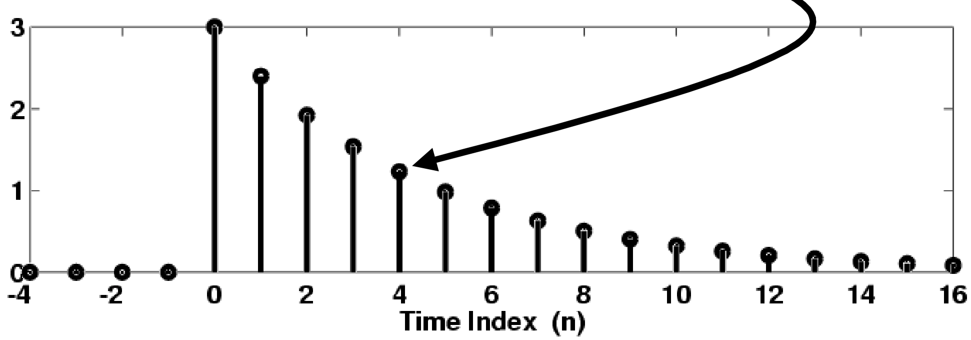
$$h[n] = 3(0.8)^n u[n]$$

- **CONVOLUTION in TIME-DOMAIN**



PLOT IMPULSE RESPONSE

$$h[n] = b_0 (a_1)^n u[n] = 3(0.8)^n u[n]$$



Infinite-Length Signal: h[n]

- **POLYNOMIAL Representation**

$$H(z) = \sum_{n=-\infty}^{\infty} h[n] z^{-n}$$

APPLIES to Any SIGNAL

- **COMPACT FORM for H(z)**

$$H(z) = \sum_{n=0}^{\infty} b a^n z^{-n} = \sum_{n=0}^{\infty} b (a z^{-1})^n$$

$$= \frac{b}{1 - a z^{-1}} \quad \text{if } |z| > |a|$$

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}$$