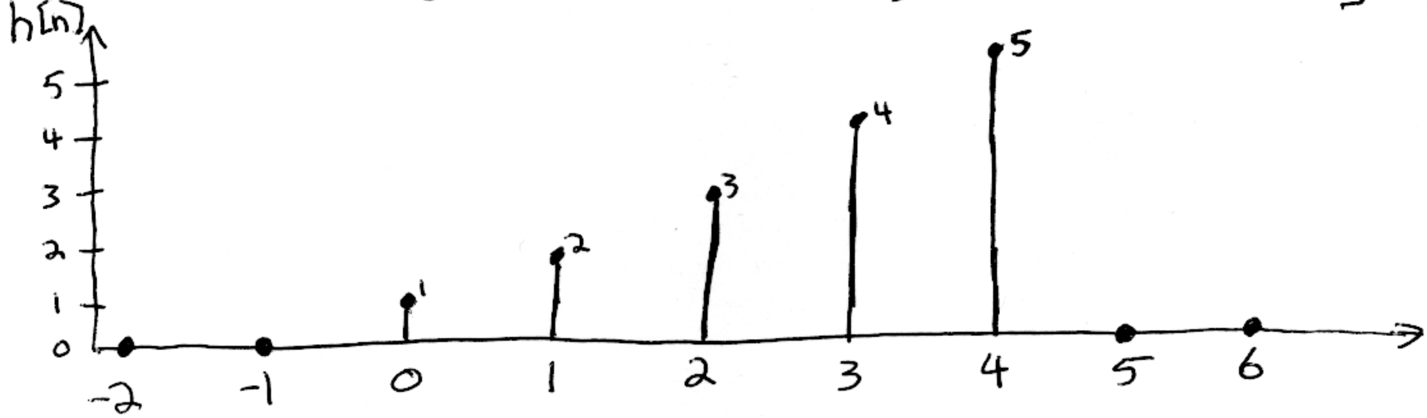
6.1

a) $y[n] = 1x[n] + 2x[n-1] + 3x[n-2] + 4x[n-3] + 5x[n-4]$

Filter coefficients $\boxed{b_0 = 1 \quad b_1 = 2 \quad b_2 = 3 \quad b_3 = 4 \quad b_4 = 5}$

($b_n = 0$ for $n < 0$ and $n > 4$)

b) $h[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2] + 4\delta[n-3] + 5\delta[n-4]$



c) $y[n] = \sum_{k=0}^{4} h[k]u(n-k)$

n	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10
u(n)	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1
h(n)	0	0	0	0	0	1	2	3	4	5	0	0	0	0	0	0
h(0) u(n)	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1
h(1) u(n-1)	0	0	0	0	0	0	2	2	2	2	2	2	2	2	2	2
h(2) u(n-2)	0	0	0	0	0	0	0	3	3	3	3	3	3	3	3	3
h(3) u(n-3)	0	0	0	0	0	0	0	0	4	4	4	4	4	4	4	4
h(4) u(n-4)	0	0	0	0	0	0	0	0	0	5	5	5	5	5	5	5
y[n]	0	0	0	0	0	1	3	6	10	15	15	15	15	15	15	15

y[n] for n < 0

y[0] y[1] y[2] y[3]

y[n] for n ≥ 4

6.2 $h[n] = \sum_{k=8}^{20} b_k \delta[n-k]$ (zero for $n<8$ and $n>20$)

a) $y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=0}^{33} x[k]h[n-k]$

$y[n]=0$ whenever $h[n-k]=0$ for all $0 \le k \le 33$.
Given what we know about h (above), this situation arises when $n-k<8$ or when $n-k>20$ for all $0\le k \le 33$.
Equivalent conditions are given by $n-0<8$ or $n-33>20$. Thus $y[n]$ can only be nonzero for $8+0 \le n \le 20+33$ or $\boxed{8 \le n \le 53.}$ Note that $53 = P-1$, where $\boxed{P=54}$ is equal to the finite length of the input $x[n]$ (34 samples) plus the order of $h[n]$ (20).

b) $y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=100}^{200} x[k]h[n-k]$

