GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 1999
Problem Set #6

SOLUTION

Date: 8 October 99 (FRIDAY)

PROBLEM 6.1*:

A linear time-invariant filter is described by the difference equation

$$y[n] = -x[n] + 2x[n-1] - x[n-2]$$

(a) Obtain an expression for the frequency response of this system.

For this problem $\{b_k\} = \{-1, 2, -1\}$ and $M = 2$. The frequency response is computed as follows:

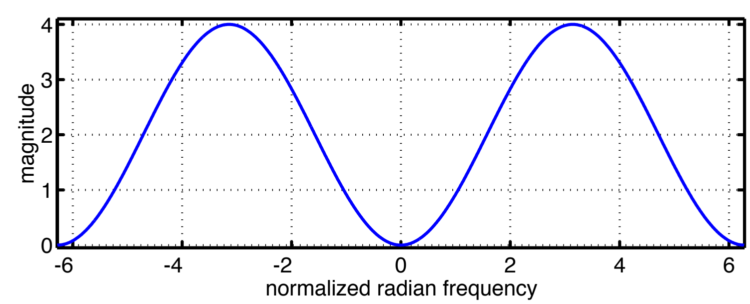
$$\begin{aligned}\mathcal{H}(\hat{\omega}) &= \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k} \\ &= -1 + 2e^{-j\hat{\omega}} - e^{-j2\hat{\omega}}\end{aligned}$$

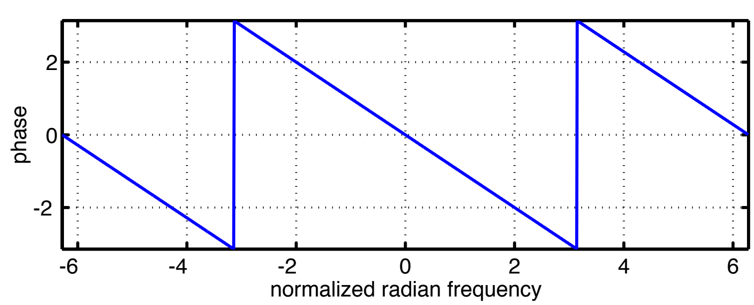
(b) Make a sketch of the frequency response (magnitude and phase) as a function of frequency.

Using symmetry to simplify the expression for $\mathcal{H}(\hat{\omega})$:

$$\begin{aligned}\mathcal{H}(\hat{\omega}) &= -1 + 2e^{-j\hat{\omega}} - e^{-j2\hat{\omega}} \\ &= e^{-j\hat{\omega}}\left(-e^{j\hat{\omega}} + 2 - e^{-j\hat{\omega}}\right) \\ &= e^{-j\hat{\omega}}\left(2 - 2\cos\hat{\omega}\right)\end{aligned}$$

Because $(2 - 2\cos\hat{\omega}) \geq 0$ for all frequencies, the magnitude of the frequency response is $|\mathcal{H}(\hat{\omega})| = (2 - 2\cos\hat{\omega})$, and the phase is $\angle\mathcal{H}(\hat{\omega}) = -\hat{\omega}$ (in the range $-\pi < \hat{\omega} \leq \pi$).





(c) What is the output if the input is $x[n] = 5 + 5\cos(0.5\pi n + \pi/2)$?

First convert $x[n]$ to a sum of complex exponentials as follows:

$$\begin{aligned} x[n] &= 5 + 5\cos\left(\frac{\pi}{2}n + \frac{\pi}{2}\right) \\ &= 5e^{j0n} + \frac{5}{2}e^{j\pi/2}e^{j(\pi/2)n} + \frac{5}{2}e^{-j\pi/2}e^{-j(\pi/2)n} \end{aligned}$$

Thus, there are three input frequencies: $\hat{\omega} = 0, \pi/2, -\pi/2$. We can evaluate $\mathcal{H}(\hat{\omega})$ at each of these frequencies to get

$$\begin{aligned} \mathcal{H}(0) &= e^{-j0}\left(2 - 2\cos(0)\right) = 0 \\ \mathcal{H}(\pi/2) &= e^{-j\pi/2}\left(2 - 2\cos(\pi/2)\right) = 2e^{-j\pi/2} \\ \mathcal{H}(-\pi/2) &= e^{j\pi/2}\left(2 - 2\cos(-\pi/2)\right) = 2e^{j\pi/2} \end{aligned}$$

