EE-2025 Fall-99

LECTURE #3

Phasor Addition Theorem

30-Aug-99

Web-CT Info

- Check the Bulletin Board for msgs
  - MAKE YOUR OWN POSTINGS
- Lectures are being posted
  - PDF format (4 per page)
- Quiz Dates:
  - Quiz #1 on 20–Sept (Monday)
    - Quiz #2 on 25–Oct
    - Quiz #3 on 22–Nov

Homework Info

- Prob Set #1 due FRIDAY in CLASS
  - At the beginning of class
- On–Line HW #1 ends at 11 AM Friday

- HW will be posted on Friday/Saturday
  - Due the following Friday
- Solutions will be posted on Weekend

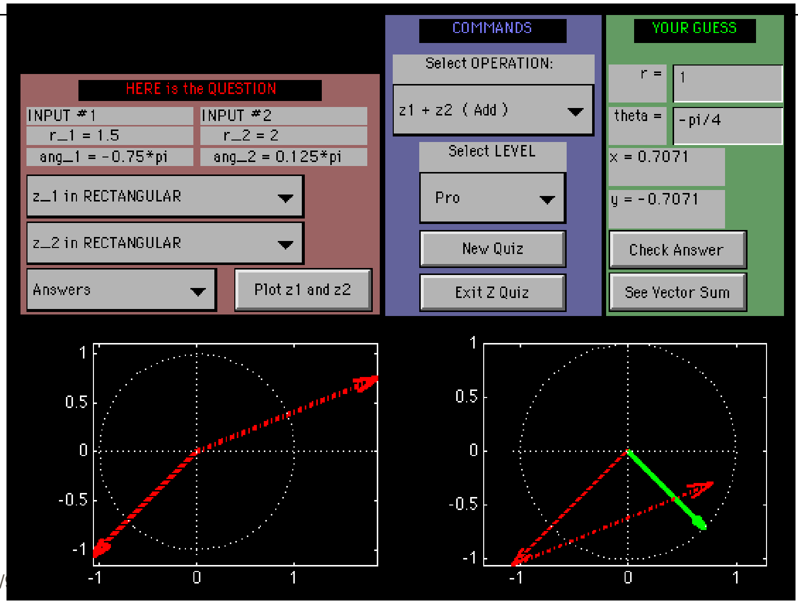
Lab Info

- NT passwd = SSN or old passwd
- MATLAB Help: M,T,Wed 6PM VL–456
- Lab #1 Report
  - Turn in during your lab time
  - Write–up sections 2 and 3
  - Include INSTRUCTOR VERIFICATION
- Lab #2 will be posted Fri/Sat

READING ASSIGNMENTS

- **This Lecture:**
  - **Chapter 2, pp. 31–43**
- **Other Reading:**
  - **Appendix A: Complex Numbers**
  - **Appendix B: MATLAB**
  - **Next Lecture: start Chapter 3**

Z DRILL (Complex Arith)



LECTURE OBJECTIVES

- **Phasors = Complex Amplitude**
  - **Add Sinusoids = Complex Addition**
  - **PHASOR ADDITION THEOREM**

$$z(t) = Xe^{j\omega t} = (Ae^{j\varphi})e^{j\omega t}$$

- **Develop the ABSTRACTION:**
  - **Complex Numbers represent Sinusoids**

AVOID Trigonometry

- **Algebra, even complex, is EASIER !!!**
- **Can you recall $\cos(\theta_1+\theta_2)$?**
- **Use the real part of $e^{j\theta_1} e^{j\theta_2}$**

