2.1(a) $x_a(t) = 2\cos\left(222\pi t - \frac{5\pi}{3}\right)$

$+ \cos\left(222\pi t + \frac{5\pi}{6}\right)$

These can be represented by the vectors

$2\angle{-300^\circ} + 1\angle{150^\circ} \leftarrow 2e^{-j5\pi/3} + 1e^{j5\pi/6}$

$1 + j3 \quad -0.866 + j0.5$

$0.1340 \quad + j2.2321$

$2.2361\angle{86.56^\circ} = 2.236e^{j0.481\pi}$

$x_a(t) = 2.2361\cos\left(222\pi t + \frac{86.56\pi}{180}\right)$

2.1(b) In a like manner to the previous problem:

$1\angle{17\pi} + \sqrt{2}\angle{17.5\pi} + \sqrt{2}\angle{18\pi}$

$1e^{j17\pi} + \sqrt{2}e^{j17.5\pi} + \sqrt{2}e^{j18\pi} = e^{j\pi} + \sqrt{2}e^{j3\pi/2} + \sqrt{2}e^{j0}$

$-1 + j0 \quad +0 - j\sqrt{2} \quad +\sqrt{2} + j0$

$0.414 - j2 = 1.4736\angle{-73.68^\circ} = 1.4736e^{-j0.409\pi}$

$x_b(t) = 1.4736\cos\left(33.33\pi t - \frac{73.68\pi}{180}\right)$

2.1(c) $x_c(t) = \cos\left(60\pi t + \frac{3\pi}{4}\right)$

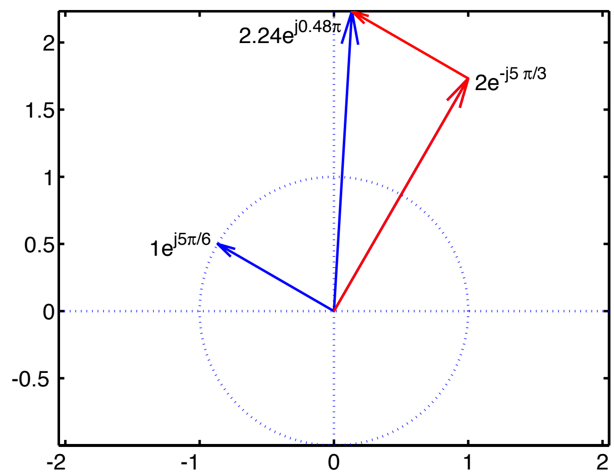
$+ \cos\left(60\pi t + \frac{5\pi}{4}\right)$

$+ 2\cos\left(60\pi t + \frac{\pi}{4}\right)$

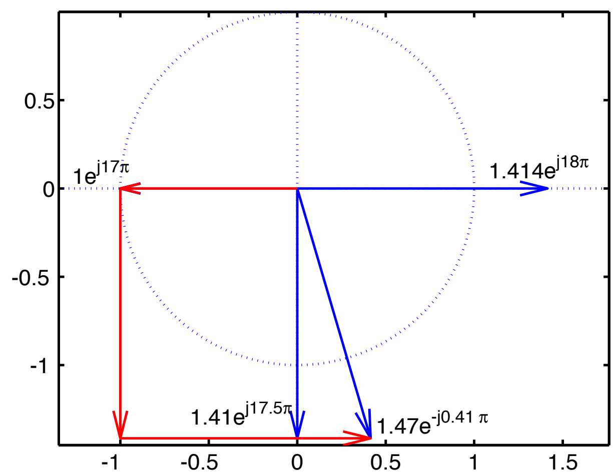
(continued next page)

PROBLEM 2.1*:
Draw the vector diagram of the complex amplitudes (phasors) to show how you obtained the answer.

