ECE 2025 HW DUE WEEK OF 7 FEB 2k

4.1 $30\pi k t = 2\pi k f_0 t \Rightarrow f_0 = 15\,\text{Hz} \Rightarrow T_0 = \frac{1}{f_0} \approx 66.7\,\text{m Sec}$

$$a_{k=-3} = \frac{1}{4+j2\cdot(-3)} = 0.139e^{j0.313\pi}$$

$$a_{-2} = \frac{1}{4+j2(-2)} = 0.178\,e^{j\pi/4}$$

$$a_{-1} = \frac{1}{4+j2(-1)} = 0.224\,e^{j0.148\pi}$$

$$a_0 = \frac{1}{4} = 0.25$$

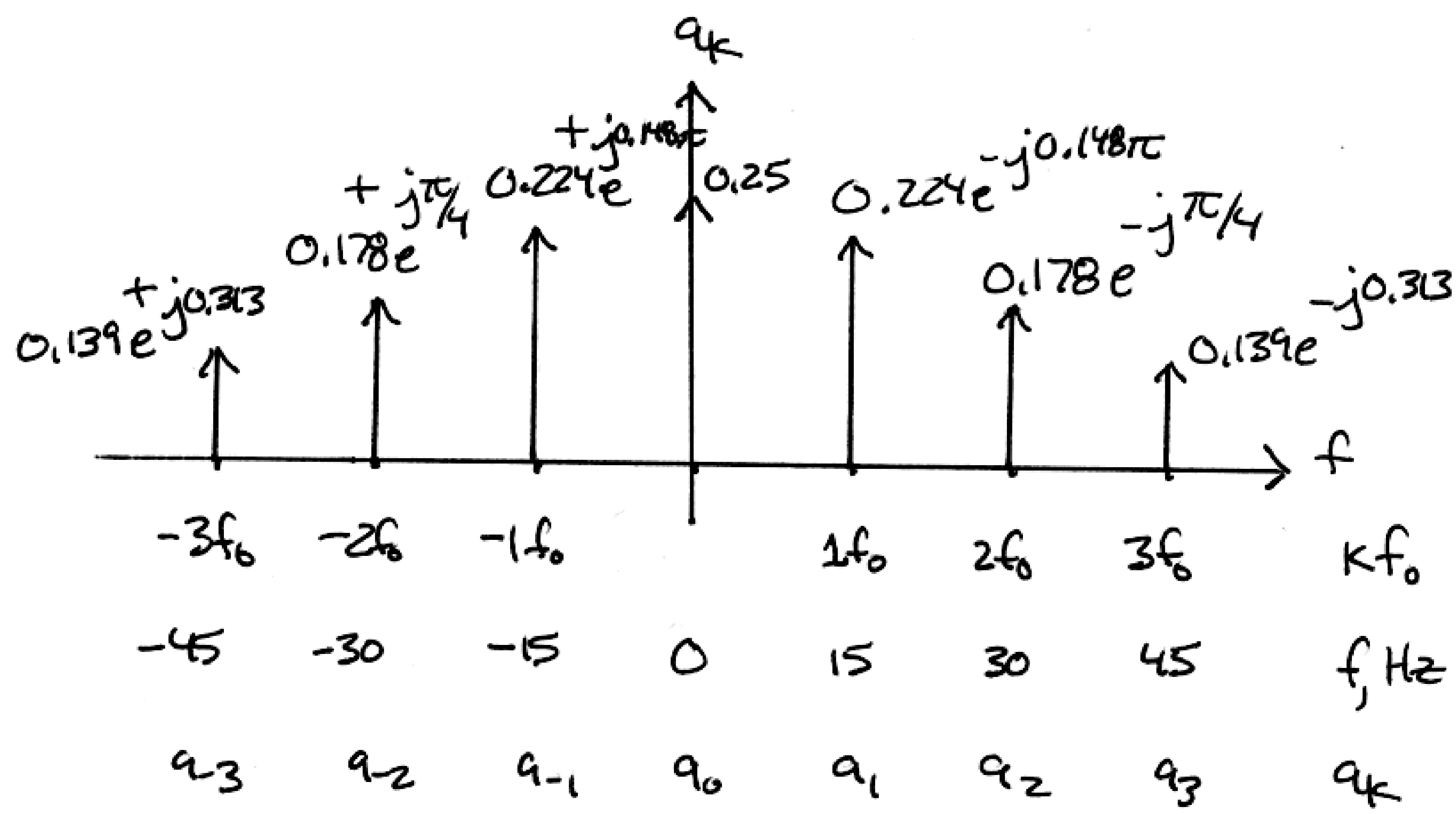
$$a_1 = \frac{1}{4+j2\cdot 1} = 0.224\,e^{-j0.148\pi}$$