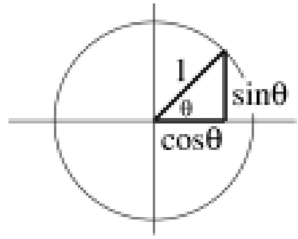
$$e^{j(\theta_1+\theta_2)} = e^{j\theta_1}e^{j\theta_2}$$

$$= (\cos\theta_1 + j\sin\theta_1)(\cos\theta_2 + j\sin\theta_2)$$

$$= \cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2 + j(\ldots)$$

Euler's FORMULA

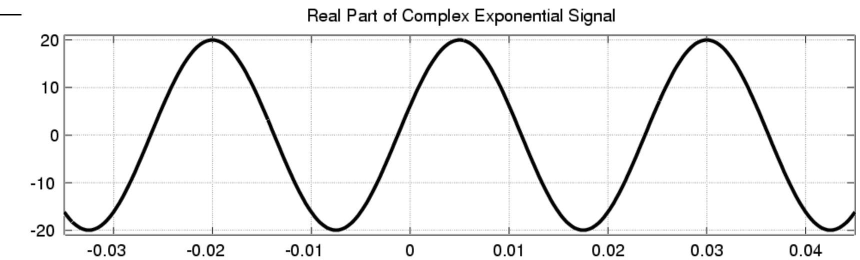
- **Complex Exponential**
  - Real part is cosine
  - Imaginary part is sine
  - Magnitude is one



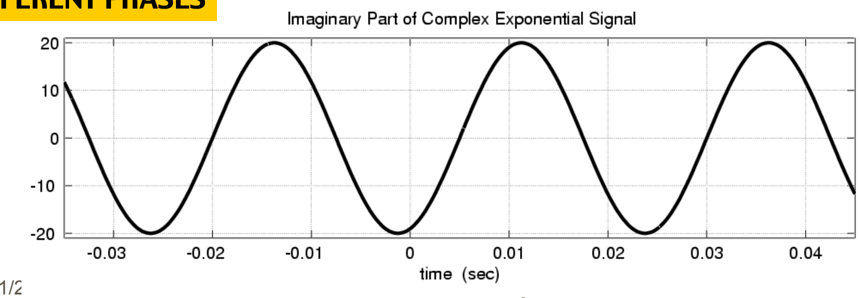
$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

$$e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$$

Real & Imaginary Part Plots



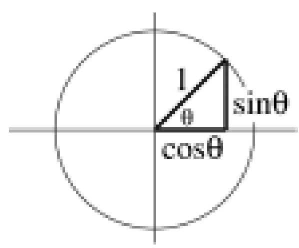
DIFFERENT PHASES



COMPLEX EXPONENTIAL

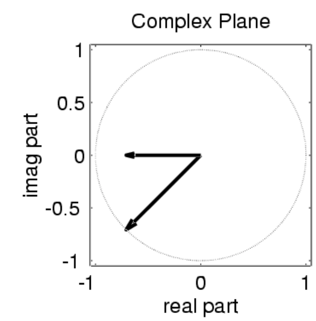
$$e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$$

- **Rotating Vector**
  - Angle changes vs. time
  - $\theta = \omega t$
  - ex: $\omega = 10\pi$
  - Rotates 0.1π in 0.01 secs

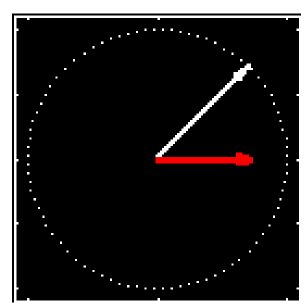


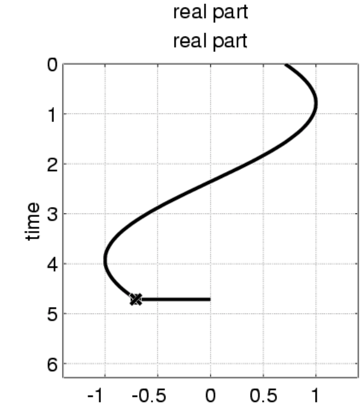
$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

