EE-2025 Fall-99

Lecture 15

H(z) & Frequency Response

22-Oct-99

Info: Web-CT, Lab, HW

- Calendar:
  - Quiz #2 is Monday, 25–Oct
    - CALCULATOR & 1 PAGE HAND–WRITTEN NOTES
  - Review Sessions: SUN, 7pm, 24–Oct
    - SATURDAY 11–1, 23–Oct (ECE Auditorium)
- Prob Set #8 due TODAY
  - On–Line HW #6 due Saturday nite
- Lab #8 is DTMF: already posted

NOTES on FOURIER

- Four New Chapters
- PDF on Web
- Copies in Bookstore (est. $6)
- How many of each?

READING ASSIGNMENTS

- This Lecture:
  - Chapter 8, pp. 263–279
- Other Reading:
  - Recitation: Ch. 8, pp. 261–272
    - POLES & ZEROS
  - Next Lecture: Chapter 8, pp. 279–300

LECTURE OBJECTIVES

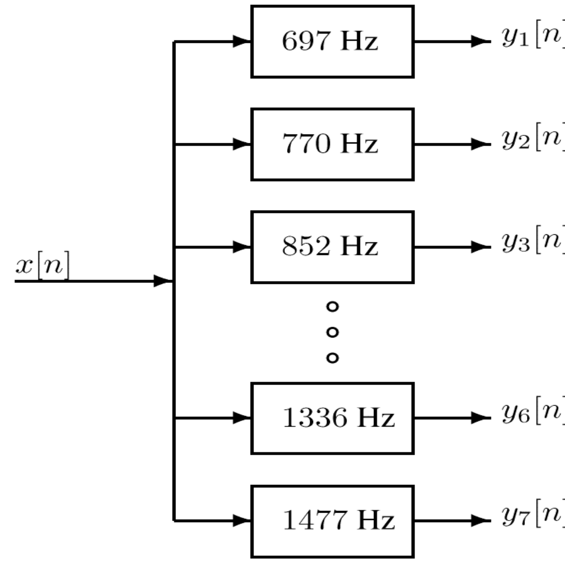
- SYSTEM FUNCTION: H(z)
- H(z) has POLES and ZEROS
- FREQUENCY RESPONSE of IIR
  - Get H(z) first

$$H(e^{j\hat{\omega}}) = H(z)\Big|_{z=e^{j\hat{\omega}}}$$

- THREE–DOMAIN APPROACH

$$h[n] \leftrightarrow H(z) \leftrightarrow H(e^{j\hat{\omega}})$$

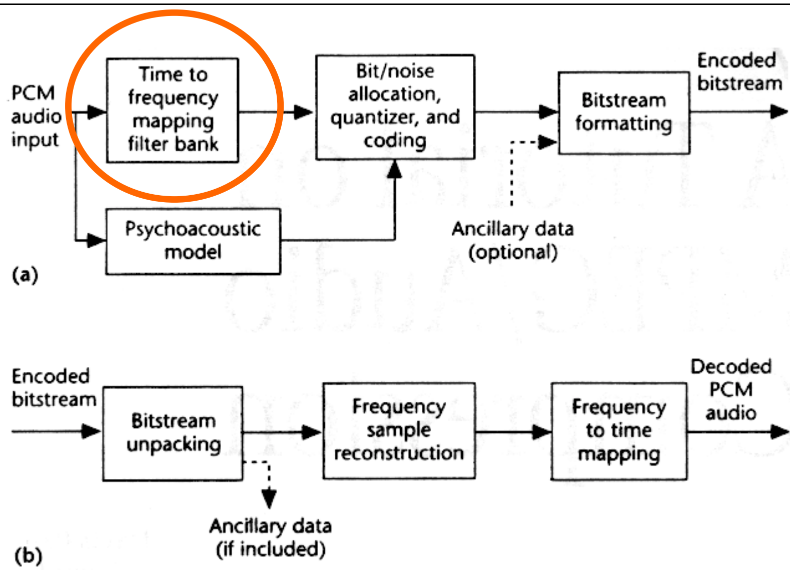
FILTER BANK in LAB (BPFs)