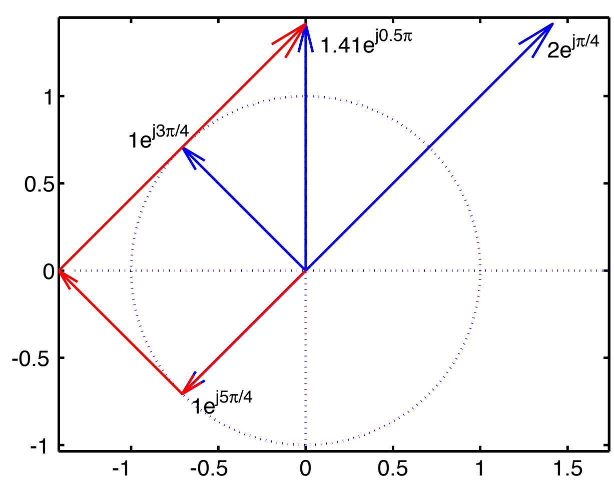
(a) Plot of $2e^{-j5\pi/3} + 1e^{j5\pi/6}$.



(b) Plot of $1e^{j17\pi} + \sqrt{2}e^{j17.5\pi} + \sqrt{2}e^{j18\pi}$.



(c) Plot of $1e^{j5\pi/4} + 1e^{j3\pi/4} + 2e^{j\pi/4}$.



$$1e^{j3\pi/4} + 1e^{j5\pi/4} + 2e^{j\pi/4} = 1\angle 135^\circ + 1\angle 225^\circ + 2\angle 45^\circ$$

$$-0.707 + j\,0.707 \quad -0.707 - j\,0.707 \quad +1.414 + j\,1.414$$

$$0 + j\,1.414 = \sqrt{2}\,\angle 90^\circ = \sqrt{2}e^{j\pi/2}$$

$$x_c(t) = \sqrt{2}\cos\left(60\pi t + \frac{\pi}{2}\right)$$

2.2(a)

$$x(t) = \sqrt{2}\cos\left(\omega_0 t + \frac{3\pi}{4}\right) + \cos\left(\omega_0 t + \frac{\pi}{2}\right)$$

$$\mathcal{Re}\{z_1(t)\} = \sqrt{2}\cos\left(\omega_0 t + \frac{3\pi}{4}\right)$$

$$z_1(t) = \sqrt{2}\,e^{j\omega_0 t}\,e^{j\frac{3\pi}{4}}$$

Complex Amplitude $= \sqrt{2}\,e^{j3\pi/4}$

2.2(b) $x(t) = \mathcal{Re}\{z(t)\}$

first simplify $x(t)$

$$\sqrt{2}\,\angle 135^\circ + 1\angle 90^\circ = \sqrt{2}\,e^{j3\pi/4} + 1e^{j\pi/2}$$

Complex Amp.

$$-1 + j1 \quad + 0 + j1$$

$$-1 + j2 = 2.236\,\angle 116.56^\circ = 2.236\,e^{j0.648\pi}$$

$$x(t) = 2.236\cos\left(\omega_0 t + 116.56\pi/180\right)$$

$$z(t) = 2.236\,e^{j\omega_0 t}\,e^{j\,116.56\pi/180}$$

2.2(c).

$\omega_0 = 0.2\pi$

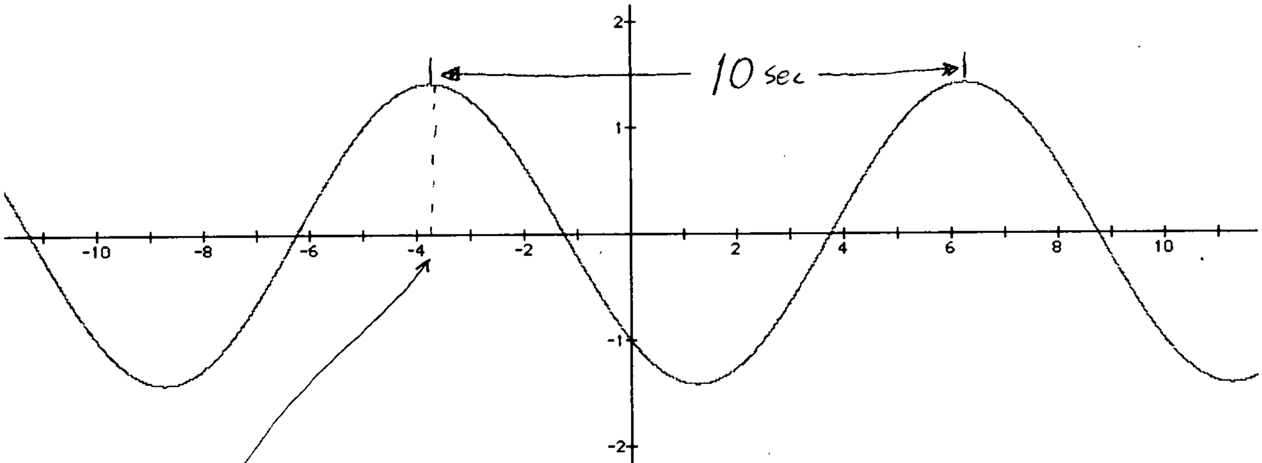
$\mathcal{R}e\left\{(-1+j)e^{j\omega_0 t}\right\} \quad -10 \le t \le 10\ \text{sec.}$

