EE-2025 Fall-99

Lecture 18

Continuous-Time Systems and Convolution

5-Nov-99

Info: Web-CT, Lab, HW

- Calendar:
  - Quiz #3 is 22-Nov
- Get NEW CHAPTERS
  - PDF or Bookstore on FRIDAY!
- Prob Set #10 is due today
- Lab #10 on SIMULATION
  - Then 2 more Labs

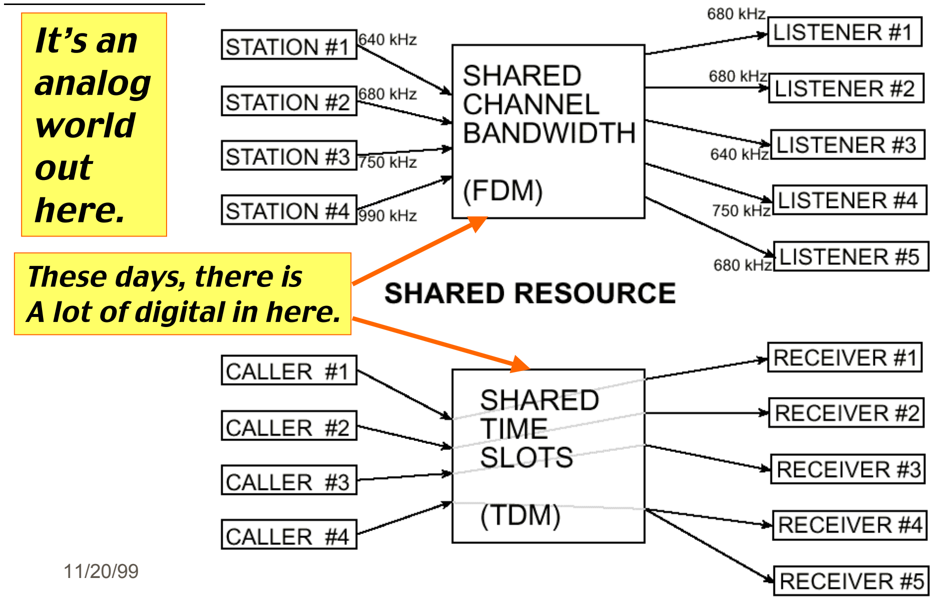
READING ASSIGNMENTS

- This Lecture:
  - Chapter 10, pp. 1000-1020
- Other Reading:
  - Recitation: Ch. 10, pp. 1020-1029
  - Next Lecture: Chapter 10, all

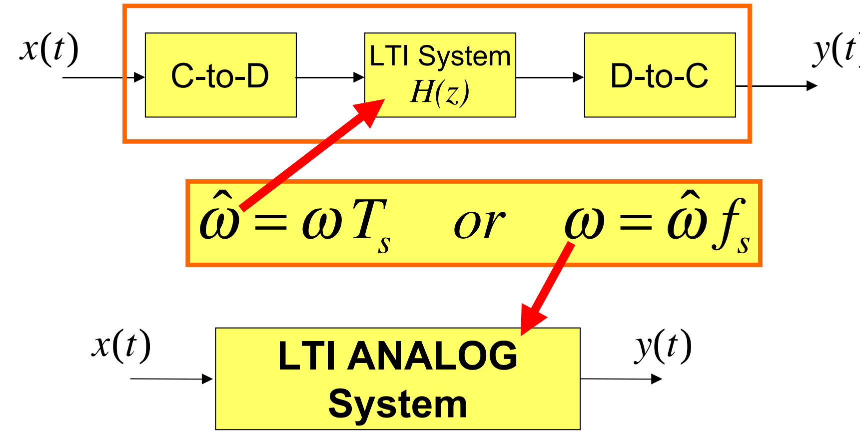
LECTURE OBJECTIVES

- Bye bye to D-T Systems for a while
- Continuous-time signals and systems
  - Review: Linearity and Time-Invariance
  - Example systems
- The UNIT IMPULSE signal
  - Definition
  - Properties
- Convolution integral: impulse response