CONSTANT BANDWIDTH
- if L is same
- 1/L controls BW

VARIABLE BANDWIDTH ?
- MP3
- match hearing
- Change L

MP-3 AUDIO CODING



IIR FILTER REVIEW

- ADD PREVIOUS OUTPUTS

$$y[n] = 0.8y[n-1] + 3x[n]$$

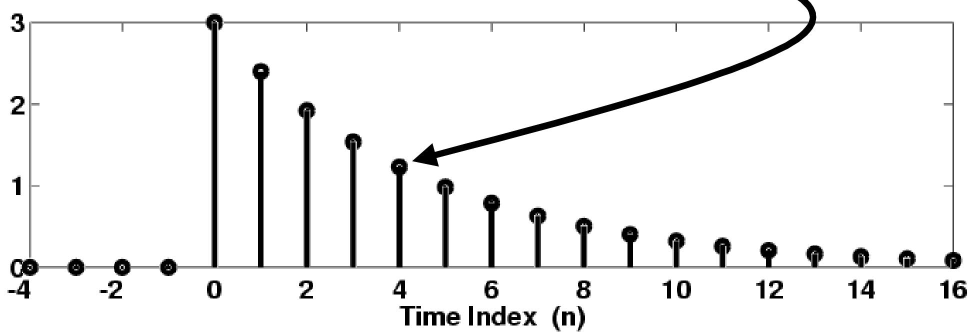
FEEDBACK COEFFICIENT

SIGN CHANGE

- MATLAB
  - `yy = filter([3],[1,-0.8],xx)`

PLOT IMPULSE RESPONSE

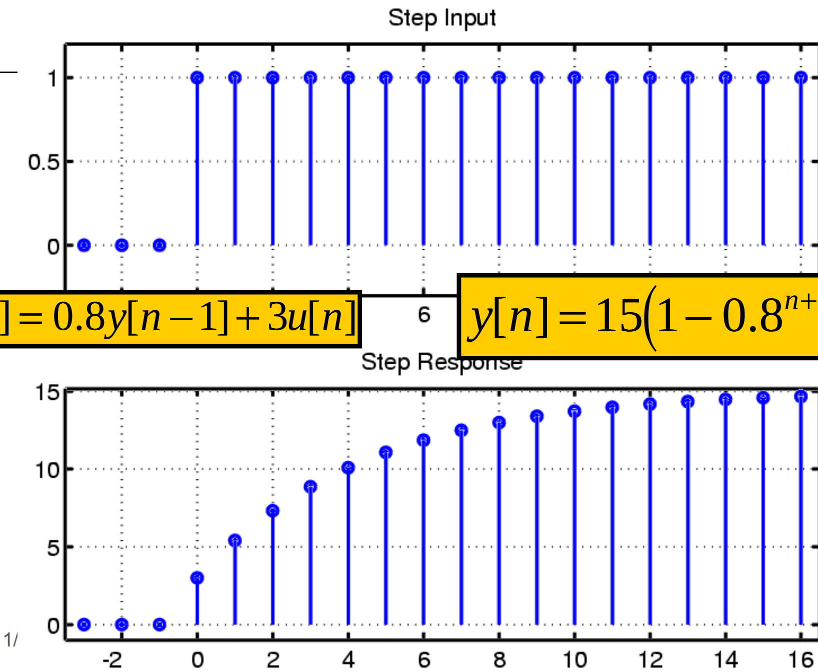
$$h[n] = b_0 (a_1)^n u[n] = 3(0.8)^n u[n]$$



