GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2000
Problem Set #4

Assigned: 31-Jan-00
Due Date: Week of 7-Feb-00

Quiz #1 will be held in lecture on Friday 4-Feb-00. It will cover material from Chapters 2 and 3, as represented in Problem Sets #1, #2 and #3.
Closed book, calculators permitted, and one hand-written formula sheet ($8\frac{1}{2}''\times 11''$**, both sides)**

Reading: The notes on *Fourier Series*. In *DSP First*, all of Chapter 3 on *Spectrum Representation,* and also start reading Chapter 4.

$\Longrightarrow$ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

PROBLEM 4.1*:

A periodic signal is represented by the Fourier Series synthesis formula:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k\, e^{j30\pi kt} \qquad \text{where} \quad a_k = \begin{cases} \dfrac{1}{4+j2k} & \text{for } k = -3,-2,-1,0,1,2,3 \\ 0 & \text{for } |k| > 3 \end{cases}$$

(a) Sketch the two-sided spectrum of this signal. Label all complex amplitudes in **polar form.**

(b) Determine the fundamental frequency (in Hz) and the fundamental period (in secs.) of this signal.

PROBLEM 4.2*:

A periodic signal $x(t) = x(t+T_0)$ is described *over one period,* $0 \le t \le T_0$, by the equation

$$x(t) = \begin{cases} t & 0 \le t \le t_c \\ 0 & t_c < t \le T_0 \end{cases}$$

where $0 < t_c < T_0$.

(a) Sketch the periodic function $x(t)$ for $-T_0 < t < 2T_0$ for the specific case $t_c = \frac{1}{2}T_0$.

(b) Determine the D.C. coefficient of the Fourier Series, a_0. Once again, use the specific case of $t_c = \frac{1}{2}T_0$.

PROBLEM 4.3*:

Continuation of Prob. 4.2. Use the same signal $x(t)$ defined in problem 4.2 with the specific case of $t_c = \frac{1}{2}T_0$.

(a) Use the Fourier *analysis* integral[1] (for $k \neq 0$)

$$a_k = \frac{1}{T_0}\int_0^{T_0} x(t)e^{-jk\omega_0 t}dt$$

to determine a general formula[2] for the Fourier Series coefficients a_k. Your final result for a_k should depend on k. Note: the frequency ω_0 would be given in rads/sec, but it does not have a specific value. However, you can simplify your formulas by using the identity $\omega_0 T_0 = 2\pi$.

(b) Use the Fourier Series coefficients to sketch the spectrum of $x(t)$ for the case $\omega_0 = 2\pi(\frac{1}{4})$ rad/sec and $t_c = \frac{1}{2}T_0$. Include *only* those frequency components corresponding to $k = 0, \pm 1, \pm 2, \pm 3$. Label each component with its frequency and its complex amplitude (i.e., numerical values of magnitude and phase).

PROBLEM 4.4*:

A linear-FM "chirp" signal is one that sweeps in frequency from $\omega_1 = 2\pi f_1$ to $\omega_2 = 2\pi f_2$ as time goes from $t = 0$ to $t = T_2$. We can define the *instantaneous frequency* of the chirp as the derivative of the "angle" of the sinusoid:

$$x(t) = A\cos(\alpha t^2 + \beta t + \phi) \qquad (1)$$

where the cosine function operates on a time-varying angle argument

$$\psi(t) = \alpha t^2 + \beta t + \phi$$

The derivative of the angle argument $\psi(t)$ is the *instantaneous frequency,* which is also the audible frequency heard from the chirp.[3]

$$\omega_i(t) = \frac{d}{dt}\psi(t) \qquad \text{radians/sec} \qquad (2)$$

(a) For the "chirp" signal

$$x(t) = \Re e\left\{e^{j2\pi(-75t^2 + 900t + 33)}\right\}$$

derive a formula for the *instantaneous* frequency versus time.

(b) For the signal in part (b), make a plot of the *instantaneous* frequency (in Hz) versus time over the range $0 \le t \le 2$ sec.

PROBLEM 4.5*:

A linear-FM "chirp" signal is one that sweeps in frequency from $\omega_1 = 2\pi f_1$ to $\omega_2 = 2\pi f_2$ as time goes from $t = 0$ to $t = T_2$.

(a) Determine the formula for a signal $x(t)$ that sweeps from $f_1 = 5000$ Hz at $T_1 = 0$ secs. to $f_2 = 1000$Hz at $T_2 = 2$ secs.

(b) Sketch the time-frequency diagram showing the instantaneous frequency versus time for the signal in part (a).

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[1] The Fourier integral can be done over any period of the signal; in this case, the most convenient choice is from 0 to T_0.

[2] The Fourier integral requires integration by parts—an opportunity to use your calculus skills.

[3] The instantaneous frequency is the frequency heard by the human ear when the chirp rate is relatively slow. There are cases of FM where the audible signal is quite different, but these happen when the chirp rate is very high.