

Lecture 23

Amplitude Modulation (AM)

29-Nov-99

Info: Web-CT, Lab, HW

- Calendar: Final Exams
 - 11am section-Period 9, Weds, 12/15
 - 12pm section-Period 13, Fri, 12/17
 - NO switching allowed
- Prob Set #13 - due 3-Dec
- Reading Assignment
 - Read Chapter 13 of Notes.

LECTURE OBJECTIVES

- Review of FT properties
 - Convolution <--> multiplication
 - Frequency shifting
- Sinewave Amplitude Modulation
 - AM radio
- Frequency-division multiplexing
 - FDM

The way communication systems work

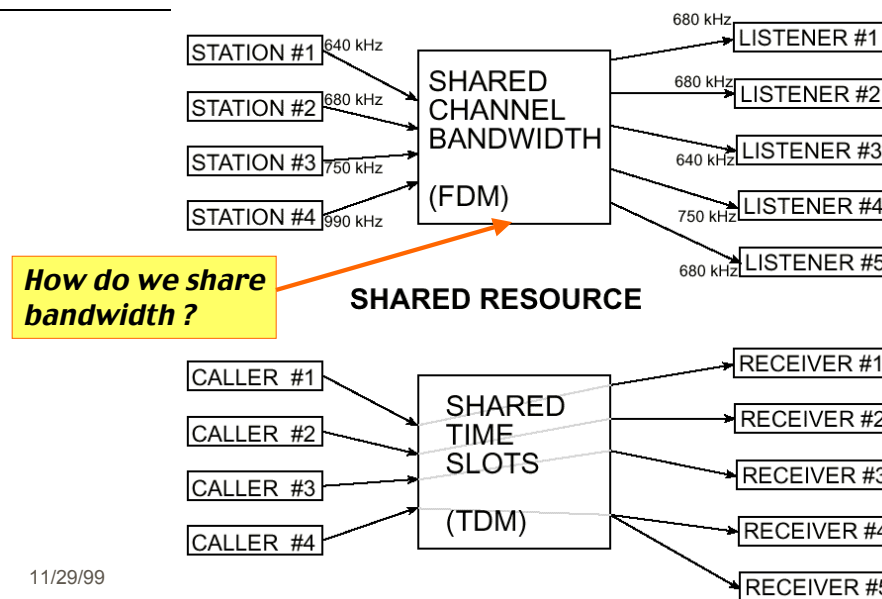


Table of Easy FT Properties

Linearity Property

$$ax_1(t) + bx_2(t) \Leftrightarrow aX_1(j\omega) + bX_2(j\omega)$$

Delay Property

$$x(t - t_d) \Leftrightarrow e^{-j\omega t_d} X(j\omega)$$

Frequency Shifting

$$x(t)e^{j\omega_0 t} \Leftrightarrow X(j(\omega - \omega_0))$$

Table of FT Properties

$$x(t) * h(t) \Leftrightarrow H(j\omega)X(j\omega)$$

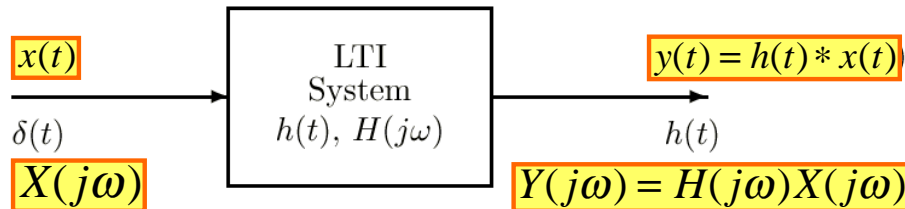
$$x(t)p(t) \Leftrightarrow \frac{1}{2\pi} X(j\omega) * P(j\omega)$$

$$x(t)e^{j\omega_0 t} \Leftrightarrow X(j(\omega - \omega_0))$$

Differentiation Property

$$\frac{dx(t)}{dt} \Leftrightarrow (j\omega)X(j\omega)$$

Convolution Property



Convolution in the time-domain

$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau)x(t - \tau)d\tau$$

corresponds to **MULTIPLICATION** in the frequency-domain $Y(j\omega) = H(j\omega)X(j\omega)$