$$a_2 = \frac{1}{4+j2\cdot 2} = 0.178\,e^{-j\pi/4}$$

$$a_3 = \frac{1}{4+j3\cdot 2} = 0.139\,e^{-j0.313\pi}$$

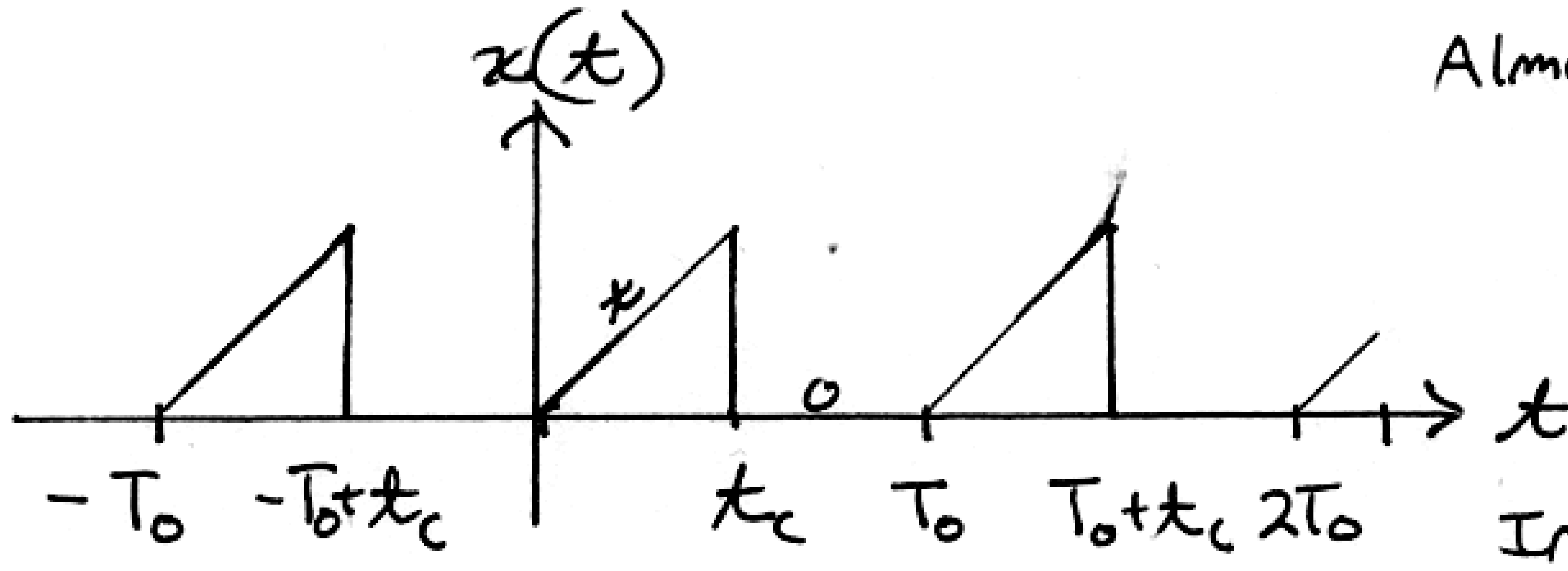
Note: Complex Conjugates

$a_{else} = 0$



Use MATLAB or Calculator to compute values of a_k for various k

(4.2)



Almost a "sawtooth" waveform

In General

$-T_0 \quad -\frac{T_0}{2} \quad 0 \quad \frac{T_0}{2} \quad T_0 \quad \frac{3T_0}{2} \quad 2T_0 \quad \leftarrow$ For $t_c = \frac{T_0}{2}$

$$a_0 = \frac{1}{T_0}\int_{\substack{\text{ONE}\\ \text{PERIOD}}} x(t)\,dt = \frac{1}{T_0}\int_0^{\frac{1}{2}T_0} t\,dt + \frac{1}{T_0}\int_{\frac{1}{2}T_0}^{T_0} 0\,dt$$

