GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2000
Problem Set #2

Assigned: 17-Jan-00
Due Date: Week of 24-Jan-00

Reading: In *DSP First*, all of Chapter 2 on *Sinusoids*; and start reading in Chapter 3: *Spectrum Representation,* especially pp. 48–61.

The web site: `http://classweb.gatech.edu:8080/public/ECE2025MJ/index.html`
You should change your password; look under `COURSE TOOLS`.

$\Longrightarrow$ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

PROBLEM 2.1*:

Simplify the following and give the answer as a single sinusoid: $A\cos(\omega t + \phi)$. Draw the vector diagram of the complex amplitudes (phasors) to show how you obtained the answer.

(a) $x_a(t) = 2\cos(222\pi t - 5\pi/3) + \cos(222\pi t + 5\pi/6)$

(b) $x_b(t) = \cos(33.33\pi t + 17\pi) + \sqrt{2}\cos(33.33\pi t + 17.5\pi) + \sqrt{2}\cos(33.33\pi t + 18\pi)$

(c) $x_c(t) = \cos(60\pi t + 3\pi/4) + \cos(60\pi t + 5\pi/4) + 2\cos(60\pi t + \pi/4)$

PROBLEM 2.2*:

Define $x(t)$ as

$$x(t) = \sqrt{2}\cos(\omega_0 t + 3\pi/4) + \cos(\omega_0 t + \pi/2)$$

(a) Find a complex-valued signal $z_1(t)$ such that $\Re e\{z_1(t)\} = \sqrt{2}\cos(\omega_0 t + 3\pi/4)$.

(b) Find a complex-valued signal $z(t)$ such that $x(t) = \Re e\{z(t)\}$. Simplify $z(t)$ as much as possible, so that you can identify its complex amplitude.

(c) Assume that $\omega_0 = 0.2\pi$ rad/sec. Make a plot of $\Re e\{(-1 + j)e^{j\omega_0 t}\}$ over the range $-10 \le t \le 10$ secs. How many periods are included in the plot?

PROBLEM 2.3*:

Complex exponentials obey the expected rules of algebra when doing integrals and derivatives. Consider the complex signal $z(t) = Ze^{j\pi t}$ where $Z = e^{-j\pi/8}$.

(a) Show that the first derivative of $z(t)$ with respect to time can be represented as a new complex exponential $Qe^{j\pi t}$, i.e., $\frac{d}{dt}z(t) = Qe^{j\pi t}$. Determine the value for the complex amplitude Q, and show that the angle of Q is 90° greater than that of Z.

(b) Evaluate the definite integral of $z(t)$ over the range $0 \le t \le 1$: $\int_0^1 z(t)dt = ?$

Note that integrating a complex quantity follows the expected rules of algebra: you could integrate the real and imaginary parts separately, but you can also *use the integration formula for an exponential* directly on $z(t)$.

(c) Evaluate the definite integral of $z(t)$ over the range $-1 \le t \le 1$: $\int_{-1}^1 z(t)dt = ?$

(d) Evaluate the integral of the magnitude squared $|z(t)|^2$ over the range $-1 \le t \le 1$: $\int_{-1}^1 |z(t)|^2 dt = ?$

PROBLEM 2.4*:

Solve the following simultaneous equations by using complex amplitudes. Show how to convert the sinusoidal equations into complex-number equations. If we assume that the amplitudes are positive, will the answers for A_1 and A_2 be unique? How about ϕ_1 and ϕ_2; are there other answers for the phases?

$$\begin{aligned} 2\cos(\omega_0 t - 2\pi/3) &= A_1\cos(\omega_0 t + \phi_1) - A_2\cos(\omega_0 t + \phi_2) \\ 2\cos(\omega_0 t - 3\pi) &= A_1\cos(\omega_0 t + \phi_1) + A_2\cos(\omega_0 t + \phi_2) \end{aligned}$$

PROBLEM 2.5*:

Suppose that $x(t)$ is formed via the following sum:

$$x(t) = 10\sqrt{3}\cos(77\pi t + \pi/6) + A\cos(77\pi t + \phi) \tag{1}$$

where A is a *positive* number. In addition, assume that $x(t)$ has a phase of zero, so it is also given by the sinusoidal definition:

$$x(t) = B\cos(77\pi t), \tag{2}$$

where B is a *positive* number.

(a) In this part assume that $B = 20$, and solve for A and ϕ.

(b) Now assume that B can vary. Solve for A, B and ϕ so that the value of A is *minimized* among all possible choices that satisfy equations (1) and (2). Draw a plot of the complex amplitudes to prove (via a geometrical argument) that you have found the minimum for A. (Remember that $A > 0$)
Hint: To solve this problem try a graphical approach with plots of the complex amplitudes. Also, recall the geometrical theorem that tells you how to find the shortest distance between a line and a point not on the line.