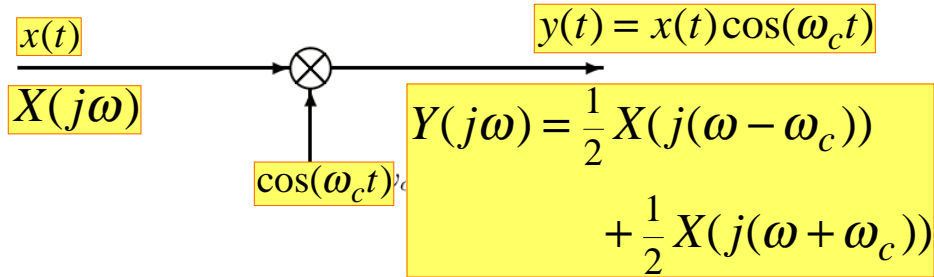
Frequency Shifting Property

$$x(t)e^{j\omega_0 t} \Leftrightarrow X(j(\omega - \omega_0))$$

$$\begin{aligned} \int_{-\infty}^{\infty} e^{-j\omega_0 t} x(t)e^{-j\omega t} dt &= \int_{-\infty}^{\infty} x(t)e^{-j(\omega - \omega_0)t} dt \\ &= X(j(\omega - \omega_0)) \end{aligned}$$

$$y(t) = \frac{\sin 7t}{\pi t} e^{j\omega_0 t} \Leftrightarrow Y(j\omega) = \begin{cases} 1 & \omega_0 - 7 < \omega < \omega_0 + 7 \\ 0 & \text{elsewhere} \end{cases}$$

Amplitude Modulator



- $x(t)$ **modulates** the amplitude of the cosine wave. The result in the frequency-domain is two shifted copies of $X(j\omega)$.

$$y(t) = x(t) \cos(\omega_c t) \Leftrightarrow$$

$$Y(j\omega) = \frac{1}{2} X(j(\omega - \omega_c)) + \frac{1}{2} X(j(\omega + \omega_c))$$

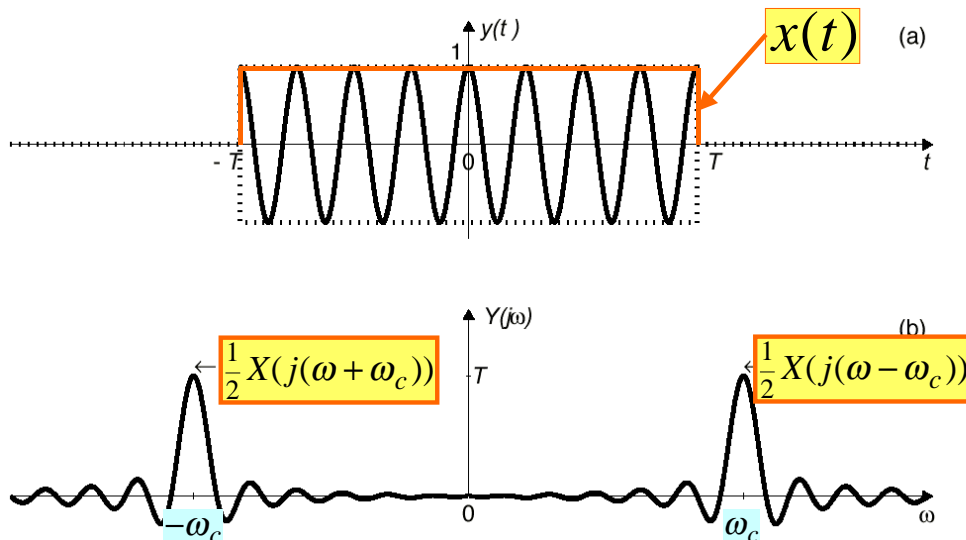
$$x(t) = \begin{cases} 1 & |t| < T \\ 0 & |t| > T \end{cases} \Leftrightarrow X(j\omega) = 2 \frac{\sin(\omega T)}{\omega}$$

