GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 1999
Problem Set #11

Assigned: 5 November
Due Date: 12 November 99 (FRIDAY)

Quiz #3 will be given on Monday, November 22 in your regular class time.

Reading: Read Chapter 10 in Notes.

$\Longrightarrow$ The five (5) **STARRED** problems will have to be turned in for grading. The other problems are all from EE2201, Spring 1999, Problem Set #1. Try working these problems as practice.

Next week a solution will be posted. Some similar problems solutions can be found on the CD-ROM and in old homeworks, especially the "unstarred" problems.

PROBLEM 11.1*:

A linear time-invariant system has system function

$$H(z) = \frac{0.0795(1+z^{-1}+z^{-2}+z^{-3})}{1-1.556z^{-1}+1.272z^{-2}-0.398z^{-3}} \quad (1)$$

$$= \frac{0.0795(1+z^{-1})(1-e^{j0.5\pi}z^{-1})(1-e^{-j0.5\pi}z^{-1})}{(1-0.556z^{-1})(1-0.846e^{j0.3\pi}z^{-1})(1-0.846e^{-j0.3\pi}z^{-1})} \quad (2)$$

$$= -0.2 + \frac{A}{1-0.556z^{-1}} + \frac{0.17e^{j0.96\pi}}{1-0.846e^{j0.3\pi}z^{-1}} + \frac{0.17e^{-j0.96\pi}}{1-0.846e^{-j0.3\pi}z^{-1}} \quad (3)$$

(a) Use equation (1) to determine the difference equation that relates the input $x[n]$ to the output $y[n]$ for this system.

(b) Starting with equation(2), use the partial fraction expansion method to show that $A = 0.62$ in equation (3).

(c) Determine the impulse response of this system by finding the inverse z-transform of equation (3). Express your answer in the form

$$h[n] = A_0\delta[n] + A_1a^nu[n] + A_2r^n\cos(\hat{\omega}_0n+\phi)u[n].$$

(d) Use equation (2) to plot the pole and zero locations for this system.

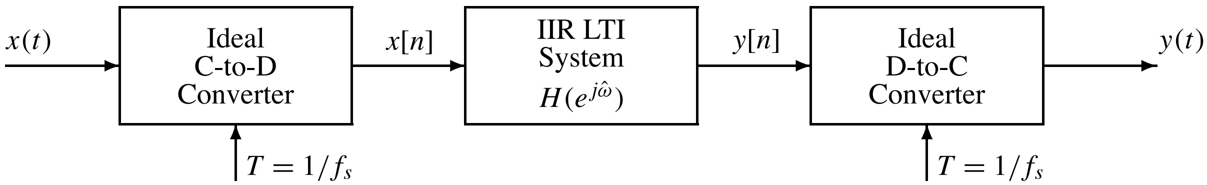
(e) The frequency response of the system can be computed in MATLAB using the following statements:

```
omegahat = -pi:(pi/200):pi;
aa =
bb =
HH=freqz(bb,aa,omegahat);
```

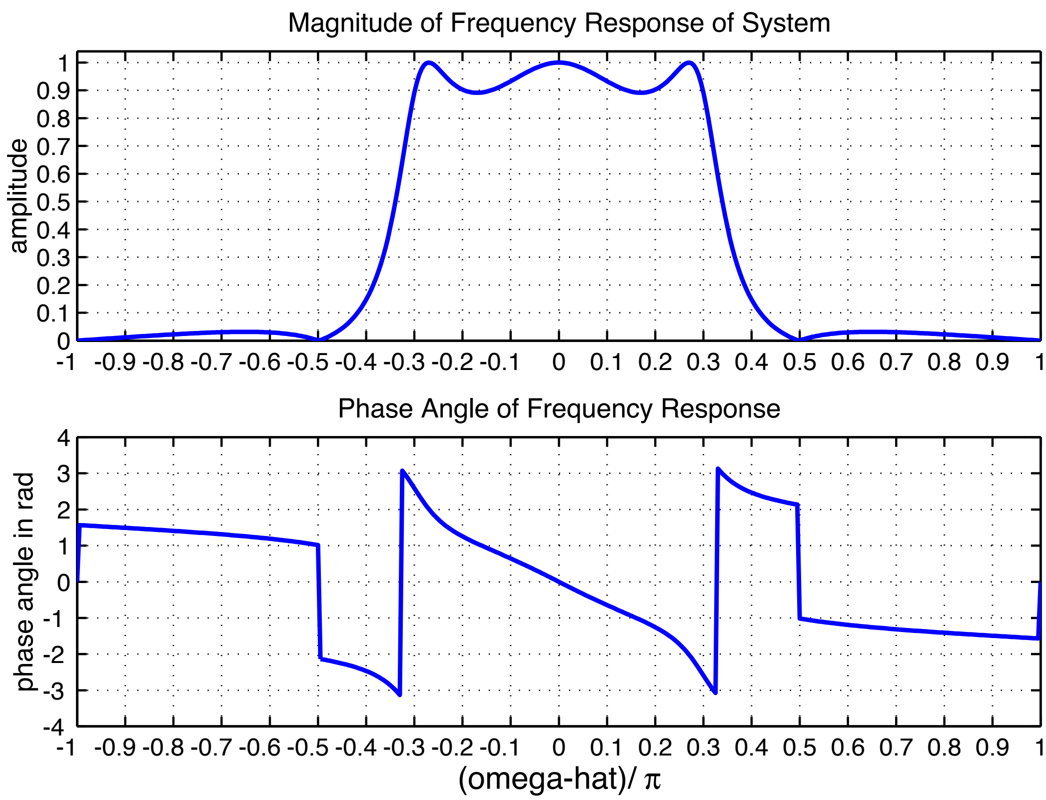
Determine the coefficient vectors aa and bb.