PLOT STEP RESPONSE

$$y[n] = 0.8y[n-1] + 3u[n]$$

$$y[n] = 15\left(1 - 0.8^{n+1}\right)u[n]$$



THREE DOMAINS

Use H(z) to get Freq. Response

$$z = e^{j\hat{\omega}}$$

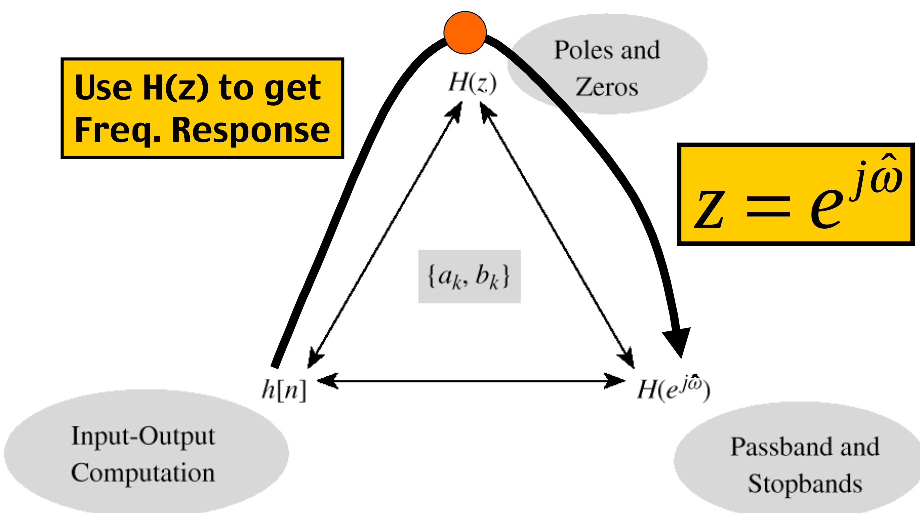


Figure 8.13 Relationship among the n-, z-, and $\hat{\omega}$-domains. The filter coefficients $\{a_k,\ b_k\}$ play a central role.

H(z) = z-Transform{ h[n] }

- **POLYNOMIAL Representation**

$$H(z) = \sum_{n=-\infty}^{\infty} h[n] z^{-n}$$

SYSTEM FUNCTION is Z-Transform of h[n]

- **COMPACT FORM for H(z)**

$$H(z) = \sum_{n=0}^{\infty} b a^n z^{-n} = \sum_{n=0}^{\infty} b(a z^{-1})^n$$

$$= \frac{b}{1 - a z^{-1}} \quad \text{if } |az^{-1}| < 1$$

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r} \quad \text{if } |r| < 1$$

First-Order Denominator

- GEOMETRIC SEQUENCE:

$$h[n] = ba^n u[n] \quad \leftrightarrow \quad H(z) = \frac{b}{1 - az^{-1}}$$

- USE KNOWN TRANSFORM:

$$h[n] = ba^n u[n] = 3(0.8)^n u[n]$$

$$H(z) = \sum_n 3(0.8)^n u[n] z^{-n}$$

$$= \sum_{n=0}^{\infty} 3(0.8)^n z^{-n} = \frac{3}{1 - 0.8z^{-1}}$$

Z-Transform of Infinite Length

- POLYNOMIAL Representation

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

APPLIES to ANY SIGNAL

- COMPACT FORM for H(z)

$$x[n] = (-0.5)^{n-1} u[n-1]$$

$$X(z) = \sum_{n=1}^{\infty} \left((-0.5)^{n-1} u[n-1] \right) z^{-n}$$

$$= z^{-1} - 0.5z^{-2} + 0.25z^{-3} - \ldots = \frac{z^{-1}}{1 + 0.5z^{-1}}$$

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r} \quad \text{if } |r| < 1$$

DELAY PROPERTY of X(z)

- DELAY in TIME<-->Multiply X(z) by z^{-1}

$$x[n] \leftrightarrow X(z)$$

$$x[n-1] \leftrightarrow z^{-1} X(z)$$

Proof: $$\sum_{n=-\infty}^{\infty} x[n-1] z^{-n} = \sum_{\ell=-\infty}^{\infty} x[\ell] z^{-(\ell+1)}$$

$$= z^{-1} \sum_{\ell=-\infty}^{\infty} x[\ell] z^{-\ell} = z^{-1} X(z)$$

Z-Transform of IIR Filter

- DERIVE the SYSTEM FUNCTION H(z)
  - Use DELAY PROPERTY

$$y[n] = a_1 y[n-1] + b_0 x[n] + b_1 x[n-1]$$

$$Y(z) = a_1 z^{-1} Y(z) + b_0 X(z) + b_1 z^{-1} X(z)$$

EASIER with DELAY PROPERTY

Time delay of n_0 samples multiplies the z-transform by z^{-n_0}

$$x[n - n_0] \quad \Longleftrightarrow \quad z^{-n_0} X(z)$$

SYSTEM FUNCTION of IIR

- NOTE the FILTER COEFFICIENTS

$$Y(z) - a_1 z^{-1} Y(z) = b_0 X(z) + b_1 z^{-1} X(z)$$

$$(1 - a_1 z^{-1}) Y(z) = (b_0 + b_1 z^{-1}) X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1}}{1 - a_1 z^{-1}} = \frac{B(z)}{A(z)}$$