$$= \frac{1}{T_0}\int_0^{\frac{T_0}{2}} t\,dt + 0$$

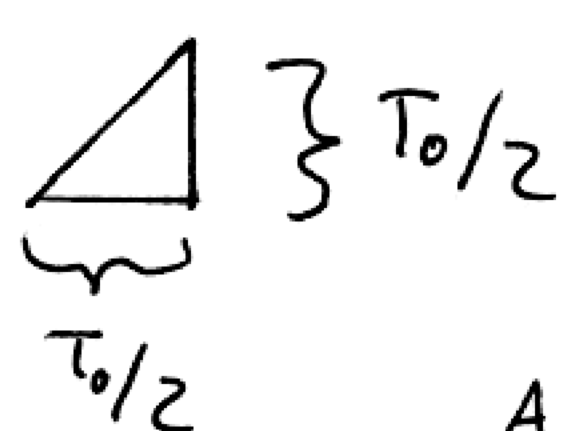
$$= \frac{1}{T_0}\cdot\frac{t^2}{2}\Big|_0^{\frac{T_0}{2}} =$$

$$= \frac{1}{T_0}\cdot\frac{(\frac{1}{2}T_0)^2}{2}$$

$$\boxed{a_0 = \frac{T_0}{8}}$$ = AVERAGE (DC) LEVEL OF $x(t)$

= AREA UNDER ONE OF THE TRIANGLES / T_0

=



$$\frac{A_\Delta}{T_0} = \frac{\frac{1}{2}\left(\frac{T_0}{2}\right)\left(\frac{T_0}{2}\right)}{T_0} = \frac{T_0}{8}$$

4.3 (a) $a_{k\neq 0} = \frac{1}{T_0}\int_{\text{ONE PERIOD}} x(t)\, e^{-jk\omega_0 t}\, dt$

$t_c = \frac{T_0}{2}$ FOR THIS PROBLEM

$$= \frac{1}{T_0}\int_0^{\frac{T_0}{2}} t\, e^{-jk\omega_0 t}\, dt + \cancelto{0}{\frac{1}{T_0}\int_{\frac{T_0}{2}}^{T_0} 0\, dt}$$

NOTE: $T_0 = \frac{\omega_0}{2\pi}$

= HARD:
1) INTEGRATE BY PARTS
2) CONSULT INTEGRAL TABLE
3) LOOK ON PAGES 8-10 OF SUPPLEMENTARY FOURIER SERIES NOTES (WebCT) FOR A DERIVATION OF a_k FOR A TRIANGLE WAVE (/\/\/\/\)

FROM NOTES:

$$\int_0^{\frac{T_0}{2}} t\, e^{-jk\omega_0 t}\, dt = \left.\frac{t\, e^{-jk\omega_0 t}}{(-jk\omega_0)}\right|_0^{\frac{T_0}{2}} - \left.\frac{e^{-jk\omega_0 t}}{(-k^2\omega_0^2)}\right|_0^{\frac{T_0}{2}} \quad \left(\text{Eqn } 33.5.8\right)$$

$$= \frac{T_0}{2}\left[-\frac{e^{-jk\pi}}{(j2\pi k)} + \frac{2e^{-jk\pi}}{(4\pi^2k^2)} - \frac{2}{4\pi^2k^2}\right] = \left(\text{Eqn } 33.5.9\right), \frac{T_0}{2}$$

So:

$$\underset{(k\neq 0)}{a_k} = \frac{T_0}{2}\left[-\frac{e^{-jk\pi}}{(j2\pi k)} + \frac{2e^{-jk\pi}}{4\pi^2k^2} - \frac{2}{4\pi^2k^2}\right]$$

(b)

$\omega_0 = 2\pi\frac{1}{4}\ \frac{\text{rad}}{\text{sec}} \Rightarrow f_0 = \frac{1}{4}\text{Hz} = \frac{1}{T_0 = 4\text{ sec}}$

FUND FREQ / PERIOD

Evaluate a_k ($k = \pm1, \pm2, \pm3$)