PROBLEM 11.2*:

Consider the following system for filtering continuous-time signals using an IIR digital filter.



The frequency response of the IIR system, which is the IIR system of Problem 11.1, is plotted in the following figure:



If the sampling rate is $f_s = 10000$ samples/second, and the input to the C-to-D converter is

$$x(t) = 10 + 10\cos(2000\pi t) + 10\cos(5000\pi t - 2\pi/3),$$

use the above frequency response plots to determine an expression for $y(t)$, the output of the D-to-C converter.

PROBLEM 11.3*:

A continuous-time system is defined by the input/output relation

$$y(t) = \int_{t-2}^{t-1} x(\tau) d\tau.$$

(a) Show that this is a linear time-invariant system.

(b) Determine the impulse response, $h(t)$, of this system. Plot it.

(c) Is this a stable system? Is it a causal system?

(d) Use convolution to determine the output of the system when the input is

$$x(t) = u(t) - u(t-1) = \begin{cases} 1 & 0 \le t < 1 \\ 0 & \text{elsewhere.} \end{cases}$$

Plot your answer.

(e) Use convolution to determine the output when the input is

$$x(t) = e^{-2t} u(t).$$

PROBLEM 11.4*:

A linear time-invariant system has impulse response:

$$h(t) = \begin{cases} e^t & -1 < t < 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Plot $h(\tau)$ and $h(t-\tau)$ as a functions of τ for $t = -2, 0,$and 2.

(b) Is the system stable? Justify your answer.

(c) Is the system causal? Justify your answer.

(d) Find the output $y(t)$ when the input is $x(t) = \delta(t-2)$.

(e) Use convolution to determine the output $y(t)$ when the input is $x(t) = \begin{cases} e^{-t} & 0 < t < 3 \\ 0 & \text{otherwise.} \end{cases}$

PROBLEM 11.5:

This is Problem 1.2 of Problem Set #1, EE2201, Spring 1999.

The impulse response of an LTI continuous-time system is such that $h(t) = 0$ for $t \le T_1$ and for $t \ge T_2$. By drawing appropriate figures as recommended for evaluating convolution integrals, show that if $x(t) = 0$ for $t \le T_3$ and for $t \ge T_4$ then $y(t) = x(t) * h(t) = 0$ for $t \le T_5$ and for $t \ge T_6$. In the process of proving this result you should obtain expressions for T_5 and T_6 in terms of T_1, T_2, T_3, and T_4.

PROBLEM 11.6:

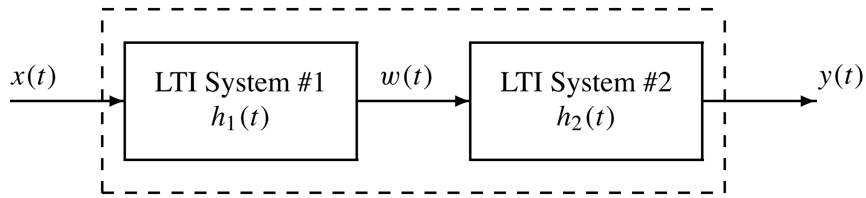
This is Problem 1.4 of Problem Set #1, EE2201, Spring 1999.
A linear time-invariant system has impulse response

$$h(t) = \begin{cases} e^t & 2 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$

The input to this system is $x(t) = u(t-1)$. Find and plot the output $y(t)$ for $-\infty < t < \infty$.