$\mathcal{R}e\left\{\sqrt{2}e^{j\frac{3\pi}{4}} e^{j\omega_0 t}\right\}$

$f_0 = \frac{\omega_0}{2\pi} = \frac{0.2\pi}{2\pi} = 0.1$ cycles/sec.

$T_0 = \frac{1}{0.1} = 10$ sec/cycle

Obviously, there are 2 periods in 20 seconds.



Note: $t_m = -\varphi/\omega = -0.75\pi/0.2\pi = -3.75$ ses

2.3 $z(t) = Ze^{j\pi t} \qquad Z = e^{-j\pi/8}$

(a) Using the rule for the derivative of a product, we have

$$z'(t) = Z'e^{j\pi t} + Zj\pi e^{j\pi t}$$

$$Z' = 0$$

$$z'(t) = Zj\pi e^{j\pi t} = j\pi e^{j\pi t} e^{-j\pi/8}$$

$$= \pi e^{j\pi t} e^{j\pi/2} e^{-j\pi/8}$$

$$Q = \pi e^{j\pi/2} e^{-j\pi/8}$$

$\angle Z = -\pi/8$
$\angle Q = \pi/2 - \pi/8$
$\Rightarrow \angle Q = 90°$ more than $\angle Z$

(b) $$\int_0^1 z(t)\,dt = e^{-j\pi/8}\int_0^1 e^{j\pi t}\,dt$$

$$= e^{-j\pi/8}\frac{1}{j\pi}e^{j\pi t}\Big|_0^1 = \frac{e^{-j\pi/8}e^{-j\pi/2}}{\pi}\left[-1-(1)\right]$$

$$= \frac{e^{-j\pi/8}e^{-j\pi/2}}{\pi}(-2) = \frac{2}{\pi}e^{-j\pi/8}e^{j\pi/2}$$

(c) $$\int_{-1}^1 z(t)\,dt = e^{-j\pi/8}\int_{-1}^1 e^{j\pi t}\,dt = e^{-j\pi/8}\frac{1}{j\pi}e^{j\pi t}\Big|_{-1}^1$$

(continued next page)

$$= \frac{e^{-j\pi/8}\, e^{-j\frac{\pi}{2}}}{\pi}\left[-1-(-1)\right] = 0$$

(d) $|z(t)|^2 = 1 \Rightarrow \int_{-1}^{1} |z(t)|^2 dt = \int_{-1}^{1} 1\, dt = t\Big|_{-1}^{1} = 1-(-1) = 2$

2.4

$$2\cos(\omega_0 t - \tfrac{2\pi}{3}) = A_1\cos(\omega_0 t + \phi_1) - A_2\cos(\omega_0 t + \phi_2)$$

$$2\cos(\omega_0 t - 3\pi) = A_1\cos(\omega_0 t + \phi_1) + A_2\cos(\omega_0 t + \phi_2)$$

Convert to phasor notation

$$2e^{-j2\pi/3} = A_1 e^{j\phi_1} - A_2 e^{j\phi_2}$$

$$2e^{-j3\pi} = A_1 e^{j\phi_1} + A_2 e^{j\phi_2}$$

Add the two equations to eliminate A_2

$$2e^{-j2\pi/3} + 2e^{-j3\pi} = 2A_1 e^{j\phi_1}$$

Combine the two terms on the left using phasors

$$-1 - j1.732 - 2 = 2\sqrt{3}e^{-j5\pi/6} = 2A_1 e^{j\phi_1}$$

$\Rightarrow A_1 = \sqrt{3}$ and $\phi_1 = -5\pi/6$

Note: phase is ambiguous, so $\phi_1 = -5\pi/6 + 2\pi k$, k=integer

Substituting $A_1 e^{j\phi_1}$ back into the first equation

$$2e^{-j2\pi/3} = \sqrt{3}e^{-j5\pi/6} - A_2 e^{j\phi_2} \Rightarrow A_2 e^{j\phi_2} = \sqrt{3}e^{-j5\pi/6} - 2e^{-j2\pi/3}$$