$$y(t) = x(t) \cos(\omega_c t) \Leftrightarrow$$

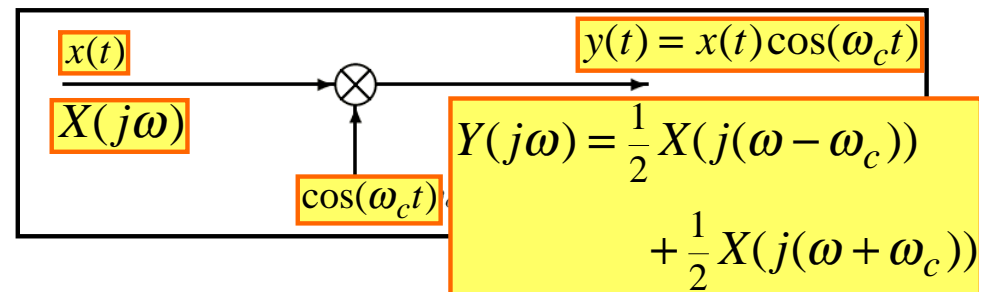
$$Y(j\omega) = \frac{\sin((\omega - \omega_c))}{(\omega - \omega_c)} + \frac{\sin((\omega + \omega_c))}{(\omega + \omega_c)}$$

$$y(t) = x(t) \cos(\omega_c t) \Leftrightarrow$$

$$Y(j\omega) = \frac{1}{2} X(j(\omega - \omega_c)) + \frac{1}{2} X(j(\omega + \omega_c))$$



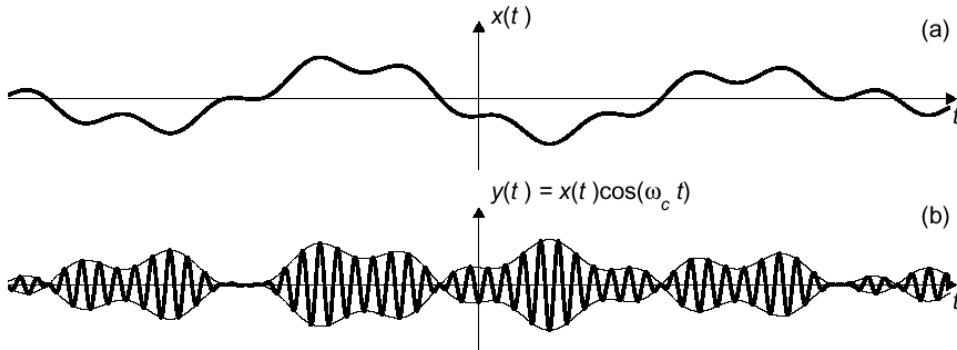
DSBAM Modulation



- If $X(j\omega) = 0$ for $|\omega| > \omega_b$ and $\omega_c > \omega_b$, the result in the frequency-domain is two shifted and scaled **exact** copies of $X(j\omega)$.

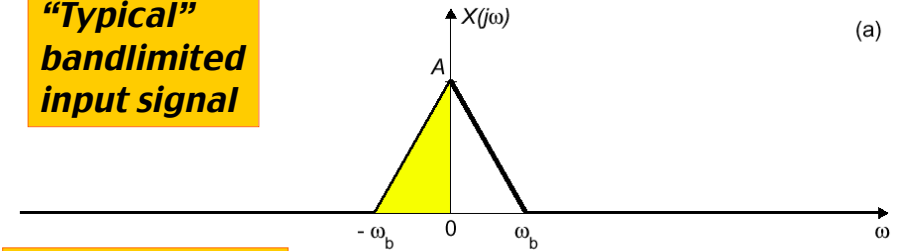
DSBAM Waveform

- In the time-domain, the envelope of the sinewave peaks follows $|x(t)|$

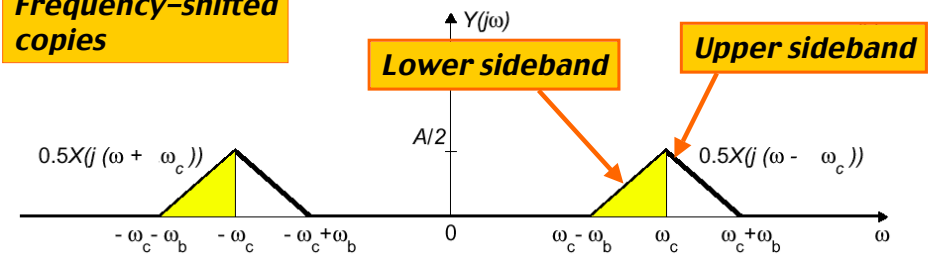


Double Sideband AM (DSBAM)

