GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

# ECE 2025 Spring 2000
# Problem Set #6

Assigned: 11-Feb-00
Due Date: Week of 21-Feb-00

---

There will be a lab quiz at the beginning of Lab #6 (21-24 Feb).

Reading: In *DSP First*, Chapter 5 on *FIR Filters*.

⟹ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

**ALL** of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

---

**Your homework is due in recitation at the beginning of class.** After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

---

## PROBLEM 6.1*:

This problem is concerned with finding the output of an FIR filter for a given input signal. A linear time-invariant system is described by the difference equation

$$y[n] = \sum_{k=0}^{4} (k+1)x[n-k]$$

The input to this system is *unit step* signal, denoted by $u[n]$, i.e., $x[n] = u[n] = \begin{cases} 0 & n < 0 \\ 1 & n \geq 0 \end{cases}$

(a) Determine the filter coefficients $\{b_k\}$ of this FIR filter.

(b) Determine the impulse response, $h[n]$, for this FIR filter. The impulse response is a discrete-time signal, so make a (stem) plot of $h[n]$ versus $n$.

(c) Use convolution to compute $y[n]$, over the range $-5 \leq n \leq \infty$, when the input is $u[n]$. Make a plot of $y[n]$ vs. $n$. (Hint: you might find it useful to check your results with MATLAB's `conv()` function.)

## PROBLEM 6.2*:

Consider a system defined by $y[n] = \sum_{k=8}^{20} b_k x[n-k]$

Notice that the filter coefficients $b_0$, $b_1$, $b_2$, ..., $b_7$ are all zero.

(a) Suppose that the input $x[n]$ is non-zero only for $0 \leq n \leq 33$. Show that $y[n]$ is non-zero at most over a finite interval of the form $8 \leq n \leq P-1$ and determine $P$.

(b) Suppose that the input $x[n]$ is non-zero only for $100 \leq n \leq 200$. Show that $y[n]$ is non-zero at most over a finite interval of the form $N_3 \leq n \leq N_4$. Determine $N_3$ and $N_4$.

*Hint: consult Figs. 5.5 and 5.6 in the book for the sliding window interpretation of the FIR filter.*

## PROBLEM 6.3*:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

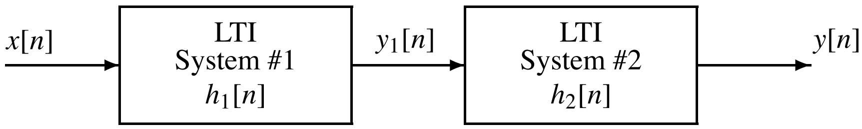



Figure 1: Cascade connection of two LTI systems.

Suppose that System #1 is an FIR filter described by the impulse response:

$$h_1[n] = \begin{cases} 0 & n < 0 \\ 2^n & n = 0, 1, 2, 3, 4, 5 \\ 0 & n > 5 \end{cases}$$

and System #2 is described by the difference equation

$$y_2[n] = y_1[n] - 2y_1[n-1]$$

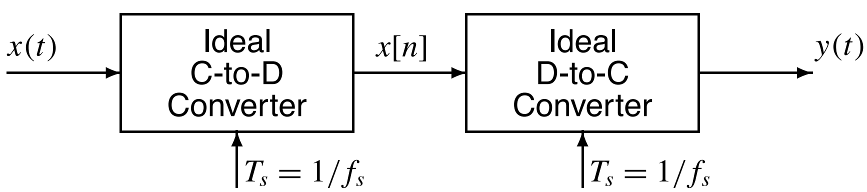
(a) Determine the filter coefficients of System #1, and also for System #2.

(b) When the input signal $x[n]$ is an impulse, $\delta[n]$, determine the signal $y_1[n]$ and make a plot.

(c) Determine the impulse response of System #2.

(d) Determine the impulse response of the overall cascade system, i.e., find $y[n]$ when $x[n] = \delta[n]$.

## PROBLEM 6.4*:




Chirps are very useful signals for probing the behavior of sampling operations and illustrating the "folding" type of aliasing (see Fig. 4.4 in the book).

