GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 1999
Problem Set #3

Assigned: 10 September 1999
Due Date: 17 September 1999 (FRIDAY)

Quiz #1 will be held in lecture on Monday 20-September-99. It will cover material from Chapters 2 and 3, as represented in Problem Sets #1, #2 and #3.
Closed book, calculators permitted, and one hand-written formula sheet ($8\frac{1}{2}'' \times 11''$, both sides)

Reading: In *DSP First*, all of Chapter 3 on *Spectrum Representation*, especially pp. 48–73.

A simpler address for the web site: `http://webct.gatech.edu/public/ECE2025`
Please check the "Bulletin Board" often.

$\Longrightarrow$ **Look for another on-line HW this week.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

After the beginning of your assigned lecture on Friday (either 11am or 12pm), the homework is considered late and will be given a zero.

PROBLEM 3.1*:

In this problem you will consider the general case of the "beating" phenomenon. Consider the signal

$$x(t) = A\cos[2\pi(f-\Delta)t] + B\cos[2\pi(f+\Delta)t]$$

(Note that for the case discussed in Section 3.2.2, $A = B = 1$.)

(a) Use phasors to obtain a complex signal $z(t)$ such that $x(t) = \Re e\{z(t)\}$.

(b) By manipulating the expression for $z(t)$ and then taking the real part, show that in the more general case above, $x(t)$ can be expressed in the form

$$x(t) = C\cos(2\pi\Delta t)\cos(2\pi f t) + D\sin(2\pi\Delta t)\sin(2\pi f t)$$

and find expressions for C and D in terms of A and B. Find values for A and B so that

$$x(t) = 2\cos(2\pi\Delta t)\cos(2\pi f t).$$

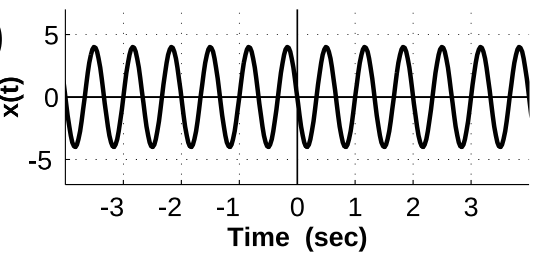
Plot the spectrum of this signal.

(c) Is it possible for $x(t)$ to be periodic? If so, what conditions on Δ and f must be satisfied?

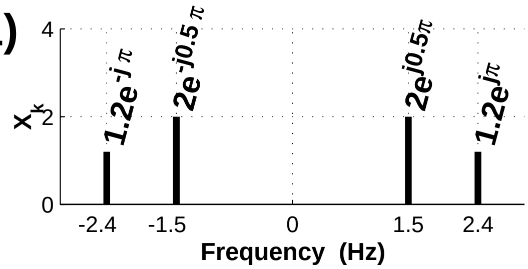
PROBLEM 3.2:

Several signals are plotted below along with their corresponding spectra. However, they are in a random order. For each of the signals (a)–(e), determine the correct spectrum (1)–(5). Explain your answers by deriving the formula for a time signal from each of the spectrum plots.

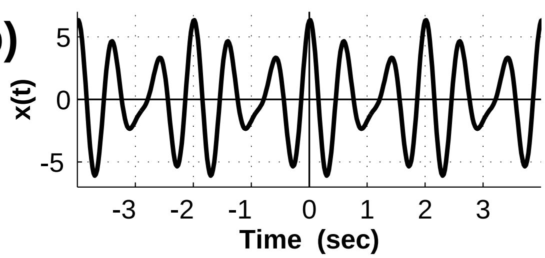
(a)



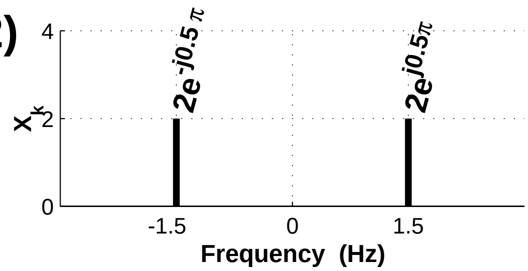
(1)



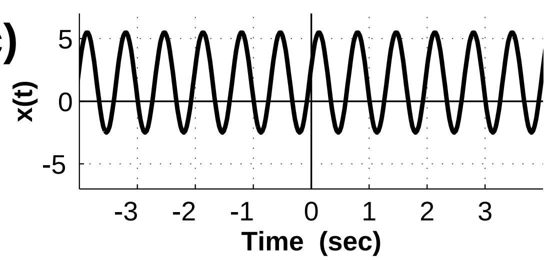
(b)