SYSTEM FUNCTION

- DIFFERENCE EQUATION:

$$y[n] = 0.8 y[n-1] + 3x[n] - 2x[n-1]$$

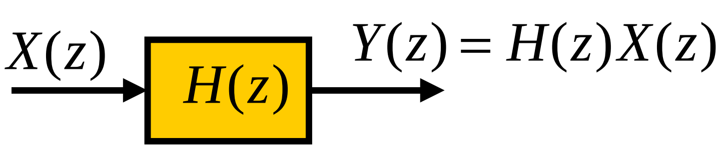
- READ the FILTER COEFFS:

$$Y(z) = \left(\frac{3 - 2z^{-1}}{1 - 0.8 z^{-1}} \right) X(z)$$

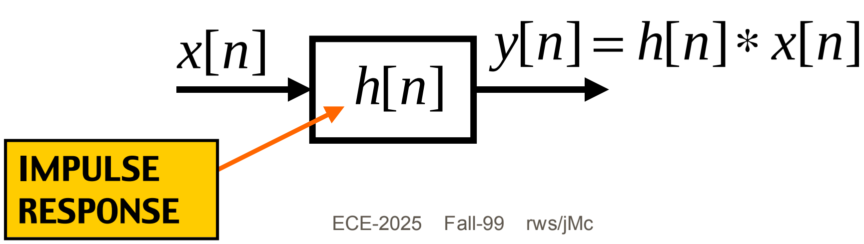
H(z)

CONVOLUTION PROPERTY

- MULTIPLICATION of z-TRANSFORMS



- CONVOLUTION in TIME-DOMAIN



POLES & ZEROS

- ROOTS of NUMERATOR & DENOMINATOR

$$H(z) = \frac{b_0 + b_1 z^{-1}}{1 - a_1 z^{-1}} \rightarrow H(z) = \frac{b_0 z + b_1}{z - a_1}$$

$$b_0 z + b_1 = 0 \quad \Rightarrow z = -\frac{b_1}{b_0}$$ ZERO

$$z - a_1 = 0 \quad \Rightarrow z = a_1$$ POLE

INTERPRET ROOTS

- VALUE of H(z) at POLES is <u>INFINITE</u>

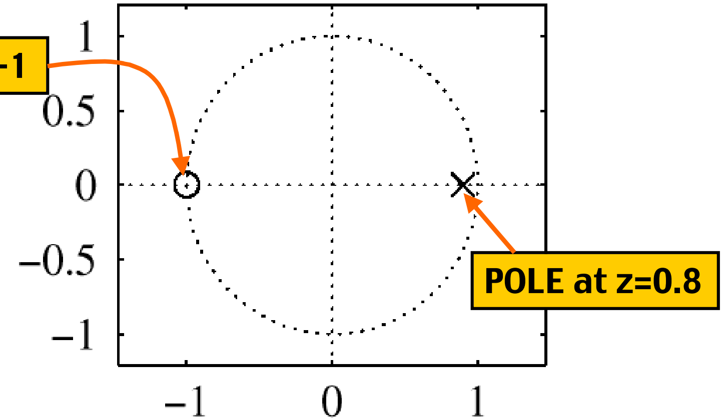
$$H(z) = \frac{b_0 + b_1 z^{-1}}{1 - a_1 z^{-1}} = \frac{b_0 z + b_1}{z - a_1}$$

$$H(z)\Big|_{z=-(b_1/b_0)} = 0$$ **ZERO**

$$H(z)\Big|_{z=a_1} \to \infty$$ **POLE**

POLE-ZERO PLOT

ZERO at z = −1



$$\frac{2+2z^{-1}}{1-0.8z^{-1}}$$

FREQUENCY RESPONSE

- SYSTEM FUNCTION: H(z)
- H(z) has DENOMINATOR
- FREQUENCY RESPONSE of IIR
  - We have H(z)