Again, given what we know about h, $y(n)=0$ whenever $h[n-k]=0$ for all $100 \le k \le 200$.
This occurs when $n-k<8$ or $n-k>20$ for all $100 \le k \le 200$, which is equivalent to saying $n-100<8$ or $n-200>20$. Thus, $y[n]$ can only be nonzero for $8+100 \le n \le 20+200$ or $\boxed{108 \le n \le 220.}$ i.e. $N_3 = 108$ and $N_4 = 220$

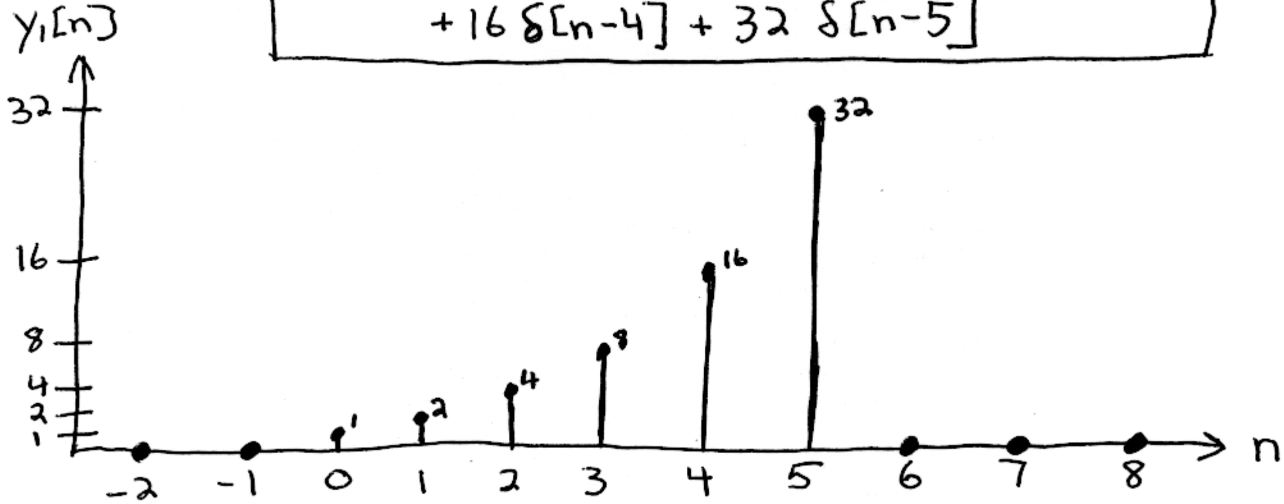
6.3 a) Filter coefficients for system #1: $a_0 = 2^0;\ a_1 = 2^1;\ a_2 = 2^2;\ a_3 = 2^3;\ a_4 = 2^4;\ a_5 = 2^5$

$\boxed{a_0 = 1,\ a_1 = 2,\ a_2 = 4,\ a_3 = 8,\ a_4 = 16,\ a_5 = 32}$

For system #2: $\boxed{b_0 = 1,\ b_1 = -2}$

b) When $x[n] = \delta[n]$, then $y_1[n] = h_1[n]$.

Thus, $\boxed{y_1[n] = \delta[n] + 2\delta[n-1] + 4\delta[n-2] + 8\delta[n-3] + 16\delta[n-4] + 32\delta[n-5]}$



c) $h_2[n] = b_0\delta[n] + b_1\delta[n-1] = \boxed{\delta[n] - 2\delta[n-1]}$

(simply plug in $y_1[n] = \delta[n]$)

d) $h[n]$ = implulse response of cascade system $= h_1[n] * h_2[n]$

$$= h_2[n] * h_1[n] = \sum_{k=-\infty}^{\infty} h_2[k]\, h_1[n-k]$$

$$= h_2[0]\, h_1[n] + h_2[1]\, h_1[n-1]$$

$$= h_1[n] - 2h_1[n-1]$$

$$= (\delta[n] + 2\delta[n-1] + 4\delta[n-2] + 8\delta[n-3] + 16\delta[n-4] + 32\delta[n-5])$$
$$-2(\delta[n-1] + 2\delta[n-2] + 4\delta[n-3] + 8\delta[n-4] + 16\delta[n-5] + 32\delta[n-6])$$

$$= \boxed{\delta[n] - 64\delta[n-6]}$$

6.4 a) $x[n] = 7\cos(1800\pi n T_s + \pi/4)$

$= 7\cos(1.8\pi n + \pi/4)$ ← since $T_s = \frac{1}{1000} = 1/f_s$

normalized frequency is NOT between $-\pi$ and π, thus the D/C converter will "see" a different normalized frequency of $-1.8\pi + 2\pi = \underline{.2\pi}$ (a case of folding)) corresponding to the signal

