EE-2025 Fall-99

Lecture 19

Convolution (Continuous-Time)

8-Nov-99

Info: Web-CT, Lab, HW

- Calendar:
  - Quiz #3 is 22–Nov
- Get NEW CHAPTERS
  - PDF or Bookstore
- Prob Set #11 is due Friday, Nov. 12
- Lab #10 on Simulation
  - Then 2 more Labs

READING ASSIGNMENTS

- This Lecture:
  - Chapter 10, pp. 1020–1038
- Other Reading:
  - Recitation: Ch. 10, pp. 1020–1029
  - Next Lecture: Start reading Chapter 11

LECTURE OBJECTIVES

- Review of C–T LTI systems
- Evaluating convolutions
  - Examples
  - Impulses
- LTI Systems
  - Cascade and parallel connections
  - Stability and causality

Linear and Time-Invariant (LTI) Systems

- If a continuous–time system is both linear and time–invariant, then the output ***y(t)*** is related to the input ***x(t)*** by a ***convolution integral***

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = x(t) * h(t)$$

where ***h(t)*** is the ***impulse response*** of the system.

Convolution of Impulses, etc.

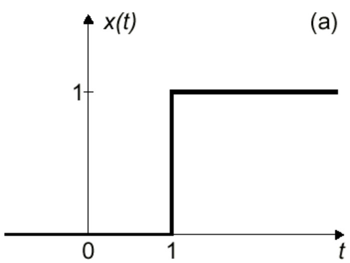
- Convolution of two impulses

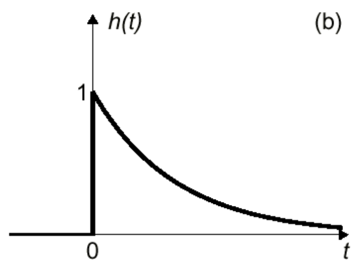
$$\delta(t-t_1) * \delta(t-t_2) = \delta(t-t_1-t_2)$$

- Convolution of step and derivative of impulse

$$u(t) * \delta^{(1)}(t) = \delta(t)$$

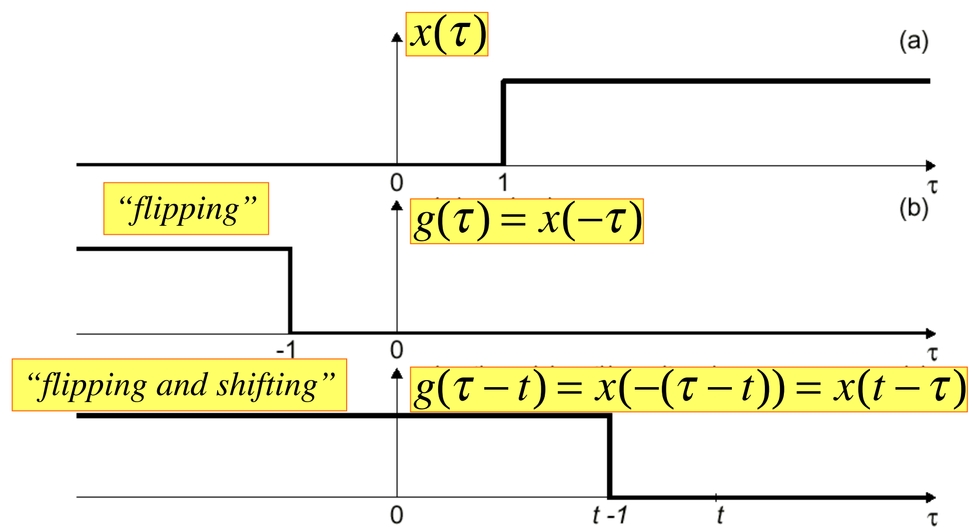
Evaluating a Convolution



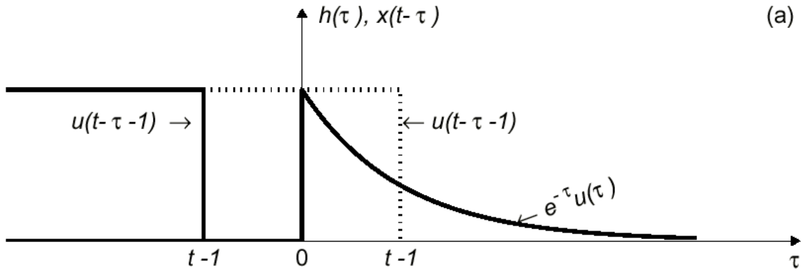


$$y(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau = h(t) * x(t)$$

"Flipping and Shifting"