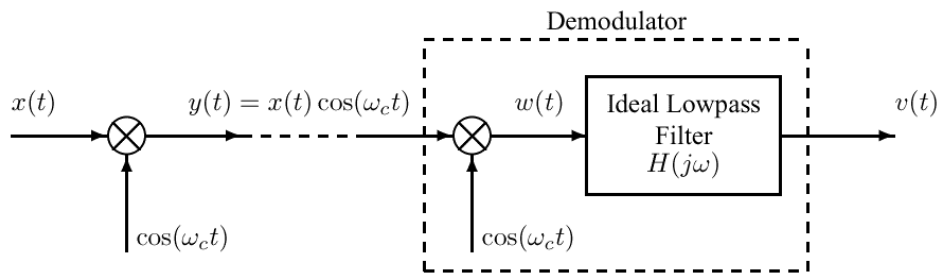
“Typical” bandlimited input signal



Frequency-shifted copies



DSBAM Demodulation

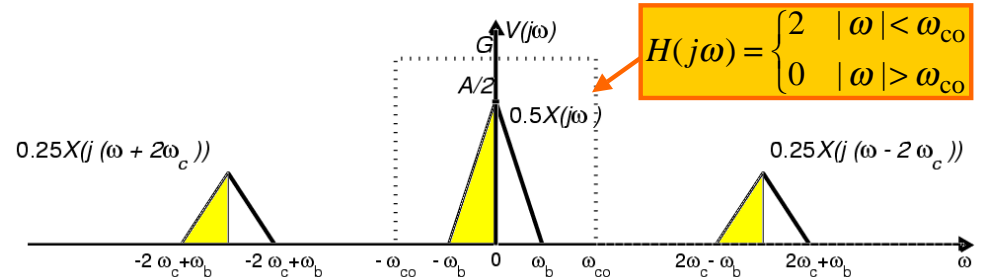
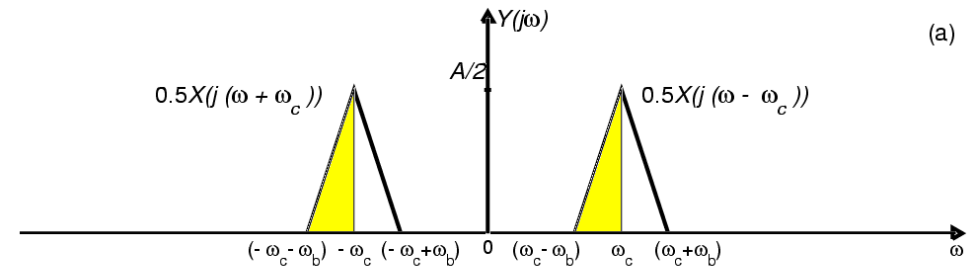


$$w(t) = x(t)[\cos(\omega_c t)]^2 = \frac{1}{2}x(t) + \frac{1}{2}x(t)\cos(2\omega_c t)$$

$$W(j\omega) = \frac{1}{2}X(j\omega) + \frac{1}{4}X(j(\omega - 2\omega_c)) + \frac{1}{4}X(j(\omega + 2\omega_c))$$