$$H(e^{j\hat{\omega}}) = H(z)\Big|_{z=e^{j\hat{\omega}}}$$

- THREE–DOMAIN APPROACH

$$h[n] \leftrightarrow H(z) \leftrightarrow H(e^{j\hat{\omega}})$$

FREQUENCY RESPONSE

- EVALUATE on the UNIT CIRCLE

$$H(e^{j\hat{\omega}}) = H(z)\Big|_{z=e^{j\hat{\omega}}}$$

$$H(z) = \frac{b_0 + b_1 z^{-1}}{1 - a_1 z^{-1}}$$

$$H(e^{j\hat{\omega}}) = H(z)\Big|_{z=e^{j\hat{\omega}}} = \frac{b_0 + b_1 e^{-j\hat{\omega}}}{1 - a_1 e^{-j\hat{\omega}}}$$

FREQ. RESPONSE FORMULA

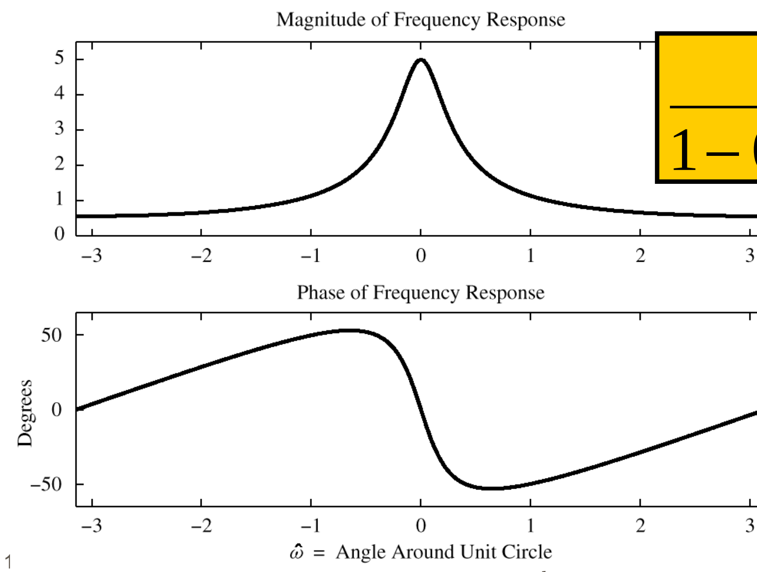
$$H(z)=\frac{1}{1-0.8z^{-1}} \rightarrow H(e^{j\hat{\omega}})=\frac{1}{1-0.8e^{-j\hat{\omega}}}$$

$$\left|\frac{1}{1-0.8e^{-j\hat{\omega}}}\right|^2=\frac{1}{1-0.8e^{-j\hat{\omega}}}\cdot\frac{1}{1-0.8e^{j\hat{\omega}}}$$

$$\frac{1}{1+0.64-0.8e^{-j\hat{\omega}}-0.8e^{j\hat{\omega}}}=\frac{1}{1.64-1.6\cos\hat{\omega}}$$

$$@\hat{\omega}=0, \quad \left|H(e^{j\hat{\omega}})\right|^2=\frac{1}{0.04}=25 \qquad @\hat{\omega}=\pi\,?$$

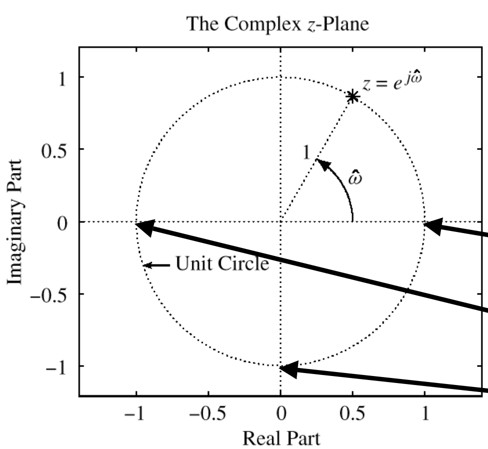
FREQ. RESPONSE from H(z)