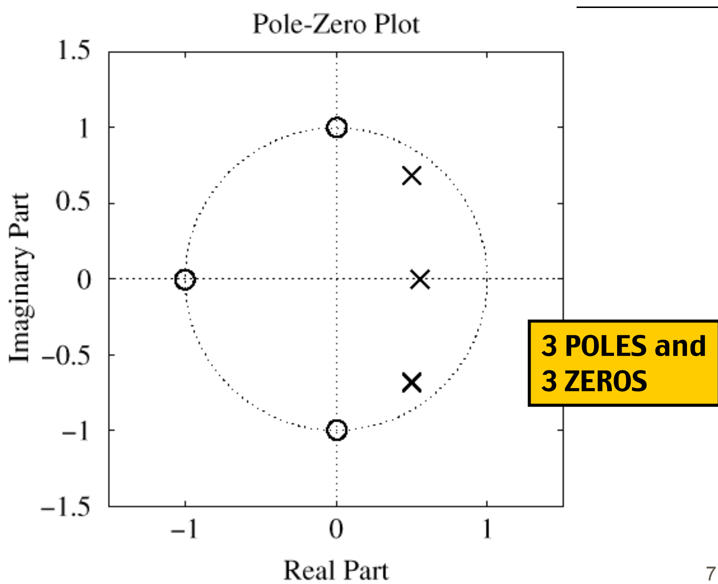
The way communication systems work



D-T Filtering of C-T Signals

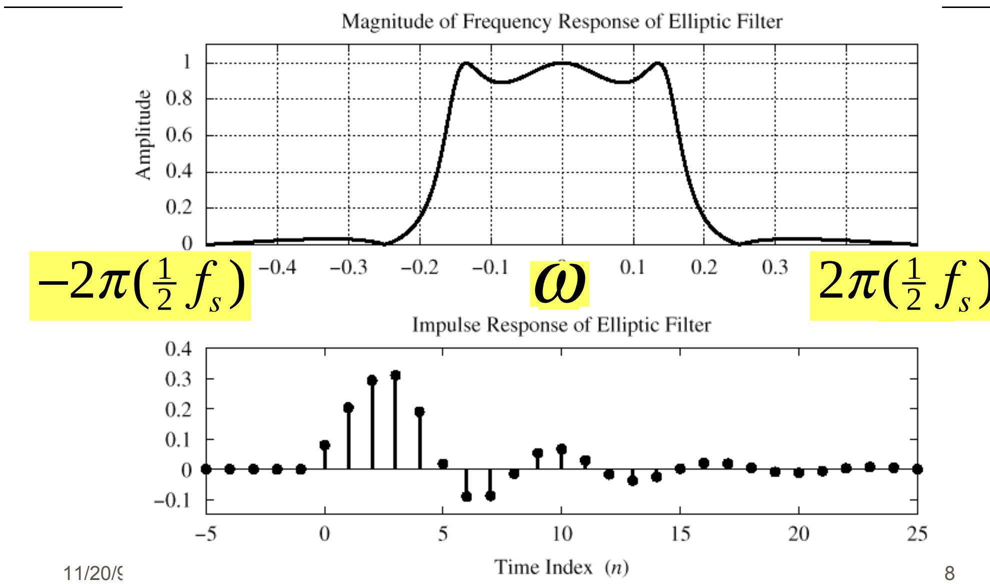


POLES & ZEROS of IIR



IIR Elliptic LPF (N=3)

3 POLES and 3 ZEROS



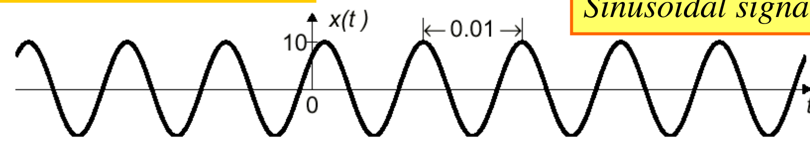
ANALOG SIGNALS x(t)

- SINUSOIDS: (t = time in secs)
- DECAYING EXPONENTIALS
- UNIT STEP
- IMPULSE SIGNAL (?)
  - VERY TRICKY to DEFINE
- DISCRETE-TIME: x[n] is list of numbers