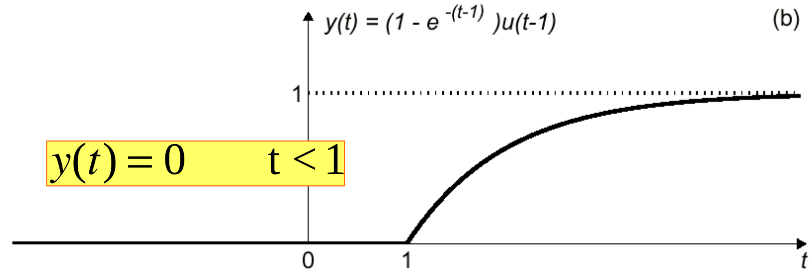
Evaluating the Integral



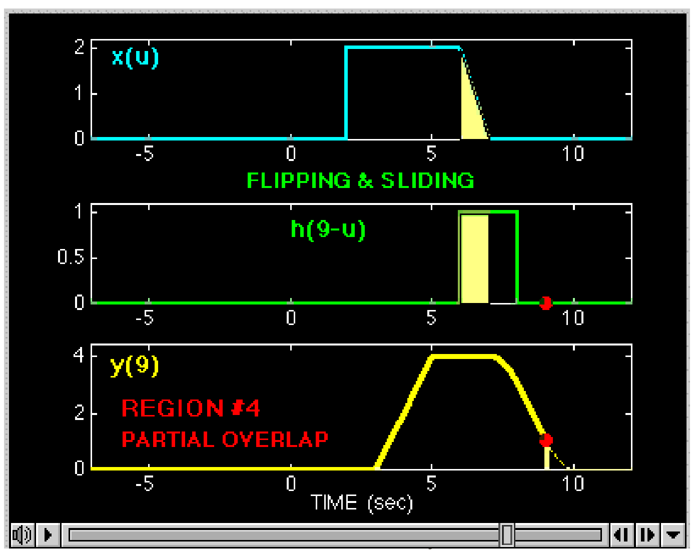
$$y(t) = 0 \qquad t-1<0$$
$$= \int_0^{t-1} e^{-\tau} d\tau \qquad t-1 \geq 0$$

Solution

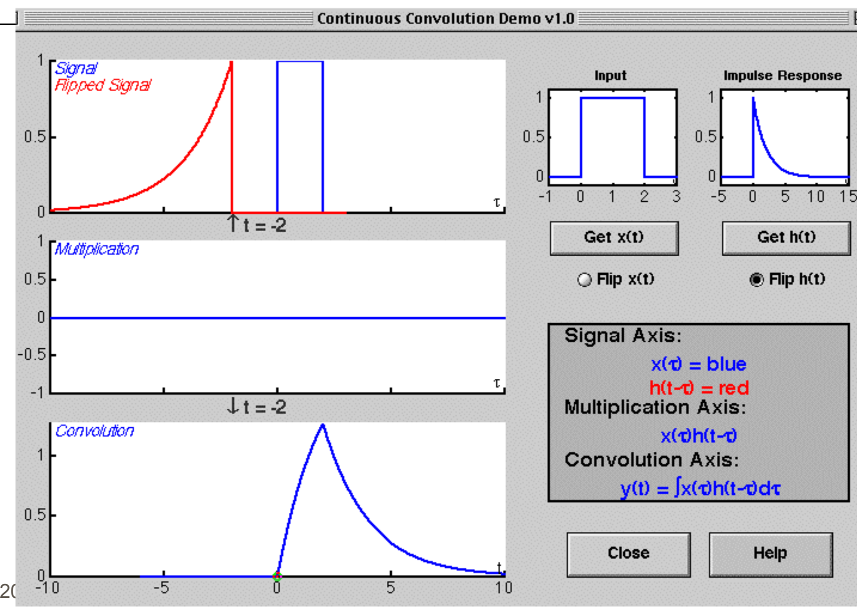
$$y(t) = \int_0^{t-1} e^{-\tau} d\tau = -e^{-\tau}\Big|_0^{t-1}$$
$$= 1-e^{-(t-1)} \qquad t \geq 1$$