$$A_2 e^{j\phi_2} = -\tfrac{1}{2} + j\tfrac{\sqrt{3}}{2} = 1e^{j2\pi/3} \Rightarrow A_2 = 1 \text{ \& } \varphi_2 = 2\pi/3 + 2\pi k \quad k=\text{integer}$$

2.5

$x(t) = 10\sqrt{3}\cos(77\pi t + \pi/6) + A\cos(77t + \phi)$

$A > 0.$ also $x(t) = B\cos(77\pi t)$

$B > 0.$

(a) $B = 20$, find A and ϕ

$20\cos(77\pi t) = 10\sqrt{3}\cos(77\pi t + \frac{\pi}{6}) + A\cos(77\pi t + \phi)$

Expressing the equation in phasor form

$20 = 10\sqrt{3}e^{j\pi/6} + Ae^{j\phi}$

$20 + j0 = 10\sqrt{3}\angle 30^\circ + Ae^{j\phi}$

$Ae^{j\phi} = 20 + j0 - 15 - j5\sqrt{3}$

(continued next page)

$$Ae^{j\phi} = 5 - j5\sqrt{3} = 10\angle{-60^\circ} = 10e^{-j\pi/3}$$

$$A = 10 \quad \phi = -60^\circ = -\pi/3$$

$$Ae^{j\phi} = 10e^{-j\,60\pi/180}$$

2.5(b)

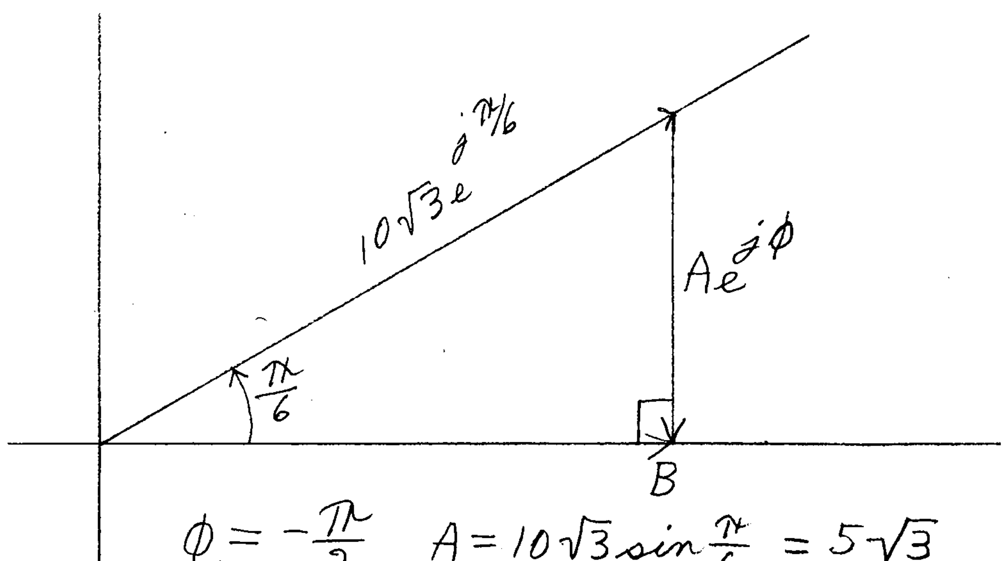
$$B\cos(77\pi t) = 10\sqrt{3}\cos(77\pi t + \pi/6) + A\cos(77\pi t + \phi)$$

Find A, B, ϕ so that A is minimized.

Expressing the equation in phasor form

$$B = 10\sqrt{3}\,e^{j\pi/6} + Ae^{j\phi}$$

The vector sum on the right side of the equation must equal B, which must be entirely real. A must be a minimum. A is a minimum if $Ae^{j\phi}$ is perpendicular to B.



$$\phi = -\frac{\pi}{2} \quad A = 10\sqrt{3}\sin\frac{\pi}{6} = 5\sqrt{3}$$