GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 1999
Problem Set #6

Assigned: 1 October 99
Due Date: 8 October 99 (FRIDAY)

Quiz #2 on 25-Oct (Monday).

Reading: In *DSP First*, Chapter 5 on *FIR Filters* and Chapter 6 on *Frequency Response*.

There will be a lab quiz at the beginning of Lab #6 (5–11 Oct).

$\Longrightarrow$ The five **STARRED** problems will have to be turned in for grading.

Next week a solution will be posted. Some similar problems solutions can be found on the CD-ROM, especially the “unstarred” problems.

PROBLEM 6.1*:

A linear time-invariant filter is described by the difference equation

$$y[n] = -x[n] + 2x[n-1] - x[n-2]$$

(a) Obtain an expression for the frequency response of this system.

(b) Make a sketch of the frequency response (magnitude and phase) as a function of frequency. *Hint: Use symmetry to simplify your expression before determining the magnitude and phase.*

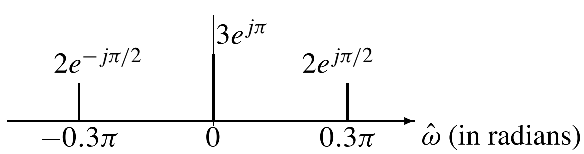
(c) What is the output if the input is $x[n] = 5 + 5\cos(0.5\pi n + \pi/2)$?

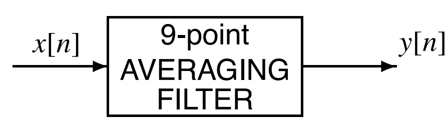
(d) What is the output if the input is the *unit impulse sequence* $x[n] = \delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0. \end{cases}$

(e) What is the output if the input is the *unit step sequence* $x[n] = u[n] = \begin{cases} 0 & n < 0 \\ 1 & n \geq 0. \end{cases}$

PROBLEM 6.2:

A discrete-time signal $x[n]$ has the two-sided spectrum representation shown below.





(a) Write an equation for $x[n]$. Make sure to express $x[n]$ as a real-valued signal.

(b) Determine the formula for the output signal $y[n]$.

See Problem 6.1 of Spring 1999 for solution to this problem.

PROBLEM 6.3*:

A discrete-time system is defined by the input/output relation

$$y[n] = x[n+1] + x[n] + x[n-1]. \tag{1}$$

(a) Determine whether or not the system defined by Equation (1) is (i) linear; (ii) time-invariant; (iii) causal.

(b) For the system of Equation (1), determine the output $y_1[n]$ when the input is [1]

$$x_1[n] = 2\cos(0.75\pi n) = e^{j0.75\pi n} + e^{-j0.75\pi n}.$$

(c) For the system of Equation (1), determine the output $y_2[n]$ when the input is

$$x_2[n] = 4 + 4\cos(0.75\pi(n-1)).$$

A second discrete-time system is defined by the input/output relation

$$y[n] = (x[n])^2. \tag{2}$$

(d) Determine whether or not the system defined by Equation (2) is (i) linear; (ii) time-invariant; (iii) causal.

(e) For the system of Equation (2), determine the output $y_1[n]$ when the input is

$$x_1[n] = 2\cos(0.75\pi n) = e^{j0.75\pi n} + e^{-j0.75\pi n}.$$

(f) For the system of Equation (2), determine the output $y_2[n]$ when the input is

$$x_2[n] = 4 + 4\cos(0.75\pi(n-1)).$$

(g) For which system does superposition hold?

(h) For which system does the output contain frequencies that are not present in the input signal?

(i) Which system can cause aliasing of sinusoidal components of the input?

[1] In parts (b), (c), (e), and (f), express your answer in terms of cosine functions. Do not leave any square powers of cosine functions in your answers.

PROBLEM 6.4*:

For the *modified Dirichlet* function:

$$\tilde{D}(\hat{\omega}, 5) = \frac{\sin(2.5\hat{\omega})}{\sin(\frac{1}{2}\hat{\omega})}$$

(a) Make a plot of $\tilde{D}(\hat{\omega}, 5)$ over the range $-2\pi \leq \hat{\omega} \leq +2\pi$. Label all the zero crossings.

(b) Determine the period of $\tilde{D}(\hat{\omega}, 5)$. Is it equal to 2π; why, or why not?

(c) Find the maximum value of the function.

Note: the unmodified *Dirichlet* function is defined via: $D(\hat{\omega}, L) = \dfrac{\sin(L\hat{\omega}/2)}{L\sin(\frac{1}{2}\hat{\omega})}$, so $\tilde{D}(\hat{\omega}, 5) = 5D(\hat{\omega}, 5)$. In MATLAB consult help on `diric` for more information.

PROBLEM 6.5*:

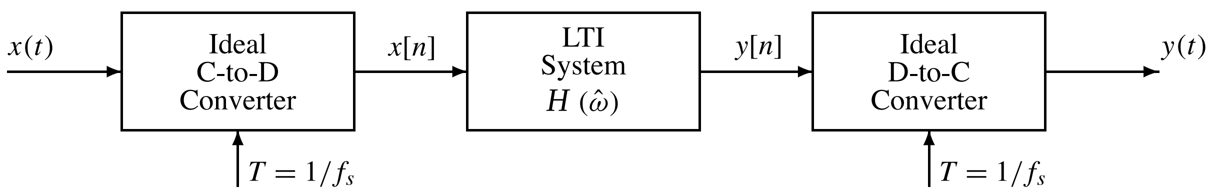
The input to the C-to-D converter in the figure below is

$$x(t) = 3 + 4\cos(2000\pi t) + 5\cos(4000\pi t - 2\pi/3)$$

The frequency response for the digital filter (LTI system) is

$$H(\hat{\omega}) = \frac{\sin(2.5\hat{\omega})}{\sin(\frac{1}{2}\hat{\omega})} e^{-j2\hat{\omega}}$$

If $f_s = 10000$ samples/second, determine an expression for $y(t)$, the output of the D-to-C converter.



PROBLEM 6.6*:

The frequency response of a linear time-invariant filter is given by the formula

$$H(\hat{\omega}) = (1 - e^{-j\hat{\omega}})(1 - e^{j\pi/3}e^{-j\hat{\omega}})(1 - e^{-j\pi/3}e^{-j\hat{\omega}}). \tag{3}$$

(a) Write the difference equation that gives the relation between the input $x[n]$ and the output $y[n]$.

(b) What is the output if the input is $x[n] = \delta[n]$?

(c) If the input is of the form $x[n] = Ae^{j\phi}e^{j\hat{\omega}n}$, for what values of $-\pi \leq \hat{\omega} \leq \pi$ will $y[n] = 0$ for all n?

(d) The frequency response in Equation (3) is written as a product of factors suggesting that it could be implemented as a cascade of several systems. By suitably grouping the factors and multiplying them together, obtain a representation as the cascade of *two* systems each of which has only *real* filter coefficients. Give the frequency responses and impulse responses of the two systems and draw a block diagram of the cascade system.