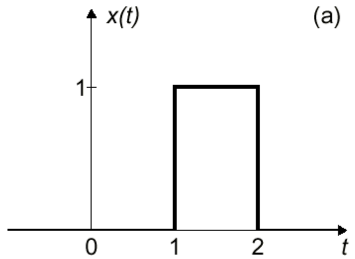
Convolution Demo

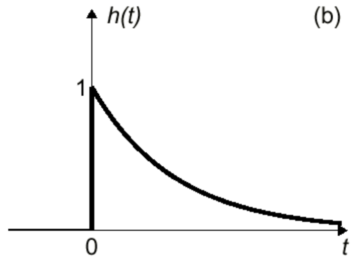


Convolution GUI



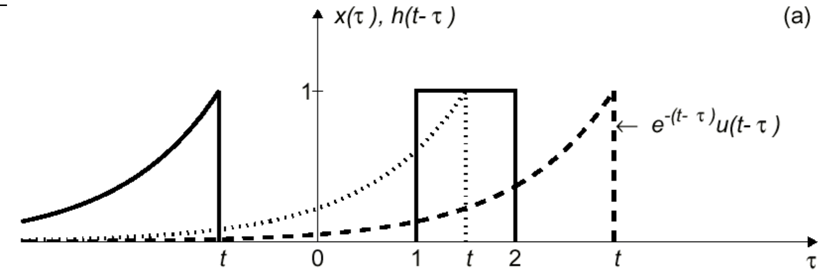
Another Convolution Example





$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = x(t) * h(t)$$

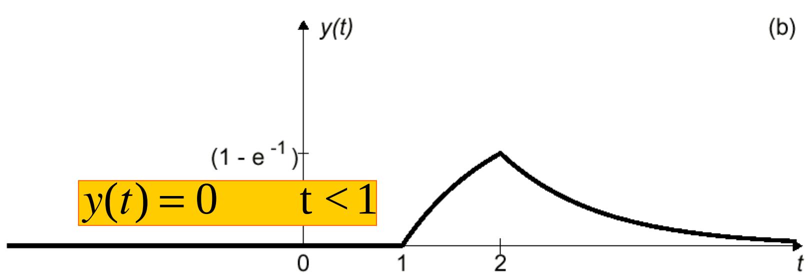
Evaluating the Integral



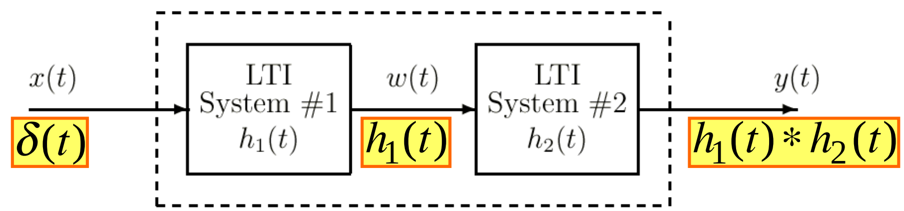
$$\begin{aligned} y(t) &= 0 & t<1 \\ &= \int_1^t e^{-(t-\tau)}d\tau & 1 \le t \le 2 \\ &= \int_1^2 e^{-(t-\tau)}d\tau & 2 \le t \end{aligned}$$

Solution

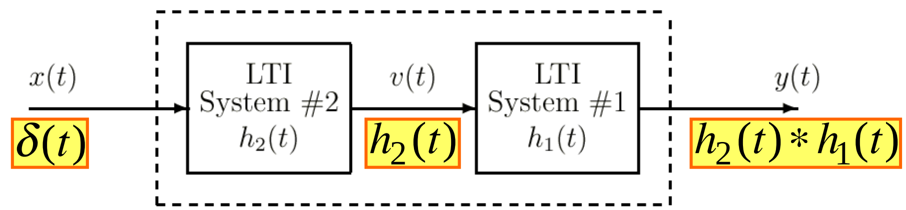
$$\begin{aligned} y(t) &= \int_1^t e^{-(t-\tau)}d\tau = e^{-(t-\tau)}\Big|_1^t = 1 - e^{-(t-1)} \quad 1 \le t \le 2 \\ &= \int_1^2 e^{-(t-\tau)}d\tau = e^{-(t-\tau)}\Big|_1^2 = e^{-(t-2)} - e^{-(t-1)} \quad 2 \le t \end{aligned}$$



Cascade of LTI Systems

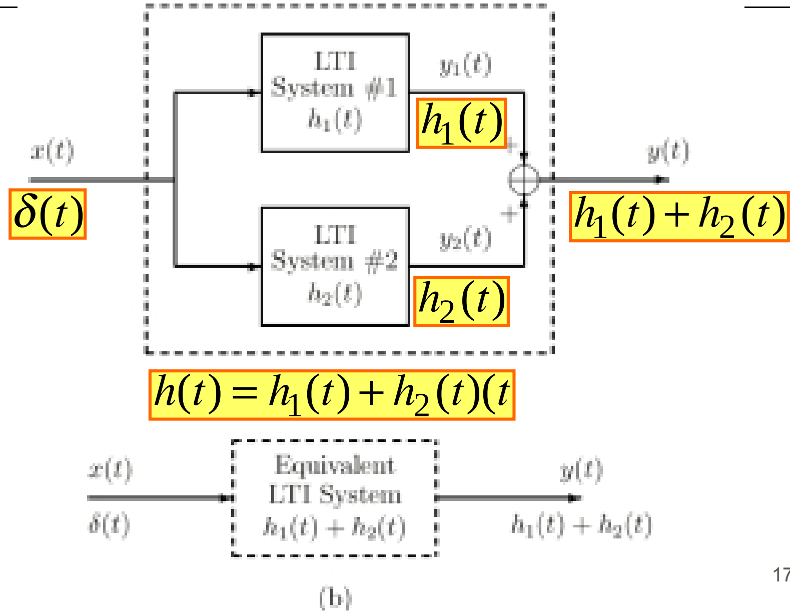


$$h(t) = h_1(t) * h_2(t) = h_2(t) * h_1(t)$$



(b)

Parallel LTI Systems



Stability

- A system is stable if every bounded input produces a bounded output.
- A continuous-time *LTI system* is stable if and only if

$$\int_{-\infty}^{\infty} |h(t)|\,dt < \infty$$

Causal Systems

- A system is causal if and only if $y(t_0)$ depends only on $x(\tau)$ for $\tau \leq t_0$.
- An LTI system is causal if and only if

$$h(t) = 0 \quad \text{for} \quad t < 0$$