GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 1999
Problem Set #12

Assigned: 12 Nov 99
Due Date: 19 Nov 99 (FRIDAY)

Quiz #3 will be given on Monday, November 22 in your regular class time.
Coverage: Material on Problem Sets #9, #10, #11 and #12.

Reading: Read Chapter 11 all, and Chapter 12, pp. 1200–1222 in Notes.

$\Longrightarrow$ The five (5) **STARRED** problems will have to be turned in for grading.

Next week a solution will be posted. Some similar problems solutions can be found in the old problems from EE-2201 on Web-CT.

PROBLEM 12.1*:

Try your hand at expressing each of the following in a simpler form:

(a) $[e^{-t}\cos(2\pi t)][\delta(t) + 2\delta(t-3)] =$

(b) $[\delta(t) - u(t)] * [\delta(t-1) + \delta(t-2)] =$

(c) $\displaystyle\int_{-\infty}^{\infty} \sin(2\pi\tau + \pi/3)\delta(t-\tau)d\tau =$

(d) $\dfrac{d}{dt}\left[e^{-(t-1)}u(t-1)\right] =$

PROBLEM 12.2*:

A continuous-time linear time-invariant system has impulse response

$$h(t) = \delta(t) - 10e^{-10t}u(t).$$

(a) Determine the frequency response $H(j\omega)$ of the system. Express your answer as a rational function with powers of $(j\omega)$ in the numerator and denominator.

(b) Plot the magnitude squared, $|H(j\omega)|^2 = H(j\omega)H^*(j\omega)$, versus ω. Likewise, plot the phase $\angle H(j\omega)$ as a function of ω.

(c) Use superposition to find the output due to an input that is the sum of three terms:

$$x(t) = 100 + 20\cos(10t) + \delta(t-3).$$

Hint: Use the easiest method (impulse response or frequency response) to find the output due to each component of the input.

PROBLEM 12.3*:

A continuous-time LTI system is defined by the following input/output relation:

$$y(t) = \tfrac{1}{2}x(t) + x(t-4) + \tfrac{1}{2}x(t-8). \tag{1}$$

(a) Determine the impulse response $h(t)$ of the overall system; i.e., determine the output when the input is an impulse.

(b) Substitute your answer for $h(t)$ into the the integral formula

$$H(j\omega) = \int_{-\infty}^{\infty} h(t)e^{-j\omega t}dt$$

to obtain the frequency response.

(c) Apply the system definition given in Eq. (1) directly to the input $x(t) = e^{j\omega t}$ for $-\infty < t < \infty$ and show that $y(t) = H(j\omega)e^{j\omega t}$, where $H(j\omega)$ is as determined in part (b).

(d) Sketch the magnitude $|H(j\omega)|$ and phase $\angle H(j\omega)$ as functions of ω.

PROBLEM 12.4:

Attend Recitation to see how to do this problem.

A *periodic impulse train* with period T_0 is defined to be the signal

$$x(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_0).$$

(a) Plot this signal for $-3T_0 \le t \le 3T_0$.

(b) What is the fundamental frequency, ω_0 if $T_0 = 10$? **Use $T_0 = 10$ in all the following parts.**

(c) Determine the Fourier coefficients a_k in the Fourier series representation

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}.$$

(d) Plot the spectrum of this signal for $-4\omega_0 \le \omega \le 4\omega_0$.

(e) The periodic impulse train $x(t)$ is the input to a system with frequency response

$$H(j\omega) = \begin{cases} e^{-j\omega 4} & |\omega| < \omega_{co} \\ 0 & |\omega| > \omega_{co}. \end{cases}$$

Determine the output signal $y(t)$ if $\omega_{co} = \pi/T_0$.

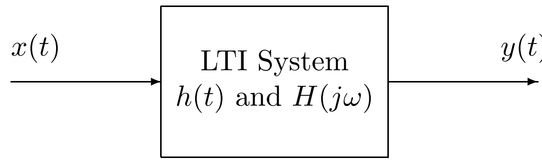
(f) The periodic impulse train $x(t)$ is the input to a system with frequency response

$$H(j\omega) = \begin{cases} e^{-j\omega 4} & |\omega| < \omega_{co} \\ 0 & |\omega| > \omega_{co}. \end{cases}$$

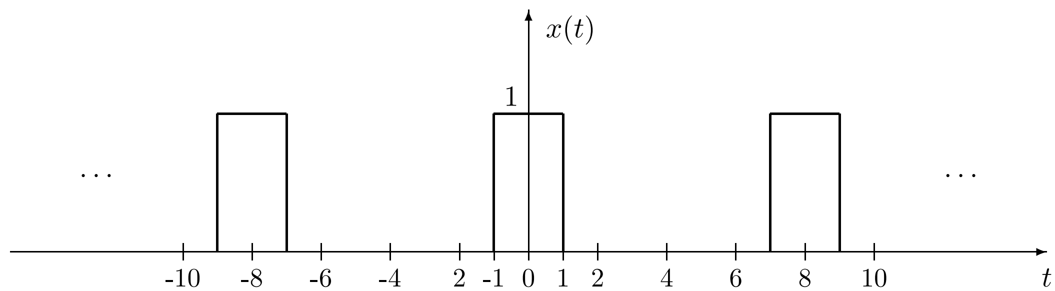
Determine the output signal $y(t)$ if $\omega_{co} = 3\pi/T_0$.

PROBLEM 12.5*:

Consider the LTI system below:



The input to this system is the periodic pulse wave $x(t)$ depicted below:



(a) Determine ω_0 and the coefficients a_k in the Fourier series representation of $x(t)$.

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

(b) Plot the spectrum of the signal $x(t)$; i.e., make a plot showing the a_k's plotted at the frequencies $k\omega_0$ for $-4\omega_0 \leq \omega \leq 4\omega_0$.

(c) If the frequency response is $H(j\omega) = e^{-j3\omega}$, plot the output of the system $y(t)$ when the input is $x(t)$ as plotted above. *Hint: What simple system has this kind of frequency response?*

(d) If the frequency response of the system is the ideal lowpass filter

$$H(j\omega) = \begin{cases} 1 & |\omega| < 0.5\pi \\ 0 & |\omega| > 0.5\pi \end{cases}$$

what is the output of the system when the input is $x(t)$ as depicted above? Give a simple equation for $y(t)$ as a real-valued function. *Use the plot of part (b) and a plot of the frequency response to help you solve this problem.*

PROBLEM 12.6*:

Use the delay property of Fourier transforms,

$$x(t - t_d) \Longleftrightarrow e^{-j\omega t_d} X(j\omega),$$

to determine the Fourier transforms of the following signals:

(a) $x(t) = \delta(t - 5)$

(b) $x(t) = 20\dfrac{\sin(200\pi(t-10))}{\pi(t-10)}$

(c) $x(t) = e^{-4t}u(t) - e^{-4t}u(t-10) = e^{-4t}u(t) - e^{-40}e^{-4(t-10)}u(t-10)$