$a_{-3} = T_0\, 0.0271\, e^{j0.567\pi}$

$a_{-2} = T_0\, 0.0398\, e^{-j\frac{\pi}{2}}$

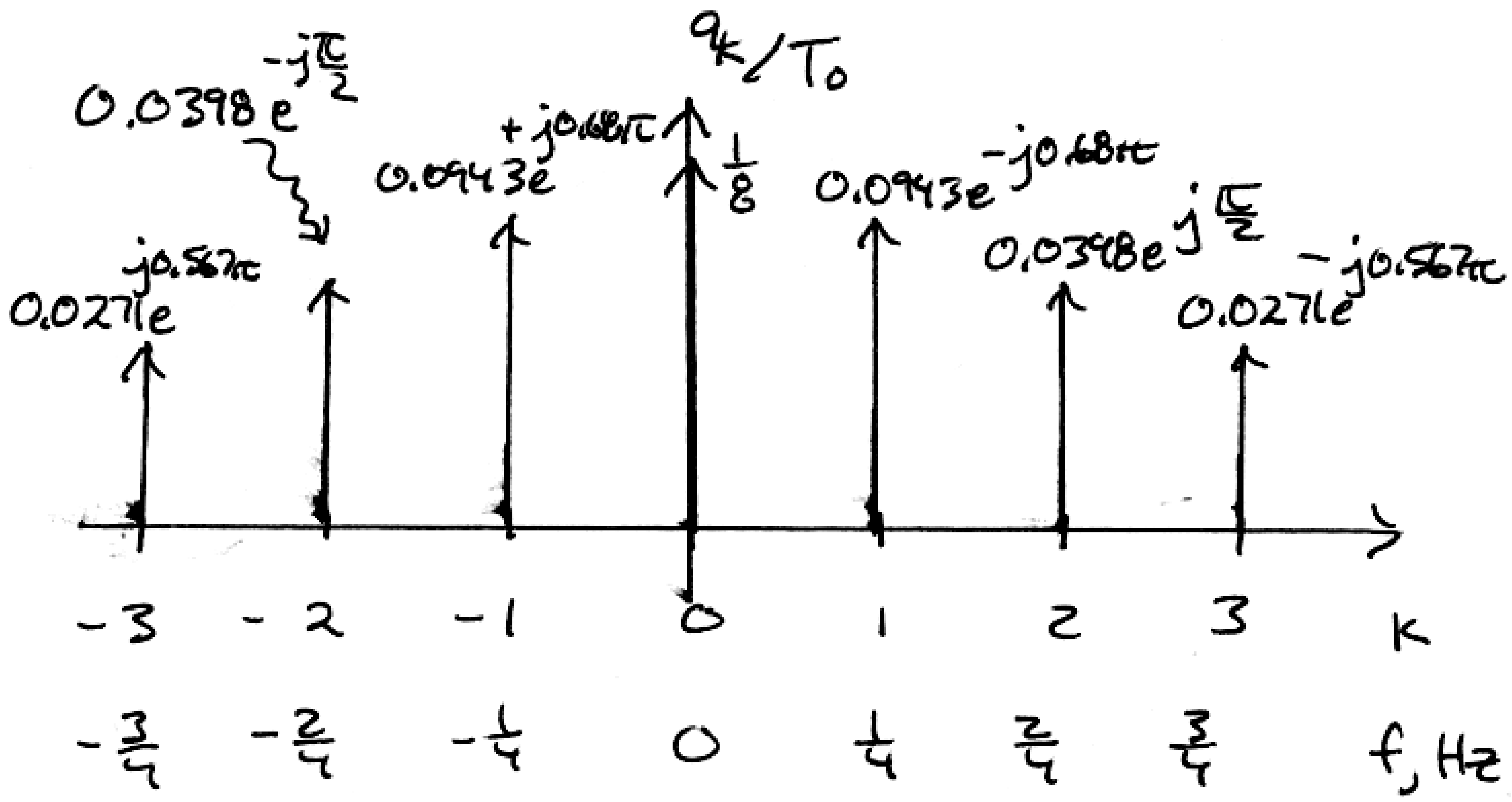
$a_{-1} = T_0\, 0.0943\, e^{j0.6805\pi}$

$a_0 = T_0/8$

$a_1 = T_0 \cdot 0.0943\, e^{-j0.6805\pi}$

$a_2 = T_0 \cdot 0.0398\, e^{j\frac{\pi}{2}}$

$a_3 = T_0 \cdot 0.0271\, e^{-j0.567\pi}$



LINEAR CHIRP

4.4

(a)