Thus, the resulting output is

$$\begin{aligned} y[n] &= \mathcal{H}(0)\left(5e^{j0n}\right) + \mathcal{H}(\pi/2)\left(\frac{5}{2}e^{j\pi/2}e^{j(\pi/2)n}\right) + \mathcal{H}(-\pi/2)\left(\frac{5}{2}e^{-j\pi/2}e^{-j(\pi/2)n}\right) \\ &= 5e^{j(\pi/2)n} + 5e^{-j(\pi/2)n} \\ &= 10\cos\left(\frac{\pi}{2}n\right) \end{aligned}$$

(d) What is the output if the input is the *unit impulse sequence* $x[n] = \delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0. \end{cases}$

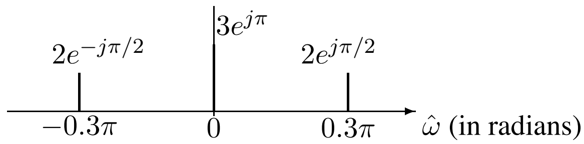
$$y[n] = \delta[n] * h[n] = h[n] = -\delta[n] + 2\delta[n-1] - \delta[n-2]$$

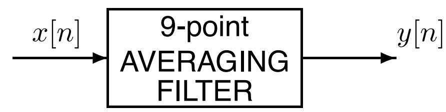
(e) What is the output if the input is the *unit step sequence* $x[n] = u[n] = \begin{cases} 0 & n < 0 \\ 1 & n \geq 0. \end{cases}$

$$y[n] = u[n] * h[n] = \begin{cases} 0 & n < 0 \\ -1 & n = 0 \\ 1 & n = 1 \\ 0 & n > 1 \end{cases}$$

PROBLEM 6.2:

A discrete-time signal $x[n]$ has the two-sided spectrum representation shown below.





(a) Write an equation for $x[n]$. Make sure to express $x[n]$ as a real-valued signal.

$$\begin{aligned} x[n] &= 2e^{-j\pi/2}e^{-j0.3\pi n} + 3e^{j\pi} + 2e^{j\pi/2}e^{j0.3\pi n} \\ &= 2e^{j\pi/2}e^{j0.3\pi n} + 2e^{-j\pi/2}e^{-j0.3\pi n} - 3 \\ &= 4\cos(3\pi n/10 + \pi/2) - 3 \end{aligned}$$

(b) Determine the formula for the output signal $y[n]$.

The frequency response for a nine-point running-average filter is defined as follows:

$$\mathcal{H}(\hat{\omega}) = \frac{1}{9}\sum_{k=0}^{8} e^{-jk\hat{\omega}} = \mathcal{D}_9(\hat{\omega})e^{-j4\hat{\omega}} = \frac{\sin(9\hat{\omega}/2)}{9\sin(\hat{\omega}/2)}e^{-j4\hat{\omega}}$$

where $\mathcal{D}_9(\hat{\omega})$ is the Dirichlet Function with $L = 9$.

The frequency response at the input frequencies ($\hat{\omega} = 0$ and $\hat{\omega} = 0.3\pi$) is:

$$\begin{aligned} \mathcal{H}(0) &= \frac{1}{9}\sum_{k=0}^{8} e^{-j0\hat{\omega}} = \frac{1}{9}(9) = 1 \\ \mathcal{H}(0.3\pi) &= \frac{\sin(1.35\pi)}{9\sin(0.15\pi)}e^{-j1.2\pi} \approx -0.218e^{-j1.2\pi} = +0.218e^{-j0.2\pi} \\ \mathcal{H}(-0.3\pi) &= \mathcal{H}^*(0.3\pi) \;=\; 0.218e^{j0.2\pi} \qquad \text{(conjugate: } \mathcal{H}(-\hat{\omega}) = \mathcal{H}^*(\hat{\omega})\text{)} \end{aligned}$$

Using these frequency-response values, we can compute the output as follows:

$$\begin{aligned} y[n] &= \mathcal{H}(0.3\pi)2e^{j0.5\pi}e^{j0.3\pi n} + \mathcal{H}(-0.3\pi)2e^{-j0.5\pi}e^{-j0.3\pi n} - \mathcal{H}(0)3 \\ &\approx (2\cdot 0.218)\left(e^{j0.5\pi - j0.2\pi}e^{j0.3\pi n} + e^{-j0.5\pi + j0.2\pi}e^{-j0.3\pi n}\right) - 1\cdot 3 \\ &= 0.872\cos(0.3\pi n + 0.3\pi) - 3 \end{aligned}$$

PROBLEM 6.3*:

A discrete-time system is defined by the input/output relation

$$y[n] = x[n+1] + x[n] + x[n-1]. \quad (1)$$

(a) Determine whether or not the system defined by Equation (1) is (i) linear; (ii) time-invariant; (iii) causal.