Rotating Phasor



See Demo on CD-ROM Chapter 2





Cos = REAL PART

- **Real Part of Euler's:**

$$\cos(\omega t) = \Re e\{e^{j\omega t}\}$$

- **General Sinusoid**

$$x(t) = A\cos(\omega t + \varphi)$$

- **So, cosine is real part of what?**

$$A\cos(\omega t + \varphi) = \Re e\{Ae^{j(\omega t+\varphi)}\}$$

$$= \Re e\{Ae^{j\varphi}e^{j\omega t}\}$$

COMPLEX AMPLITUDE

- **General Sinusoid**

$$x(t) = A\cos(\omega t + \varphi) = \Re e\{Ae^{j\varphi}e^{j\omega t}\}$$

- **Complex Exponential**

$$z(t) = Xe^{j\omega t} \qquad X = Ae^{j\varphi}$$

- **Sinusoid is REAL PART of $Xe^{j\omega t}$**

$$x(t) = \Re e\{z(t)\} = \Re e\{Xe^{j\omega t}\}$$

WANT to ADD SINUSOIDS

- **ALL SINUSOIDS HAVE SAME FREQUENCY**
- **HOW to GET {Amp,Phase} of RESULT ?**

$$x_1(t) = 1.7\cos(2\pi(10)t + 70\pi/180)$$

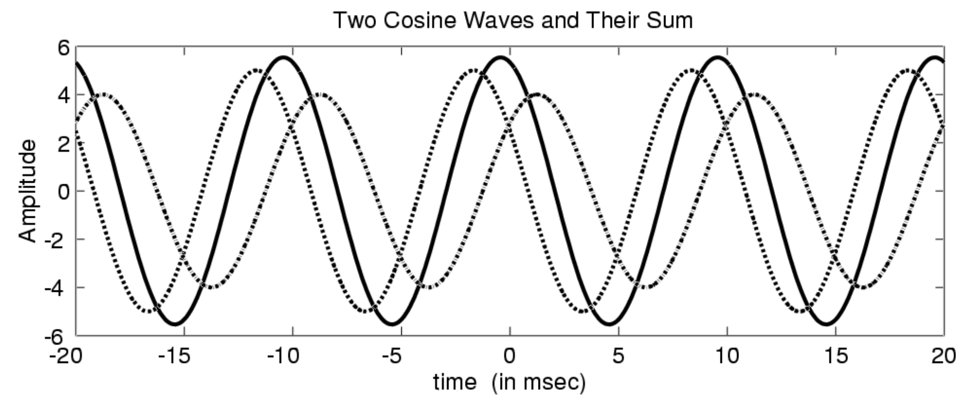
$$x_2(t) = 1.9\cos(2\pi(10)t + 200\pi/180)$$

$$x_3(t) = x_1(t) + x_2(t)$$

$$= 1.532\cos(2\pi(10)t + 141.79\pi/180)$$

ADD SINUSOIDS

- **Sum Sinusoid has same Frequency**



PHASOR ADDITION RULE

$$x(t) = \sum_{k=1}^{N} A_k \cos(\omega_0 t + \phi_k)$$

$$= A\cos(\omega_0 t + \phi)$$

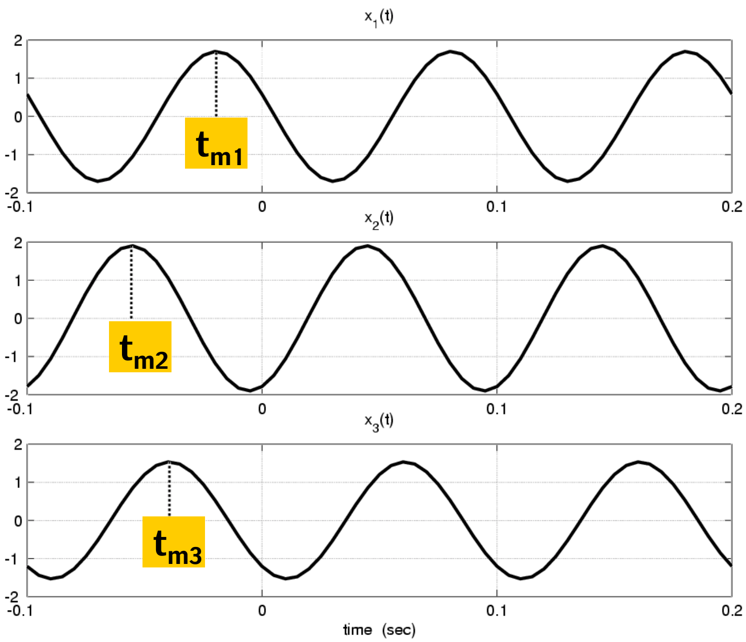
Get the new complex amplitude by complex addition