$$x(t) = \mathrm{Re}\left\{ e^{j2\pi(-75t^2 + 900t + 33)} \right\}$$

$$= \cos\left[2\pi(-75t^2 + 900t + 33)\ \text{rad}\right]$$

$$= \cos\left[\text{phase}(t)\ \text{rad}\right]$$

SIGNAL

Phase of Signal

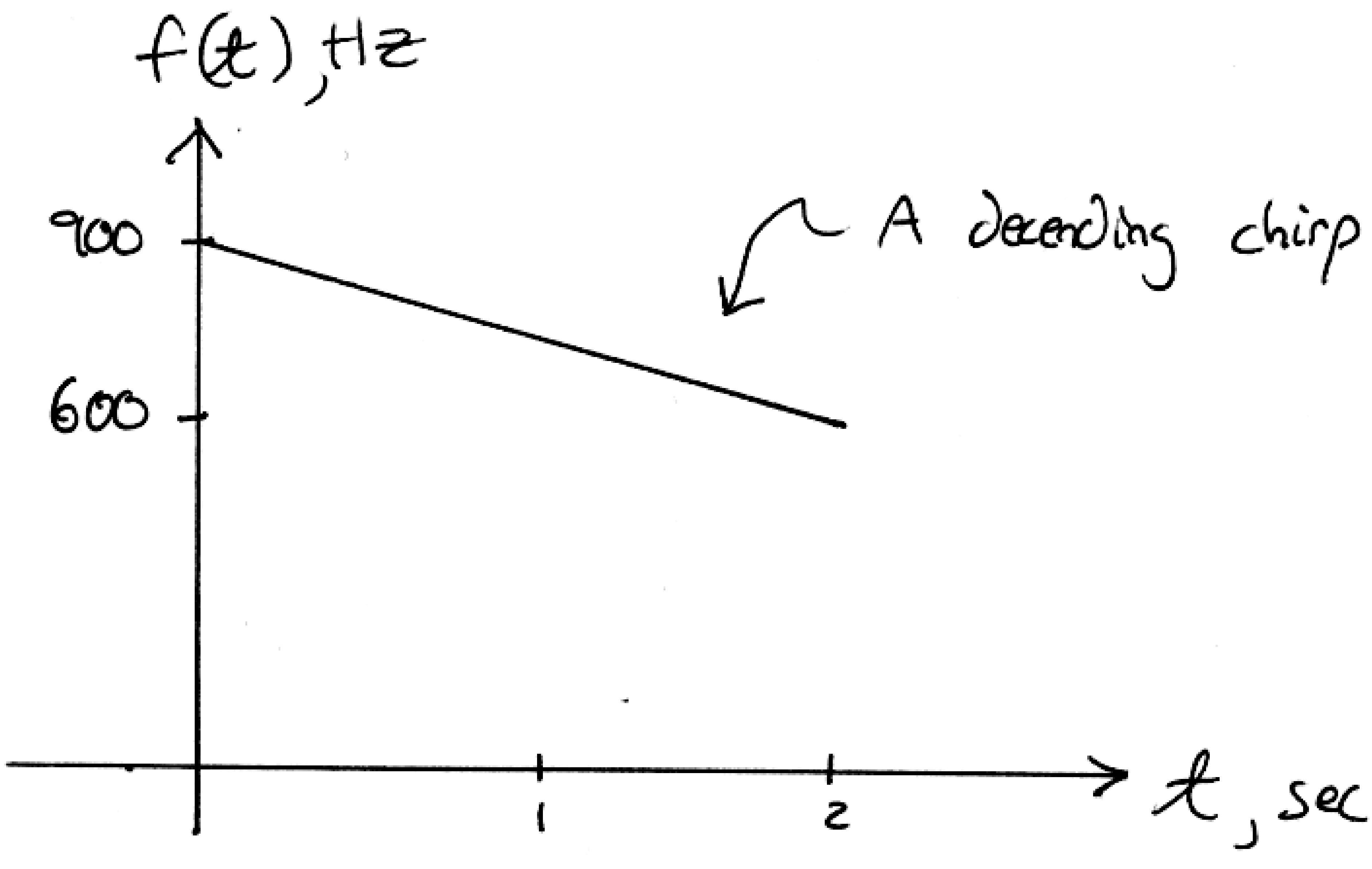
$$\frac{\partial\, \text{phase}(t)\,[\text{rad}]}{\partial t} = \omega(t)\left[\frac{\text{rad}}{\text{sec}}\right] = 2\pi(-150)t + 2\pi\, 900 + 0 = 2\pi f(t)$$

$$f(t) = -150t + 900 \quad [\text{Hz}]$$

(b)