$$V(j\omega) = H(j\omega)W(j\omega)$$

DSBAM Demodulation



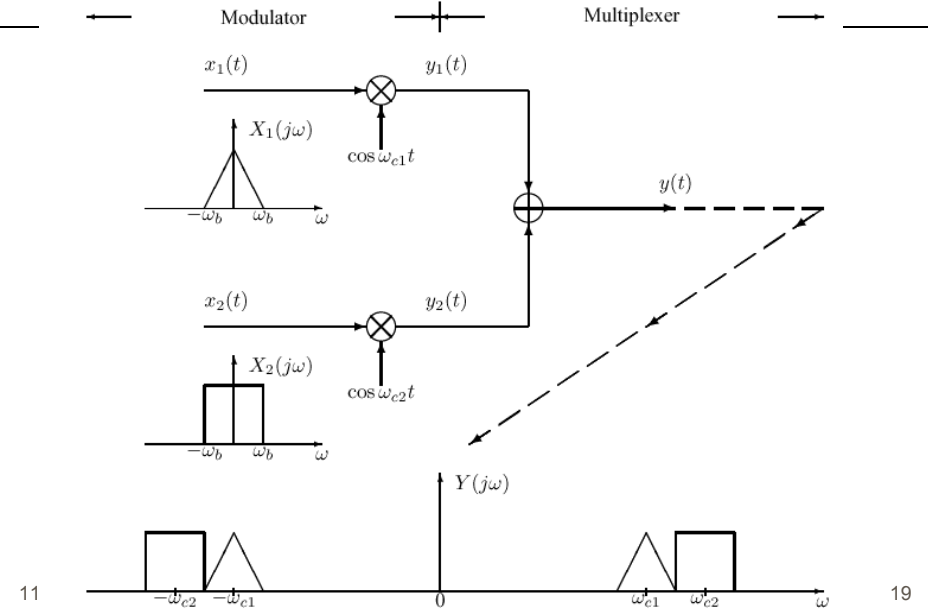
$$H(j\omega) = \begin{cases} 2 & |\omega| < \omega_{co} \\ 0 & |\omega| > \omega_{co} \end{cases}$$

$$V(j\omega) = H(j\omega)W(j\omega) = X(j\omega) \text{ if } \omega_b < \omega_{co} < 2\omega_c - \omega_b$$

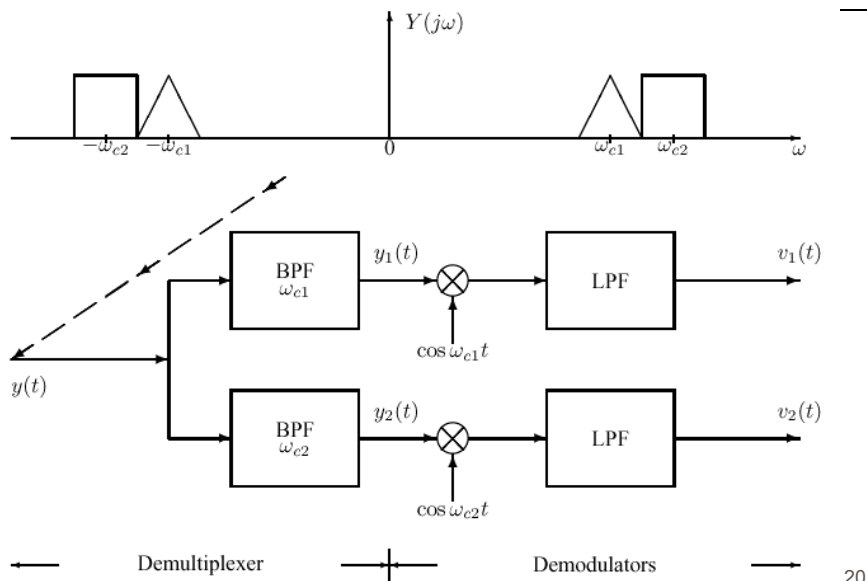
Frequency-Division Multiplexing (FDM)

- Shifting spectrum of signal to higher frequency:
 - Permits transmission of low-frequency signals with high-frequency EM waves
 - By allocating a frequency band to each signal multiple **bandlimited** signals can share the same channel
 - AM radio: 530–1620 kHz (10 kHz bands)
 - FM radio: 88.1–107.9 MHz (200 kHz bands)

FDM Block Diagram



Frequency-Division De-Mux



Bandpass Filters for De-Mux