$$\frac{1}{1-0.8e^{-j\hat{\omega}}}$$

UNIT CIRCLE

- **MAPPING BETWEEN** z and $\hat{\omega}$



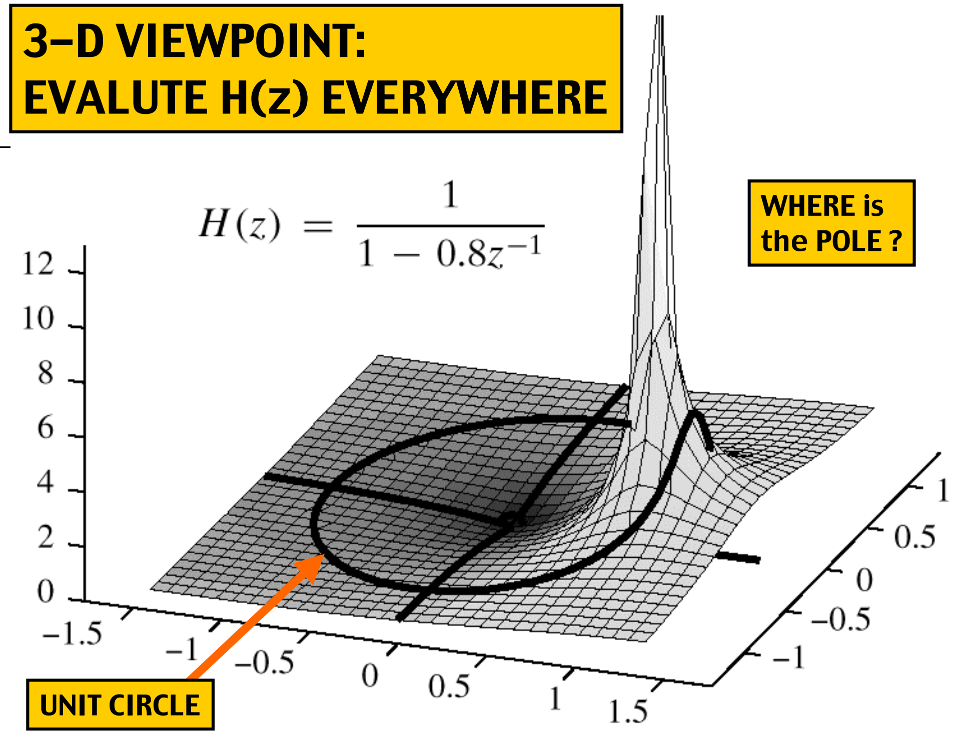
$$z=e^{j\hat{\omega}}$$

$$z=1 \leftrightarrow \hat{\omega}=0$$

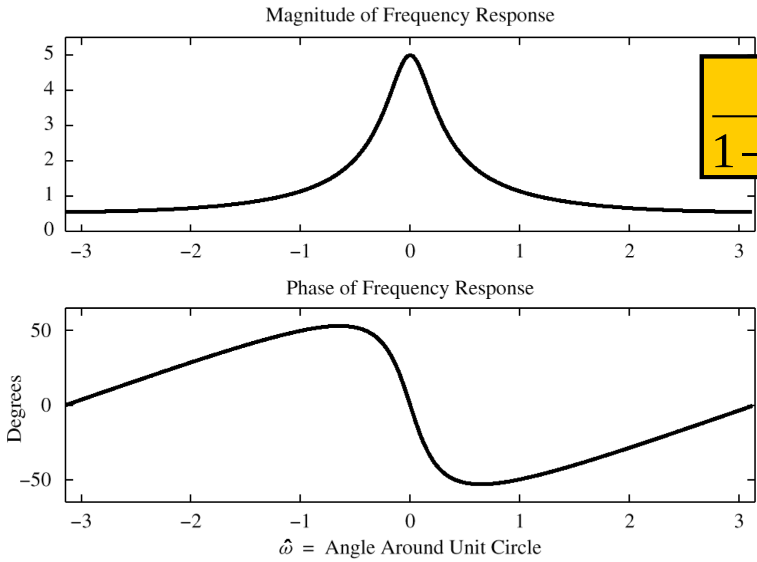
$$z=-1 \leftrightarrow \hat{\omega}=\pm\pi$$