$$f(t=0) = 900\ \text{Hz}$$

$$f(t = 2\ \text{sec}) = -150(2) + 900 = 600\ \text{Hz}$$

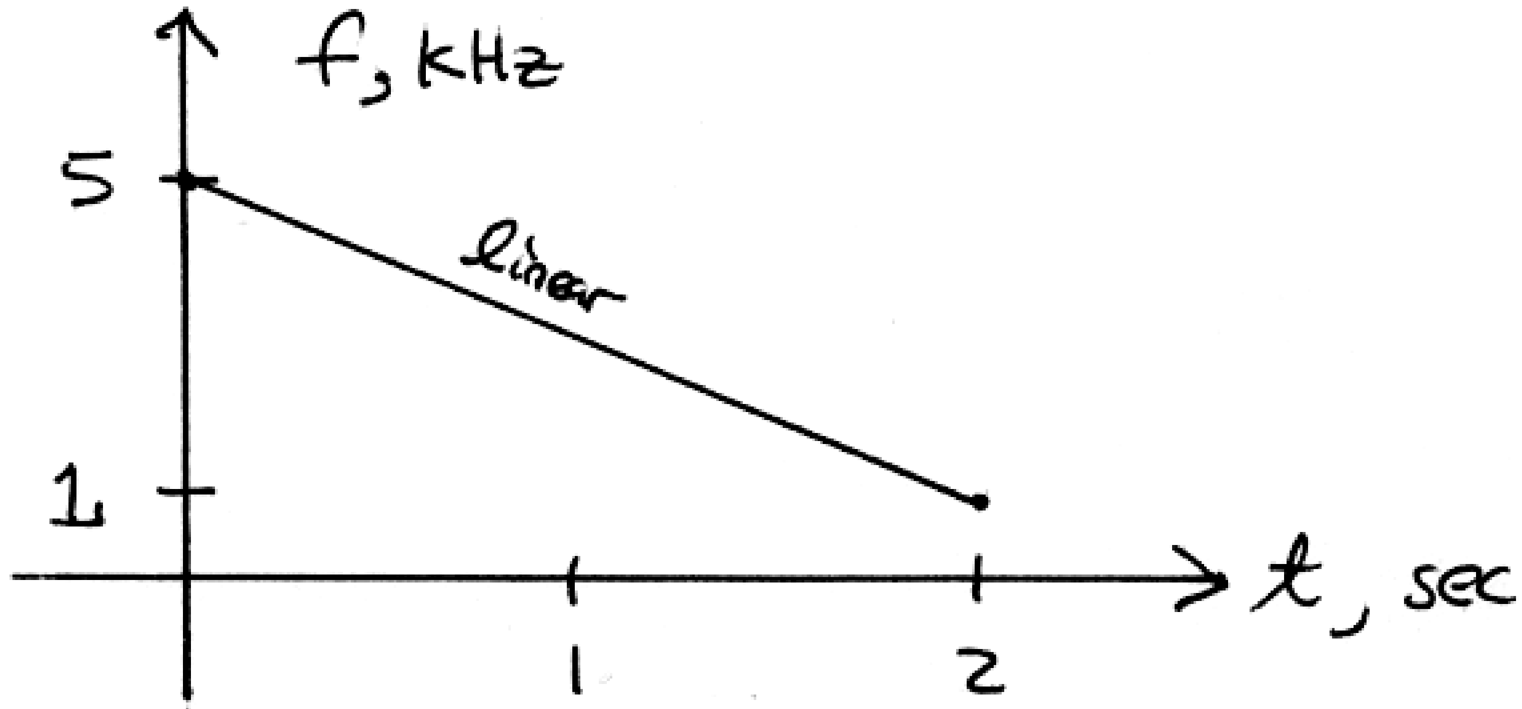


(4.5) WORK PROBLEM 4.4 BACKWARDS

LINEAR CHIRP

Frequency(t)

(b) $f(t=0) = 5000\text{ Hz}$
$f(t=2\text{sec}) = 1000\text{ Hz}$



If LINEAR: $y = mx + b$

$$f(t) = -2000t + 5000 \quad \text{Hz}$$

$$\omega(t) = 2\pi\left[-2000t + 5000\right] \quad \text{rad/sec}$$

Phase(t)

$$\text{phase}(t)\ [\text{rad}] = \int \omega(t)\,dt =$$

$$= 2\pi(-2000)\frac{t^2}{2} + 2\pi\, 5000t + \left(\text{ANY INITIAL Phase} \atop \text{Assume zero}\right)$$

signal(t) = x(t)

a) $$x(t) = A\cos\left[2\pi\left(-1000t^2 + 5000t\right)\right]$$

$$= \operatorname{Re}\left\{Ae^{j2\pi(-1000t^2 + 5000t)}\right\}$$