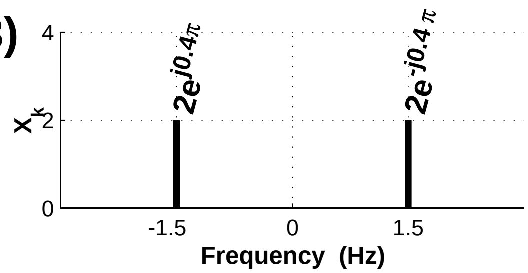
(2)



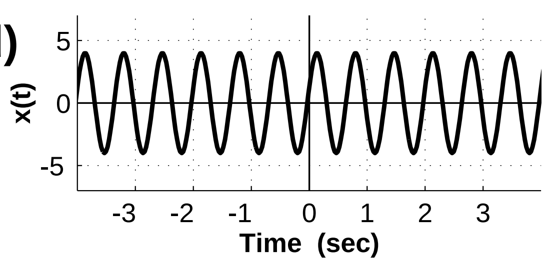
(c)



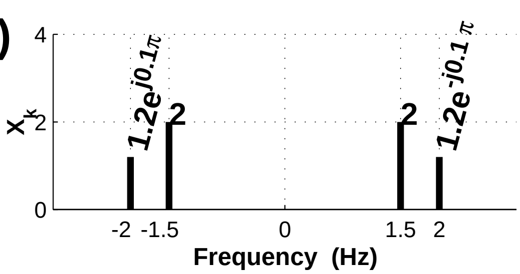
(3)



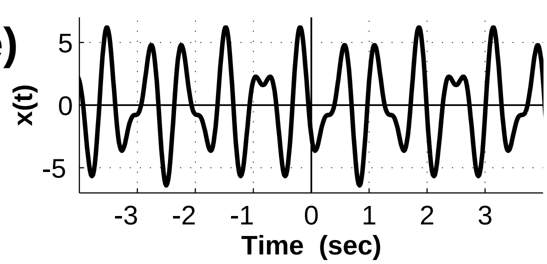
(d)



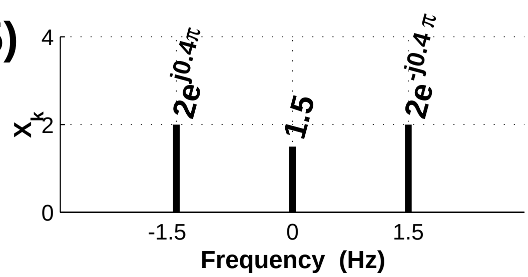
(4)



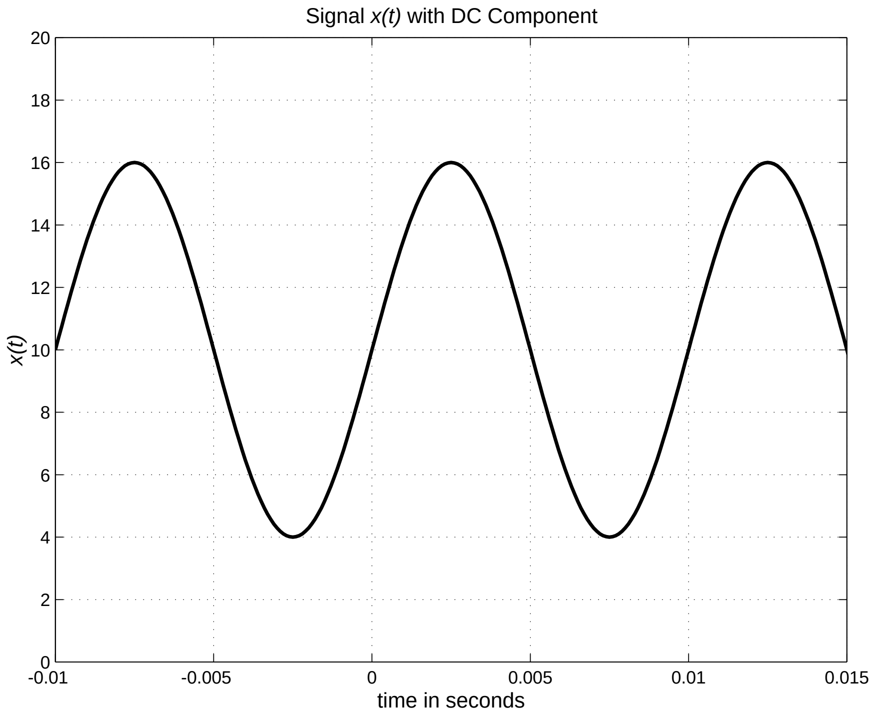
(e)



(5)



PROBLEM 3.3*:



The above signal $x(t)$ consists of a DC (or constant) component plus a cosine signal.