Linear: yes

Definition: A system is linear if $x_1[n] \to y_1[n]$ and $x_2[n] \to y_2[n]$, then

$$x[n] = \alpha x_1[n] + \beta x_2[n] \to y[n] = \alpha y_1[n] + \beta y_2[n]$$

For this system,

$$\begin{aligned} y[n] &= (\alpha x_1[n+1] + \beta x_2[n+1]) + (\alpha x_1[n] + \beta x_2[n]) + (\alpha x_1[n-1] + \beta x_2[n-1]) \\ &= \alpha(x_1[n+1] + x_1[n] + x_1[n-1]) + \beta(x_2[n+1] + x_2[n] + x_2[n-1]) \\ &= \alpha y_1[n] + \beta y_2[n] \end{aligned}$$

and thus the system is linear.

Time invariant: yes

Definition: A system is time invariant if $x[n-n_0] \to y[n-n_0]$.

For this system,

$$\begin{aligned} y[n-n_0] &= x[(n+1)-n_0] + x[n-n_0] + x[(n-1)-n_0] \\ &= x[(n-n_0)+1] + x[n-n_0] + x[(n-n_0)-1] \end{aligned}$$

where the latter statement is the result of the system when the input is $x[n-n_0]$, and thus the system is time invariant.

Causal: no

This system is not causal because it requires future input information in the term $x[n+1]$.

(b) For the system of Equation (1), determine the output $y_1[n]$ when the input is

$$x_1[n] = 2\cos(0.75\pi n) = e^{j0.75\pi n} + e^{-j0.75\pi n}.$$

First, we can compute the frequency response of the LTI system in general as

$$\begin{aligned} \mathcal{H}(\hat{\omega}) &= e^{j\hat{\omega}} + 1 + e^{-j\hat{\omega}} \\ &= 1 + 2\cos\hat{\omega} \end{aligned}$$

and for the specific frequencies in this problem as

$$\begin{aligned} \mathcal{H}(0.75\pi) &= 1 - \sqrt{2} = (\sqrt{2}-1)e^{j\pi} \approx 0.414e^{j\pi} \\ \mathcal{H}(-0.75\pi) &= 1 - \sqrt{2} = (\sqrt{2}-1)e^{-j\pi} \approx 0.414e^{-j\pi} \end{aligned}$$

This results in an output as follows:

$$\begin{aligned} y_1[n] &= \mathcal{H}(0.75\pi)e^{j0.75\pi n} + \mathcal{H}(-0.75\pi)e^{-j0.75\pi n} \\ &= (\sqrt{2}-1)\left(e^{j\pi}e^{j0.75\pi n} + e^{-j\pi}e^{-j0.75\pi n}\right) \\ &= (\sqrt{2}-1)\cdot 2\cos(0.75\pi n + \pi) \\ &\approx 0.828\cos(0.75\pi n + \pi) \end{aligned}$$

(c) For the system of Equation (1), determine the output $y_2[n]$ when the input is

$$x_2[n] = 4 + 4\cos(0.75\pi(n-1)).$$

First, we convert the input into a sum of complex exponentials:

$$\begin{aligned} x_2[n] &= 4 + 4\cos(0.75\pi n - 0.75\pi) \\ &= 4 + 2e^{-j0.75\pi}e^{j0.75\pi n} + 2e^{j0.75\pi}e^{-j0.75\pi n} \end{aligned}$$

For this problem, we must add the additional frequency of 0, which has the frequency response of $\mathcal{H}(0) = 3$. The resulting output is

$$\begin{aligned} y_2[n] &= 3\cdot 4 + (\sqrt{2}-1)e^{j\pi}\cdot 2e^{-j0.75\pi}e^{j0.75\pi n} + (\sqrt{2}-1)e^{-j\pi}\cdot 2e^{j0.75\pi}e^{-j0.75\pi n} \\ &= 12 + 2(\sqrt{2}-1)\left(e^{j0.25\pi}e^{j0.75\pi n} + e^{-j0.25\pi}e^{-j0.75\pi n}\right) \\ &= 12 + 4(\sqrt{2}-1)\cos(0.75\pi n + 0.25\pi) \\ &\approx 12 + 1.6569\cos(0.75\pi n + \pi/4) \end{aligned}$$

A second discrete-time system is defined by the input/output relation

$$y[n] = (x[n])^2. \tag{2}$$

(d) Determine whether or not the system defined by Equation (2) is (i) linear; (ii) time-invariant; (iii) causal.