$$z=\pm j \leftrightarrow \hat{\omega}=\pm\tfrac{1}{2}\pi$$

3-D VIEWPOINT: EVALUTE H(z) EVERYWHERE

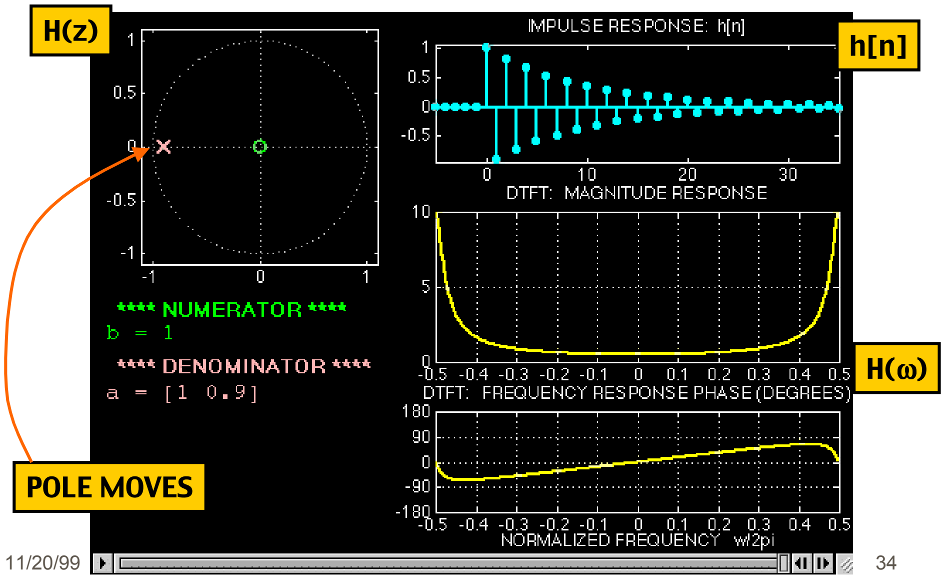


FREQ. RESPONSE from 3-D

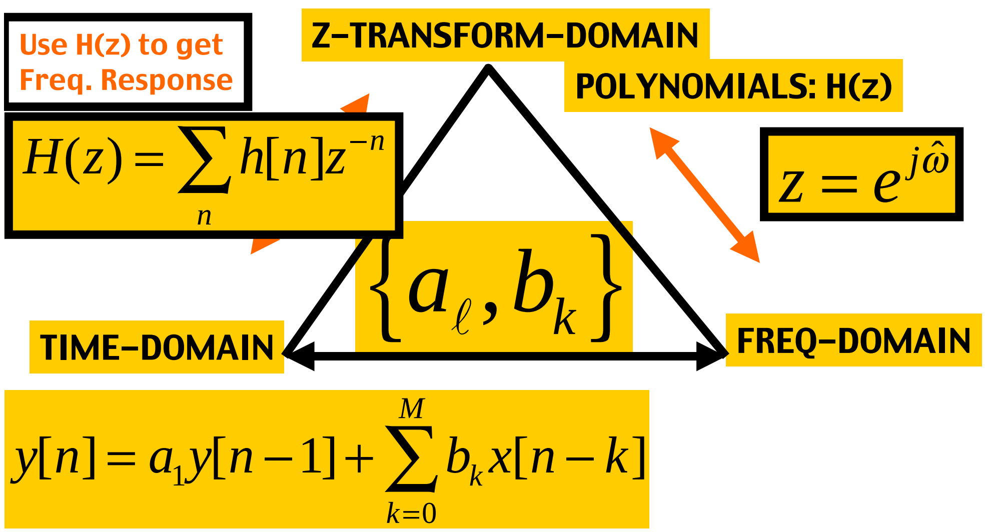


$$\frac{1}{1-0.8z^{-1}}$$

3 DOMAINS MOVIE: IIR



THREE DOMAINS



$$H(z)=\sum_{n} h[n]z^{-n}$$

$$z=e^{j\hat{\omega}}$$

$$\{a_\ell, b_k\}$$

$$y[n]=a_1 y[n-1]+\sum_{k=0}^{M} b_k x[n-k]$$

POP QUIZ

- **Given:** $H(z)=\dfrac{2+2z^{-1}}{1-0.8z^{-1}}$
- **Find the Impulse Response, h[n]**
- **Find the output, y[n]**
  - **When** $x[n]=\cos(0.25\pi n)$