Continuous-Time Signals

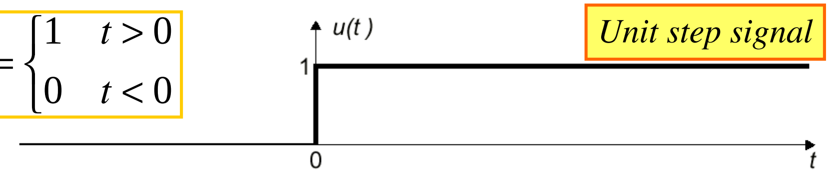
$x(t) = 10\cos(200\pi t - \pi/4)$

Sinusoidal signal



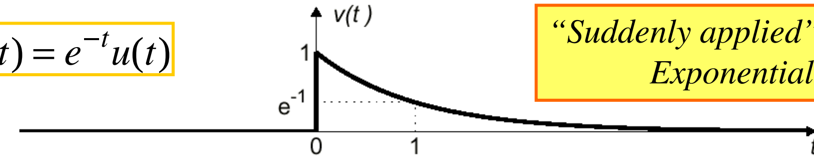
$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$

Unit step signal



$v(t) = e^{-t}u(t)$

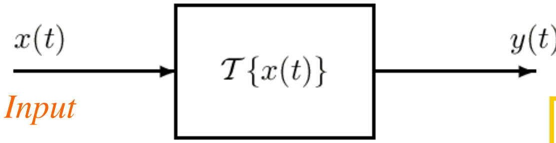
"Suddenly applied" Exponential



BUILDING BLOCKS

- INTEGRATOR (CIRCUITS)
- DIFFERENTIATOR
- DELAY by t_o
- MODULATOR
- MULTIPLIER & ADDER

Continuous-Time Systems

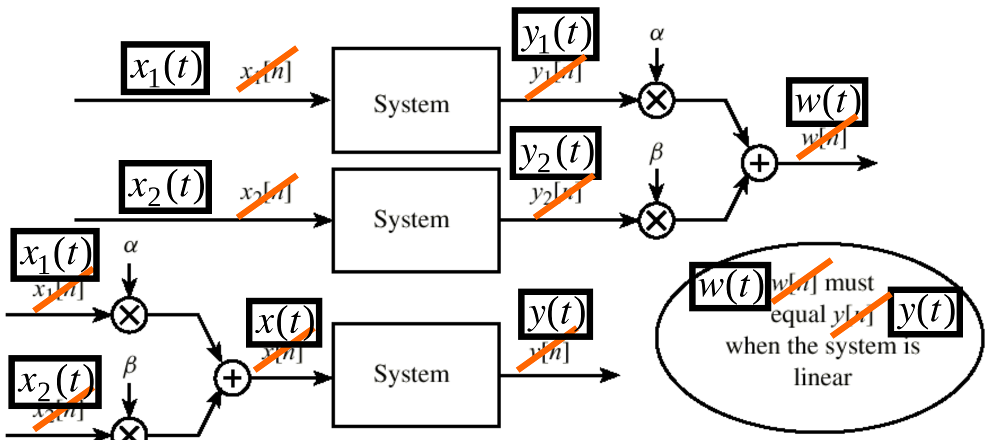


Input *Output*

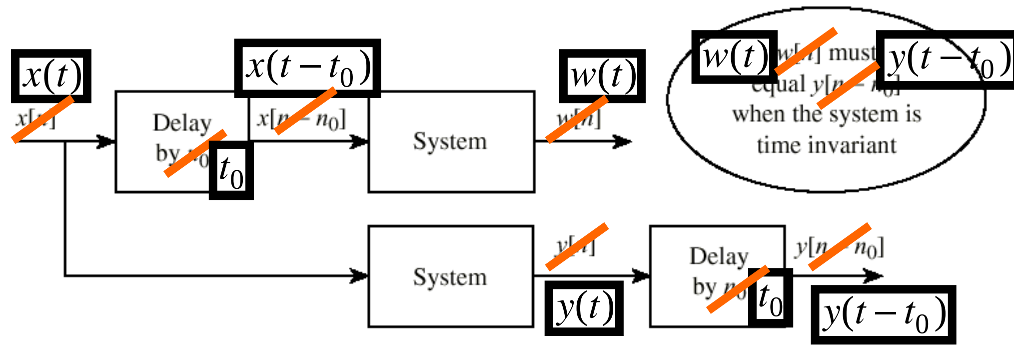
$x(t) \mapsto y(t)$

- Examples:
  - Delay $y(t) = x(t - t_d)$
  - Modulator $y(t) = [A + x(t)]\cos\omega_c t$
  - Integrator $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$

Testing for Linearity



Testing Time-Invariance



Linear and Time-Invariant (LTI) Systems

- If a continuous-time system is both linear and time-invariant, then the output ***y(t)*** is related to the input ***x(t)*** by a ***convolution integral***

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = x(t) * h(t)$$

where ***h(t)*** is the ***impulse response*** of the system.

Time Delay: $y(t) = x(t - t_d)$

- Linear

$$ax_1(t-t_d) + bx_2(t-t_d) = ay_1(t) + by_2(t)$$