$$\sum_{k=1}^{N} A_k e^{j\phi_k} = Ae^{j\phi}$$

Phasor Addition Proof

$$\begin{aligned}\sum_{k=1}^{N} A_k \cos(\omega_0 t + \phi_k) &= \sum_{k=1}^{N} \Re e\left\{A_k e^{j(\omega_0 t + \phi_k)}\right\} \\ &= \Re e\left\{\sum_{k=1}^{N} A_k e^{j\phi_k} e^{j\omega_0 t}\right\} \\ &= \Re e\left\{\left(\sum_{k=1}^{N} A_k e^{j\phi_k}\right) e^{j\omega_0 t}\right\} \\ &= \Re e\left\{\left(Ae^{j\phi}\right) e^{j\omega_0 t}\right\} = A\cos(\omega_0 t + \phi)\end{aligned}$$

ADD SINUSOIDS EXAMPLE

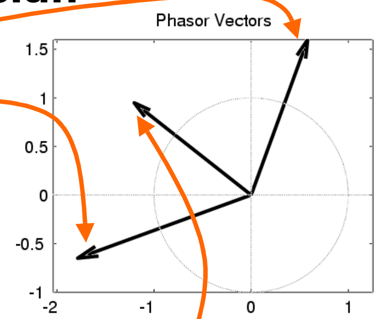


Convert Time-Shift to Phase

- Measure **peak times**:
  - t_{m1}=−0.0194, t_{m2}=−0.0556, t_{m3}=−0.0394
- Convert to **phase** (T=0.1)
  - $\phi_1 = -2\pi(t_{m1}/T) = 70\pi/180$,
  - $\phi_2 = 200\pi/180$
- Amplitudes
  - A_1=1.7, A_2=1.9, A_3=1.532

Phasor Add: Numerical

- Convert Polar to Cartesian
  - $X_1 = 0.5814 + j1.597$
  - $X_2 = -1.785 - j0.6498$
  - $X_3 = -1.204 + j0.9476$

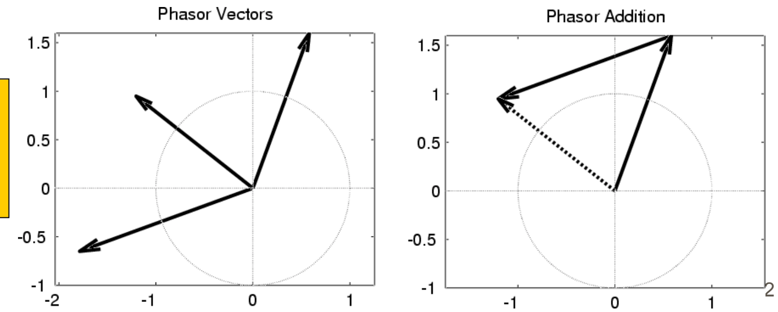


- Convert back to Polar
  - $X_3 = 1.532$ at angle $141.79\pi/180$
  - This is the sum

ADD SINUSOIDS

$$x_1(t) = 1.7\cos(2\pi(10)t + 70\pi/180)$$
$$x_2(t) = 1.9\cos(2\pi(10)t + 200\pi/180)$$
$$x_3(t) = x_1(t) + x_2(t)$$
$$= 1.532\cos(2\pi(10)t + 141.79\pi/180)$$

VECTOR (PHASOR) ADD



POP QUIZ

- ADD THESE 2 SINUSOIDS:

$$x_1(t) = \cos(77\pi t)$$
$$x_2(t) = \sqrt{3}\cos(77\pi t + 0.5\pi)$$

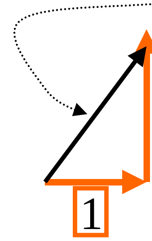
- COMPLEX ADDITION:

$$1 + \sqrt{3}e^{j0.5\pi}$$

POP QUIZ (answer)

- COMPLEX ADDITION:

$$1 + j\sqrt{3} = 2e^{j\pi/3}$$



$$j\sqrt{3} = \sqrt{3}e^{j0.5\pi}$$

- CONVERT back to cosine form:

$$x_3(t) = 2\cos(77\pi t + \tfrac{\pi}{3})$$