Linear: no

For this system,

$$\begin{aligned} y[n] &= (\alpha x_1[n] + \beta x_2[n])^2 = \alpha^2 (x_1[n])^2 + 2\alpha\beta(x_1[n]x_2[n]) + \beta^2 (x_2[n])^2 \\ &\neq \alpha (x_1[n])^2 + \beta (x_2[n])^2 \\ &= \alpha y_1[n] + \beta y_2[n] \end{aligned}$$

Because $y[n] \neq \alpha y_1[n] + \beta y_2[n]$, the system is *not* linear.

Time invariant: yes

If we use the delayed input, $x[n-n_0]$, as the input to the system, the output is equal to $(x[n-n_0])^2$. If $x[n]$ is the input, then $y[n] = (x[n])^2$, and delaying the output gives $y[n-n_0] = (x[n-n_0])^2$. Since delaying the input gives the same result as delaying the output, the system is time invariant.

Causal: yes

This system is causal because it does not require future input information (it only requires present input information).

(e) For the system of Equation (2), determine the output $y_1[n]$ when the input is

$$x_1[n] = 2\cos(0.75\pi n) = e^{j0.75\pi n} + e^{-j0.75\pi n}$$

The output can be computed as follows:

$$\begin{aligned} y_1[n] &= (x_1[n])^2 \\ &= \left(e^{j0.75\pi n} + e^{-j0.75\pi n}\right)^2 \\ &= e^{j1.5\pi n} + 2 + e^{-j1.5\pi n} \end{aligned}$$

Note that the frequencies $\hat{\omega} = 1.5\pi$ and $\hat{\omega} = -1.5\pi$ do not fall in the range $-\pi < \hat{\omega} \leq \pi$, so they have aliases within that range at $\hat{\omega} = -0.5\pi$ and $\hat{\omega} = 0.5\pi$, and the equation can be rewritten as follows:

$$\begin{aligned} y_1[n] &= e^{-j0.5\pi n} + 2 + e^{j0.5\pi n} \\ &= 2 + 2\cos(\pi n/2) \end{aligned}$$

(f) For the system of Equation (2), determine the output $y_2[n]$ when the input is

$$x_2[n] = 4 + 4\cos(0.75\pi(n-1)).$$

As in Part (c), $x_2[n]$ can be rewritten as follows:

$$x_2[n] = 4 + 2e^{-j0.75\pi} e^{j0.75\pi n} + 2e^{j0.75\pi} e^{-j0.75\pi n}$$

From this, we can compute $y_2[n]$ as follows:

$$\begin{aligned} y_2[n] &= \left(4 + 2e^{-j0.75\pi}e^{j0.75\pi n} + 2e^{j0.75\pi}e^{-j0.75\pi n}\right)^2 \\ &= 24 + 16e^{-j0.75\pi}e^{j0.75\pi n} + 16e^{j0.75\pi}e^{-j0.75\pi n} + 4e^{-j1.5\pi}e^{j1.5\pi n} + 4e^{j1.5\pi}e^{-j1.5\pi n} \\ &= 24 + 16e^{-j0.75\pi}e^{j0.75\pi n} + 16e^{j0.75\pi}e^{-j0.75\pi n} + 4e^{j0.5\pi}e^{-j0.5\pi n} + 4e^{-j0.5\pi}e^{j0.5\pi n} \\ &= 24 + 32\cos(0.75\pi n - 0.75\pi) + 8\cos(0.5\pi n - 0.5\pi) \end{aligned}$$

(g) For which system does superposition hold?

Superposition is equivalent to linearity. Thus, it holds in the first system and not in the second.

(h) For which system does the output contain frequencies that are not present in the input signal?

The output of the second system includes a frequency of $\hat{\omega} = 0.5\pi$, which was not found in the input.