- and Time-Invariant

$$\begin{aligned} w(t) &= x((t-t_0)-t_d) \\ y(t-t_0) &= x((t-t_0)-t_d) \end{aligned}$$

Integrator: $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$

- **Linear**

$$\int_{-\infty}^{t} [ax_1(\tau) + bx_2(\tau)]d\tau = ay_1(t) + by_2(t)$$

- **And Time–Invariant**

$$w(t) = \int_{-\infty}^{t} x(\tau - t_0)d\tau \qquad \text{let } \sigma = \tau - t_0$$

$$\Rightarrow \quad w(t) = \int_{-\infty}^{t-t_0} x(\sigma)d\sigma = y(t - t_0)$$

Modulator: $y(t) = [A + x(t)]\cos\omega_c t$

- ***Not*** **linear––obvious because**

$$A + ax_1(t) + bx_2(t)] \neq$$
$$[A + ax_1(t)] + [A + bx_2(t)]$$

- ***Not*** **time–invariant**

$$w(t) = [A + x(t - t_0)]\cos\omega_c t \neq y(t - t_0)$$

Linear and Time-Invariant (LTI) Systems

- **If a continuous–time system is both linear and time–invariant, then the output *y(t)* is related to the input *x(t)* by a *convolution integral***

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t - \tau)d\tau = x(t) * h(t)$$

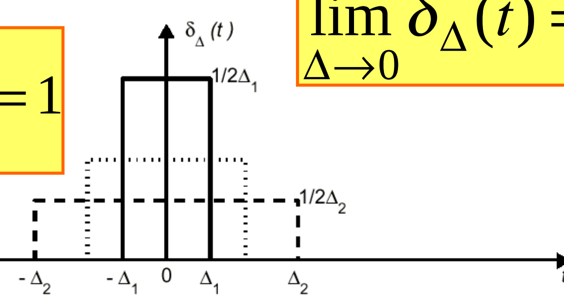
where *h(t)* is the *impulse response* of the system.

What is an Impulse?

- **A signal that is concentrated at one point.**

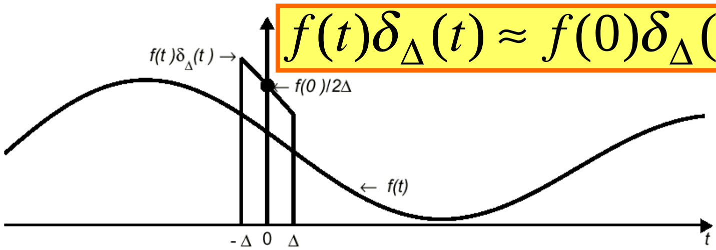
$$\int_{-\infty}^{\infty} \delta_\Delta(t)dt = 1$$