(a) If the input to the ideal C/D converter is $x(t) = 7\cos(1800\pi t + \pi/4)$, and the sampling frequency is 1000 Hz, then the output $y(t)$ is a sinusoid. Determine the formula for the output signal.

(b) Suppose that the input signal is a chirp signal defined as follows:

$$x(t) = \cos(2000\pi t - 400\pi t^2) \qquad \text{for } 0 \le t \le 5 \text{ sec.}$$

If the sampling rate is $f_s = 1000$ Hz, then the output signal $y(t)$ will have time-varying frequency content. Draw a graph of the resulting analog *instantaneous* frequency (in Hz) versus time of the signal $y(t)$ **after reconstruction.** Hint: this could be done in MATLAB by putting a sampled chirp signal into the MATLAB function `specgram()`, or the DSP-First function `plotspec()`.

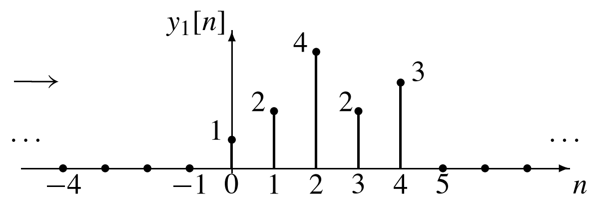
**PROBLEM 6.5*:**

Answer the following questions about the time-domain response of FIR digital filters:

$$y[n] = \sum_{k=0}^{M} b_k\, x[n-k]$$

(a) When tested with an input signal that is the sum of two shifted impulses $x_1[n] = \delta[n+1] + \delta[n-1]$, the observed output from the filter is the signal $y_1[n]$ shown below:

$x_1[n] = \delta[n+1] + \delta[n-1] \quad \longrightarrow$




Determine the filter coefficients $\{b_k\}$ of the difference equation for the FIR filter.

(b) If the input signal is

$$x[n] = \begin{cases} 0 & \text{for } n < 0 \\ (-1)^n & \text{for } n = 0, 1, 2, 3 \\ 0 & \text{for } n > 3 \end{cases}$$

use linearity and time-invariance to determine the output signal $y[n]$ for all $n$. Give your answer as either a plot or a table of values.

(c) Finally, determine the impulse response of the system. This might be difficult, because you are essentially being asked to solve the following convolution equation:

$$x_1[n] * h[n] = y_1[n]$$

for $h[n]$. In general, it is not always possible to solve such an equation.[1]

(d) Is the filter *causal?* Use the appropriate property of the impulse response that guarantees causality.

---

[1]The convolution equation can be regarded as a set of simultaneous linear equations in the unknown impulse response values $h[n]$. It is also necessary to figure out the length of the impulse response, which can be done using an approach similar to Problem 6.2.

## PROBLEM 6.6:

This problem is the same as problem 4.32 on the CD-ROM, but it is relevant to the image processing lab.[2]
A non-ideal D-to-C converter takes a sequence $y[n]$ as input and produces a continuous-time output $y(t)$ according to the relation

$$y(t) = \sum_{n=-\infty}^{\infty} y[n]p(t - nT_s)$$

where $T_s = 0.1$ second. The input sequence is given by the formula

$$y[n] = \begin{cases} 1 & 0 \le n \le 4 \\ (0.5)^{(n-4)} & 5 \le n \le 9 \\ 0 & \text{otherwise} \end{cases}$$

(a) Plot $y[n]$ versus $n$.

(b) For the pulse shape

$$p(t) = \begin{cases} 1 & -0.05 \le t \le 0.05 \\ 0 & \text{otherwise} \end{cases}$$

carefully sketch the output waveform $y(t)$ over its non-zero region.

(c) For the pulse shape

$$p(t) = \begin{cases} 1 - 10|t| & -0.1 \le t \le 0.1 \\ 0 & \text{otherwise} \end{cases}$$

carefully sketch the output waveform $y(t)$ over its non-zero region.

[2]The solution is available on the DSP-First CDROM.