(i) Which system can cause aliasing of sinusoidal components of the input?

The second system had an alias of $\hat{\omega} = 1.5\pi$ to $\hat{\omega} = -0.5\pi$.

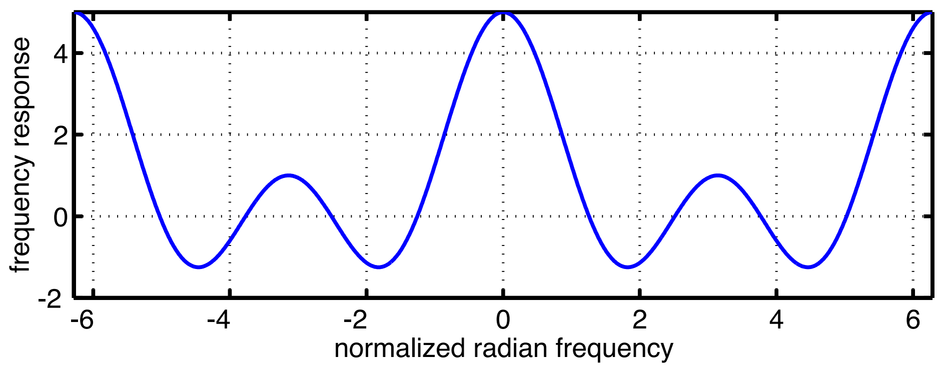
PROBLEM 6.4*:

For the *modified Dirichlet* function:

$$\tilde{\mathcal{D}}(\hat{\omega}, 5) = \frac{\sin(2.5\hat{\omega})}{\sin(\frac{1}{2}\hat{\omega})}$$

(a) Make a plot of $\tilde{\mathcal{D}}(\hat{\omega}, 5)$ over the range $-2\pi \leq \hat{\omega} \leq +2\pi$. Label all the zero crossings.

The zero crossings occur when $\sin(2.5\hat{\omega}) = 0$ and $\sin(\frac{1}{2}\hat{\omega}) \neq 0$. In the range, $-2\pi \leq \hat{\omega} \leq +2\pi$, these conditions are true at $\hat{\omega} = \pm 2\pi/5, \pm 4\pi/5, \pm 6\pi/5, \pm 8\pi/5$.



(b) Determine the period of $\tilde{\mathcal{D}}(\hat{\omega}, 5)$. Is it equal to 2π; why, or why not?

For both the numerator, $\sin(2.5\hat{\omega})$, and the denominator, $\sin(\frac{1}{2}\hat{\omega})$, of the function, the following properties hold true:

$$f(\hat{\omega} + 2\pi k) = \begin{cases} -f(\hat{\omega}) & k \text{ odd} \\ f(\hat{\omega}) & k \text{ even} \end{cases}$$

Thus, for all k, the signs cancel,

$$\frac{\sin(2.5(\hat{\omega} + 2\pi k))}{\sin(\frac{1}{2}(\hat{\omega} + 2\pi k))} = \frac{\sin(2.5\hat{\omega} + \pi k)}{\sin(\frac{1}{2}\hat{\omega} + \pi k)} = \frac{(-1)^k \sin(2.5\hat{\omega})}{(-1)^k \sin(\frac{1}{2}\hat{\omega})} = \frac{\sin(2.5\hat{\omega})}{\sin(\frac{1}{2}\hat{\omega})}$$

and the function is periodic with a period of 2π.

(c) Find the maximum value of the function.

From the figure above, we see that the maximum value of the function occurs at $\hat{\omega} = 2\pi k$, where k is an integer. At these frequencies, the numerator and denominator of the function are both equal to 0. Thus, we can use L'Hopital's Rule to determine the value:

$$\lim_{\hat{\omega} \to 2\pi k} \tilde{\mathcal{D}}(\hat{\omega}, 5) = \lim_{\hat{\omega} \to 2\pi k} \frac{\sin(2.5\hat{\omega})}{\sin(\frac{1}{2}\hat{\omega})} = \left.\frac{2.5\cos(2.5\hat{\omega})}{\frac{1}{2}\cos(\frac{1}{2}\hat{\omega})}\right|_{\hat{\omega}=2\pi k} = \frac{\pm 2.5}{\pm 0.5} = 5$$

PROBLEM 6.5*:

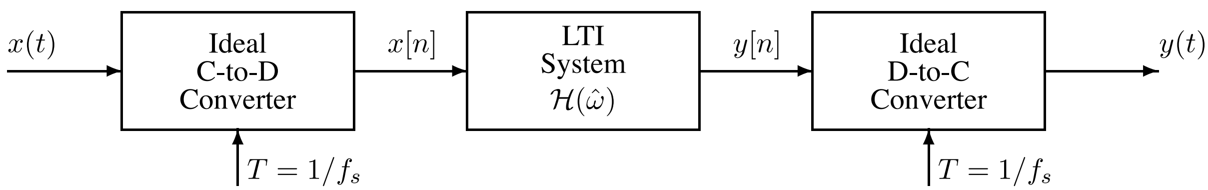
The input to the C-to-D converter in the figure below is

$$x(t) = 3 + 4\cos(2000\pi t) + 5\cos(4000\pi t - 2\pi/3)$$

The frequency response for the digital filter (LTI system) is

$$\mathcal{H}(\hat{\omega}) = \frac{\sin(2.5\hat{\omega})}{\sin(\frac{1}{2}\hat{\omega})} e^{-j2\hat{\omega}}$$

If $f_s = 10000$ samples/second, determine an expression for $y(t)$, the output of the D-to-C converter.



For $f_s = 10000$ samples/second, the output of the ideal C-to-D converter is

$$\begin{aligned} x[n] &= x(n/f_s) = 3 + 4\cos(2000\pi(n/10000)) + 5\cos(4000\pi(n/10000) - 2\pi/3) \\ &= 3 + 4\cos(0.2\pi n) + 5\cos(0.4\pi n - 2\pi/3) \\ &= 3 + 2e^{j0.2\pi n} + 2e^{-j0.2\pi n} + 2.5e^{-j2\pi/3}e^{j0.4\pi n} + 2.5e^{j2\pi/3}e^{-j0.4\pi n} \end{aligned}$$

The frequency response at each of these frequencies is

$$\begin{aligned} \mathcal{H}(0) &= 5 \\ \mathcal{H}(0.2\pi) &= \frac{\sin(0.5\pi)}{\sin(0.1\pi)} e^{-j0.4\pi} \approx 3.24e^{-j0.4\pi} \\ \mathcal{H}(-0.2\pi) &\approx 3.24e^{j0.4\pi} \\ \mathcal{H}(0.4\pi) &= \frac{\sin(\pi)}{\sin(0.2\pi)} e^{-j0.8\pi} = 0 \\ \mathcal{H}(-0.4\pi) &= 0 \end{aligned}$$

The output of the LTI system is thus

$$\begin{aligned} y[n] &= \mathcal{H}(0)3 + \mathcal{H}(0.2\pi)2e^{j0.2\pi n} + \mathcal{H}(-0.2\pi)2e^{-j0.2\pi n} + \\ &\qquad \mathcal{H}(0.4\pi)2.5e^{-j2\pi/3}e^{j0.4\pi n} + \mathcal{H}(-0.4\pi)2.5e^{j2\pi/3}e^{-j0.4\pi n} \\ &\approx 15 + 6.48e^{-j0.4\pi}e^{j0.2\pi n} + 6.48e^{j0.4\pi}e^{-j0.2\pi n} \\ &= 15 + 12.96\cos(0.2\pi n - 0.4\pi) \end{aligned}$$

Finally, if the output sampling rate is $f_s = 10000$ samples/second, the output of the ideal D-to-C converter is obtained by replacing n in $y[n]$ with $f_s t$

$$\begin{aligned} y(t) = y[f_s t] &\approx 15 + 12.96\cos(0.2\pi[10000t] - 2\pi/5) \\ &= 15 + 12.96\cos(2000\pi t - 2\pi/5) \end{aligned}$$

PROBLEM 6.6*:

The frequency response of a linear time-invariant filter is given by the formula

$$\mathcal{H}(\hat{\omega}) = (1 - e^{-j\hat{\omega}})(1 - e^{j\pi/3}e^{-j\hat{\omega}})(1 - e^{-j\pi/3}e^{-j\hat{\omega}}) \tag{3}$$

(a) Write the difference equation that gives the relation between the input $x[n]$ and the output $y[n]$.