$$\lim_{\Delta \to 0} \delta_\Delta(t) = \delta(t)$$

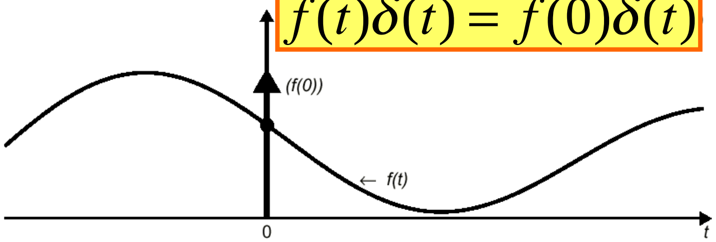


Sampling Property

$$f(t)\delta_\Delta(t) \approx f(0)\delta_\Delta(t)$$

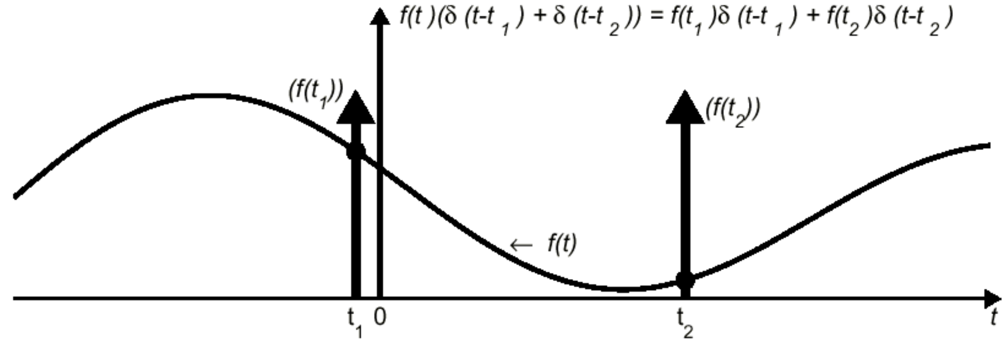


$$f(t)\delta(t) = f(0)\delta(t)$$



General Sampling Property

$$f(t)\delta(t-t_0) = f(t_0)\delta(t-t_0)$$



Integrator: $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$

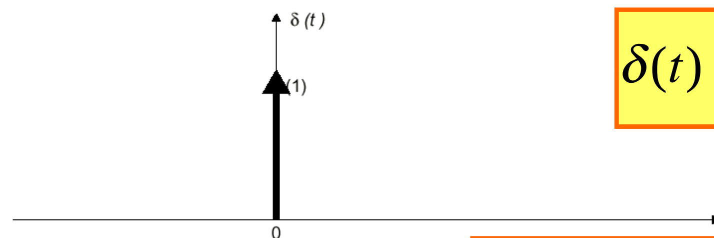
- Integrate the impulse

$$\int_{-\infty}^{t} \delta(\tau)d\tau = u(t)$$

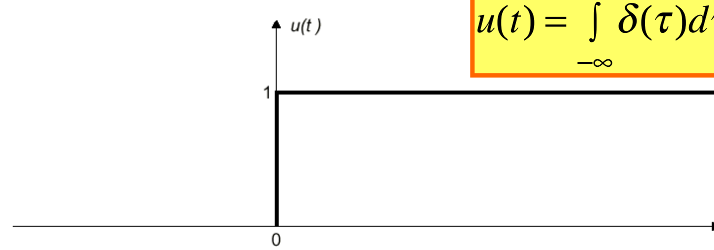
- IF t<0, we get zero
- IF t>0, we get one
  - Thus we have h(t) for the integrator

Graphical Representation

$$\delta(t) = \frac{du(t)}{dt}$$



$$u(t) = \int_{-\infty}^{t} \delta(\tau)d\tau = \begin{cases} 1 & t>0 \\ 0 & t<0 \end{cases}$$



Properties of the Impulse

$$\delta(t) = 0, \quad t \neq 0$$

Concentrated at t=0

$$f(t)\delta(t-t_0) = f(t_0)\delta(t-t_0)$$

Sampling Property

$$\int_{-\infty}^{\infty} \delta(\tau)d\tau = 1$$

Unit area

$$\int_{-\infty}^{t} \delta(\tau)d\tau = u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$