(a) What is the frequency of the constant component? What is the frequency of the cosine component?

(b) Write an equation for the signal $x(t)$. You should be able to determine numerical values for all the amplitudes, frequencies, and phases in your equation by inspection of the above graph.

(c) Expand the equation obtained in part (a) into a sum of positive and negative frequency complex exponential signals and plot the two-sided spectrum of the signal $x(t)$. Show the complex amplitudes for each positive and negative frequency contained in $x(t)$.

PROBLEM 3.4*:

DSP First, Chapter 3, Problem 8, page 80.

We have seen that musical tones can be modeled mathematically by sinusoidal signals. If you read music or play the piano you are aware of the fact that the piano keyboard is divided into octaves, with the tones in each octave being twice the frequency of the corresponding tones in the next lower octave. To calibrate the frequency scale, the reference tone is the A above middle-C, which is usually called A440 since its frequency is 440 Hz. Each octave contains 12 tones, and the ratio between the frequencies of successive tones is constant. Since middle C is 9 tones below A440, its frequency is approximately $(440)2^{-9/12} \approx 262$ Hz. The names of the tones (notes) of the octave starting with middle-C and ending with high-C are:

note name	C	$C^\#$	D	$E^\flat$	E	F	$F^\#$	G	$G^\#$	A	$B^\flat$	B	C
note number	40	41	42	43	44	45	46	47	48	49	50	51	52
frequency													

(a) Explain why the ratio of the frequencies of successive notes must be $2^{1/12}$.

(b) Make a table of the frequencies of the tones of the octave beginning with middle-C assuming that A above middle C (note #49) is tuned to 440 Hz.

(c) The above notes on a piano are numbered 40 through 52. If n denotes the note number, and f denotes the frequency of the corresponding tone, give a formula for the frequency of the tone as a function of the note number.

(d) A *chord* is a combination of musical notes sounded simultaneously. A *triad* is a three note chord. The E Minor chord is composed of the tones of $E,\ G,\ B$ sounded simultaneously. From the set of corresponding frequencies determined in part (a), make a sketch of the essential features of the spectrum of the E-Minor chord assuming that each note is realized by a pure sinusoidal tone and that each note is equally loud. (You do not have to specify the complex amplitudes precisely.)

PROBLEM 3.5*:

A signal composed of sinusoids is given by the equation

$$x(t) = 100\cos(40\pi t - \pi/4) + 80\sin(80\pi t) - 60\cos(120\pi t + \pi/6)$$

(a) Sketch the spectrum of this signal indicating the complex size of each frequency component. You do not have to make separate plots for real/imaginary parts or magnitude/phase. Just indicate the complex phasor value at the appropriate frequency.

(b) Is $x(t)$ periodic? If so, what is the period? Which harmonics are present?

(c) Now consider a new signal $y(t) = x(t) + 90\cos(60\pi t + \pi/6)$. How is the spectrum changed? Is $y(t)$ periodic? If so, what is the period?

(d) Finally, consider another new signal $w(t) = x(t) + 10\cos(280t + \pi/2)$. How is the spectrum changed? Is $w(t)$ periodic? If so, what is the period? If not, why not?

PROBLEM 3.6*:

A linear-FM "chirp" signal is one that sweeps in frequency from $\omega_1 = 2\pi f_1$ to $\omega_2 = 2\pi f_2$ as time goes from $t = 0$ to $t = T_2$. We can define the *instantaneous frequency* of the chirp as the derivative of the phase of the sinusoid:

$$x(t) = A\cos(\alpha t^2 + \beta t + \phi) \tag{1}$$

where the cosine function operates on a time-varying argument

$$\psi(t) = \alpha t^2 + \beta t + \phi$$

The derivative of the argument $\psi(t)$ is the *instantaneous frequency* which is also the audible frequency heard from the chirp *if the chirping frequency does not change too rapidly*.

$$\omega_i(t) = \frac{d}{dt}\psi(t) \qquad \text{radians/sec} \tag{2}$$

There are examples on the CD-ROM in the Chapter 3 demos.

(a) For the linear-FM "chirp" in (1), determine formulas for the beginning instantaneous frequency (ω_1) and the ending instantaneous frequency (ω_2) in terms of α, β and T_2. For this problem, assume that the starting time of the "chirp" is $t = 0$.

(b) For the "chirp" signal

$$x(t) = \Re e\left\{e^{j2\pi(25t^2 - 25t)}\right\}$$

derive a formula for the *instantaneous* frequency versus time. Should your answer for the frequency be a positive number?

(c) For the signal in part (b), make a plot of the *instantaneous* frequency (in Hz) versus time over the range $0 \le t \le 1$ sec.