First rearrange the frequency response as follows:

$$\begin{aligned}
\mathcal{H}(\hat{\omega}) &= (1 - e^{-j\hat{\omega}})(1 - e^{j\pi/3}e^{-j\hat{\omega}})(1 - e^{-j\pi/3}e^{-j\hat{\omega}}) \\
&= (1 - e^{-j\hat{\omega}})\left[1 - (e^{j\pi/3} + e^{-j\pi/3})e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}\right] \\
&= (1 - e^{-j\hat{\omega}})\left[1 - (1)e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}\right] \\
&= 1 - 2e^{-j\hat{\omega}} + 2e^{-j2\hat{\omega}} - e^{-j3\hat{\omega}}
\end{aligned}$$

From this equation, we can derive the filter coefficients: $\{b_k\} = \{1, -2, 2, -1\}$. Thus, the output of the filter is given by the following difference equation:

$$y[n] = x[n] - 2x[n-1] + 2x[n-2] - x[n-3]$$

(b) What is the output if the input is $x[n] = \delta[n]$?

When the input to the filter is the unit impulse sequence, the output is unit impulse response:

$$h[n] = \delta[n] - 2\delta[n-1] + 2\delta[n-2] - \delta[n-3]$$

(c) If the input is of the form $x[n] = Ae^{j\phi}e^{j\hat{\omega}n}$, for what values of $-\pi \le \hat{\omega} \le \pi$ will $y[n] = 0$ for all n?

For inputs of this form, the output of the filter is zero for all n when the frequency response of zero, i.e., when $\mathcal{H}(\hat{\omega}) = 0$ at a particular frequency. From Equation (3), the frequency response is zero when one of the factors is zero, i.e., when any ne of the following conditions is true:

$$\begin{aligned}
(1 - e^{-j\hat{\omega}}) &= 0 \\
(1 - e^{j\pi/3}e^{-j\hat{\omega}}) &= 0 \\
(1 - e^{-j\pi/3}e^{-j\hat{\omega}}) &= 0
\end{aligned}$$

These conditions are true when $\hat{\omega} = 0$, $\hat{\omega} = \pi/3$, and $\hat{\omega} = \pi/3$, respectively. For example, we can solve the middle one:

$$\begin{aligned}
(1 - e^{j\pi/3}e^{-j\hat{\omega}}) &= 0 \\
e^{j\hat{\omega}} - e^{j\pi/3} &= 0 \\
e^{j\hat{\omega}} &= e^{j\pi/3} \qquad \Rightarrow \quad \hat{\omega} = \pi/3
\end{aligned}$$

(d) The frequency response in Equation (3) is written as a product of factors suggesting that it could be implemented as a cascade of several systems. By suitably grouping the factors and multiplying them together, obtain a representation as the cascade of *two* systems each of which has only *real* filter coefficients. Give the frequency responses and impulse responses of the two systems and draw a block diagram of the cascade system.

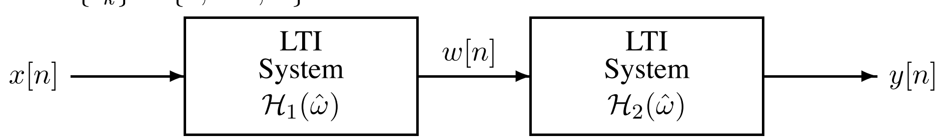
In the derivation in Part (a), we saw that

$$\mathcal{H}(\hat{\omega}) = (1 - e^{-j\hat{\omega}})(1 - e^{-j\hat{\omega}} + e^{-j2\hat{\omega}})$$

This system can be implemented by the cascade of two systems with frequency responses and impulse responses as follows:

$$\begin{aligned} \mathcal{H}_1(\hat{\omega}) &= 1 - e^{-j\hat{\omega}} \\ h_1[n] &= \delta[n] - \delta[n-1] \\ \mathcal{H}_2(\hat{\omega}) &= 1 - e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} \\ h_2[n] &= \delta[n] - \delta[n-1] + \delta[n-2] \end{aligned}$$

because the filter coefficients of the first system are $\{b_k\} = \{1,\ -1\}$, and the filter coefficients of the second system are $\{b_k\} = \{1,\ -1,\ 1\}$.



Here is the detailed FIR filter structure with all the multipliers, adders and delays to implement the cascade of the two difference equations:

$$\begin{aligned} w[n] &= x[n] - x[n-1] \\ y[n] &= w[n] - w[n-1] + w[n-2] \end{aligned}$$