Unit step is its integral

$$\frac{du(t)}{dt} = \delta(t)$$

Derivative of unit step

Ideal Delay: $y(t) = x(t - t_d)$

- To find ***h(t),*** let ***x(t)*** be an impulse, so

$$h(t) = \delta(t - t_d)$$

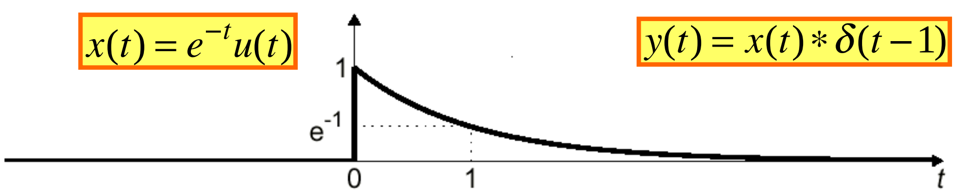
- Therefore, convolution with an impulse simply shifts ***x(t).***

$$x(t) * \delta(t - t_d) = x(t - t_d)$$

Output of Ideal Delay of 1 sec

$$x(t) = e^{-t}u(t)$$

$$y(t) = x(t) * \delta(t-1)$$



$$y(t) = x(t-1) = e^{-(t-1)}u(t-1)$$

Integrator: $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$

- To find ***h(t),*** let ***x(t)*** be an impulse, so

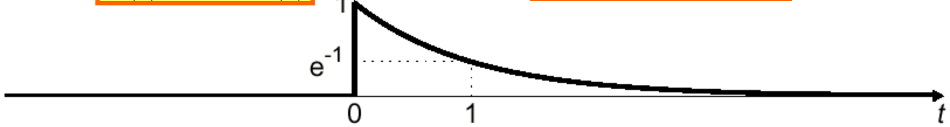
$$h(t) = \int_{-\infty}^{t} \delta(\tau)d\tau = u(t)$$

- Therefore, convolution with a unit step simply integrates ***x(t).***

$$x(t) * u(t) = \int_{-\infty}^{t} x(\tau)d\tau$$

Output of Integrator

$x(t) = e^{-t}u(t)$ $y(t) = x(t) * u(t)$



$$y(t) = \int_{-\infty}^{t} e^{-\tau}u(\tau)d\tau = \begin{cases} 0 & t < 0 \\ \int_{0}^{t} e^{-\tau}d\tau & t \geq 0 \end{cases}$$

$$= (1 - e^{-t})u(t)$$

Differentiator: $y(t) = \dfrac{dx(t)}{dt}$

- To find ***h(t),*** let ***x(t)*** be an impulse, so

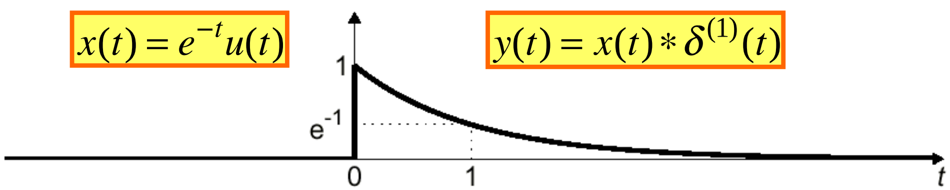
$$h(t) = \frac{d\delta(t)}{dt} = \delta^{(1)}(t)$$

- Therefore, conv. with the derivative of an impulse differentiates ***x(t).***

$$x(t) * \delta^{(1)}(t) = \frac{dx(t)}{dt}$$

Differentiator Output: $y(t) = \dfrac{dx(t)}{dt}$

$x(t) = e^{-t}u(t)$ $y(t) = x(t) * \delta^{(1)}(t)$



$$y(t) = \frac{d}{dt}\left(e^{-t}u(t)\right) = \frac{d}{dt}\left(e^{-t}\right)u(t) + e^{-t}\frac{d}{dt}(u(t))$$

$$= -e^{-t}u(t) + e^{-t}\delta(t)$$

$$= -e^{-t}u(t) + e^{-0}\delta(t) = -e^{-t}u(t) + \delta(t)$$