GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 1999
Problem Set #10

Assigned: 29 October 99
Due Date: 5 November 99 (FRIDAY)

Quiz #3 will be given on Monday, November 22 in your regular class time.
Reading: In *DSP First*, Chapter 8 on *IIR Filters.*

$\Longrightarrow$ The five (5) **STARRED** problems will have to be turned in for grading.

Next week a solution will be posted. Some similar problems solutions can be found on the CD-ROM and in old homeworks, especially the "unstarred" problems.

PROBLEM 10.1*:

Determine the z-transforms of the following sequences:

(a) $x_a[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3] + \delta[n-4]$.
Express your answer as a polynomial in z^{-1}.

(b) $x_b[n] = u[n] - u[n-5]$.
Express your answer as a ratio of polynomials in z^{-1}.

(c) $x_c[n] = (0.8)^n u[n] + (-0.8)^n u[n]$.
Express your answer as: (1) a sum of rational functions; (2) a ratio of polynomials in z^{-1}; and (3) a product of factors of the form $(1 - az^{-1})$.

(d) $x_d[n] = 2(0.8)^n \cos(0.5\pi n) u[n]$.
Express your answer as: (1) a sum of rational functions; (2) a ratio of polynomials in z^{-1}; and (3) a product of factors of the form $(1 - az^{-1})$.

PROBLEM 10.2*:

Determine the inverse z-transforms of the following:

(a) $H_a(z) = 1 + 2z^{-2} + 4z^{-4} - 2z^{-6} - z^{-8}$.

(b) $H_b(z) = \dfrac{1 + z^{-2}}{1 - 0.5z^{-1}}$.

(c) $H_c(z) = \dfrac{2}{1 - 0.4z^{-1}} - \dfrac{1}{1 + 0.8z^{-1}}$.

(d) $H_d(z) = \dfrac{1 + 2z^{-1}}{1 - 0.4z^{-1} + 0.32z^{-2}}$.

PROBLEM 10.3*:

Work Problem 8.15 in *DSP First*.

PROBLEM 10.4*:

Work Problem 8.16 in *DSP First*.

PROBLEM 10.5*:

An LTI system has the following system function:

$$H(z) = \frac{1 + z^{-2}}{1 - 0.5z^{-1}}.$$

If you can answer the following questions, you can feel confident that you understand a lot about IIR discrete-time systems.

(a) Plot the poles and zeros of $H(z)$ in the z-plane.

(b) Determine the difference equation that is satisfied by the general input $x[n]$ and the corresponding output $y[n]$ of the system.

(c) Use z-transforms to determine the impulse response $h[n]$ of the system; i.e., the output of the system when the input is $x[n] = \delta[n]$.

(d) Determine an expression for the frequency response $H(e^{j\hat{\omega}})$ of the system.

(e) Use the frequency response function to determine the output $y_1[n]$ of the system when the input is

$$x_1[n] = 2\cos(0.5\pi n) \qquad -\infty < n < \infty.$$

(f) Use the z-transform to determine the output $y_2[n]$ when the input is

$$x_2[n] = 2\cos(0.5\pi n)u[n] = \begin{cases} 2\cos(0.5\pi n) & n \geq 0 \\ 0 & n < 0. \end{cases}$$