PROBLEM 11.7:

This is Problem 1.5 of Problem Set #1, EE2201, Spring 1999.



The first system is described by the input/output relation

$$w(t) = \frac{dx(t)}{dt}$$

and the second system has impulse response

$$h_2(t) = u(t-5) - u(t-10)$$

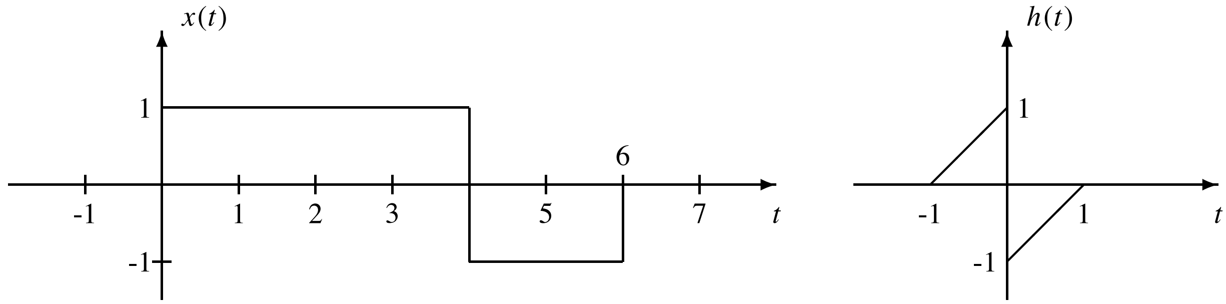
(a) Find the impulse response of the overall system; i.e., find the output $y(t) = h(t)$ when the input is $x(t) = \delta(t)$.

(b) Give a general expression for $y(t)$ in terms of $x(t)$ that holds for any input signal.

PROBLEM 11.8:

This is Problem 1.8 of Problem Set #1, EE2201, Spring 1999.

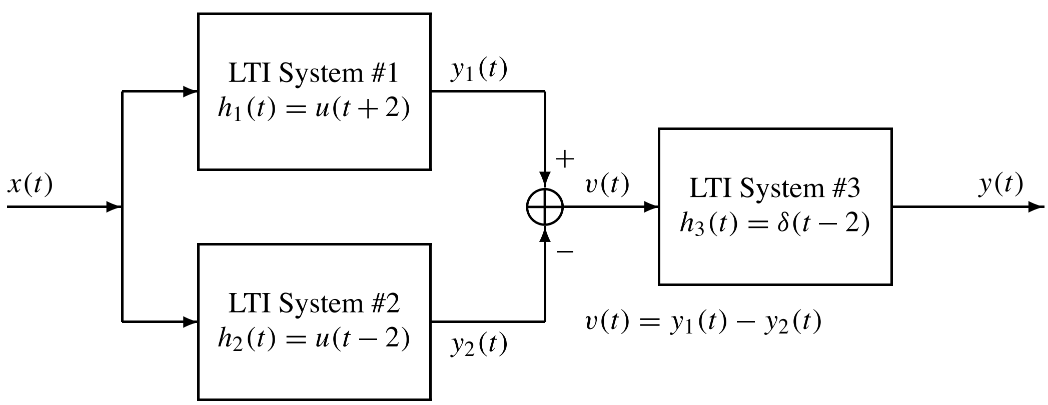
If the input $x(t)$ and the impulse response $h(t)$ of an LTI system are the following:



(a) Determine $y(0)$, the value of the output at $t = 0$.

(b) Determine the complete set of values of t such that the output $y(t) = 0$. *You do not need to find $y(t)$ at any other values of t.*

PROBLEM 11.9*:



(a) What is the impulse response of the overall LTI system (i.e., from $x(t)$ to $y(t)$)? Give your answer <u>both</u> as an equation and as a carefully labeled sketch.

(b) Is the overall system a causal system? (Explain to receive credit.) Is it a stable system? (Explain to receive credit.)

(c) Are all the subsystems causal? (Explain to receive credit.) Are all the subsystems stable? (Explain to receive credit.)