$= 7\cos(-(1.8\pi n + \pi/4) + 2\pi n)$

$= 7\cos(.2\pi n - \pi/4)$

Upon reconstruction, the normalized frequency of $.2\pi$ will give rise to an analog frequency of $.2\pi/T_s = .2\pi(1000) = 200\pi$ rad/s or 100 Hz

$$y(t) = \boxed{7\cos(200\pi t - \pi/4)}$$

b) Instantaneous frequency of $x(t) = 2000\pi - 800\pi t$

$= 1000 - 400t$ Hz $\qquad \underline{f_s = 1000 \text{ Hz}}$

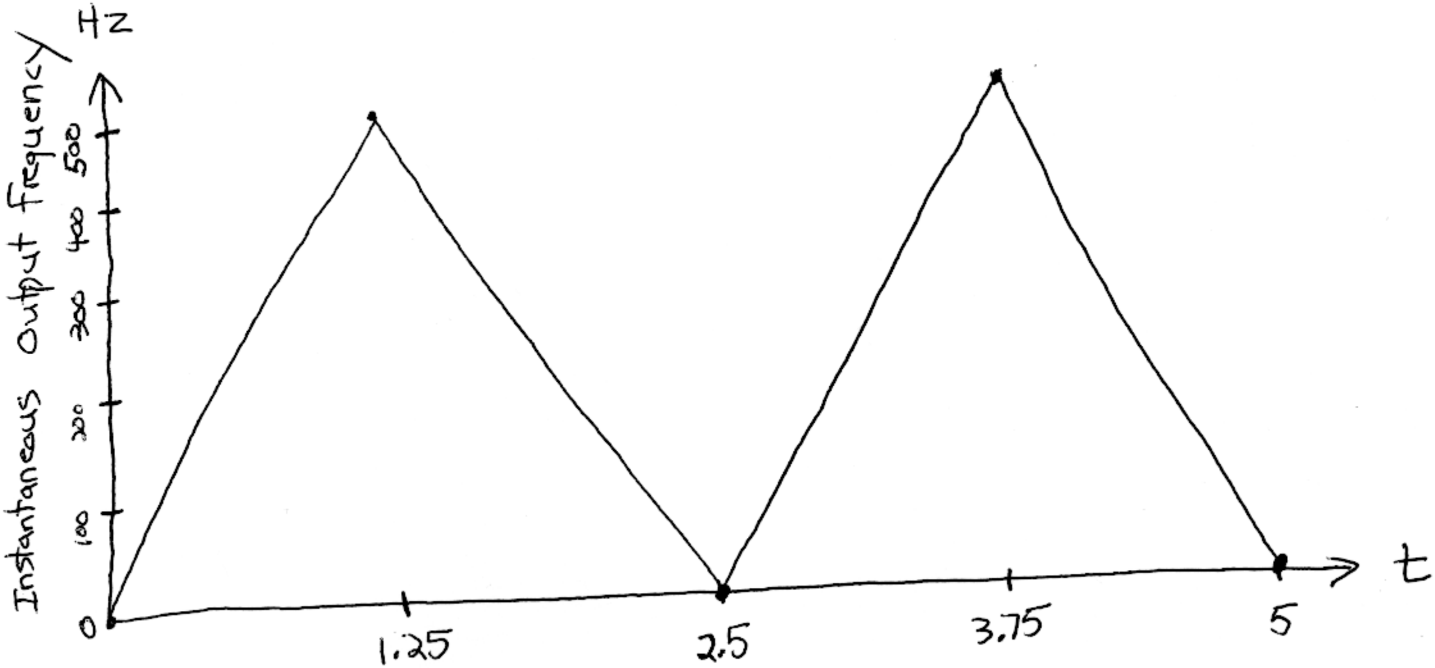
<u>For $0 \le t \le 1.25$:</u> Input frequencies: 1000 to 500 Hz
Normalized frequencies: $2\pi(1)$ to $2\pi(.5)$
After <u>folding</u>: $2\pi(0)$ to $2\pi(.5)$
Output frequencies: <u>0 to 500 Hz</u>

<u>For $1.25 \le t \le 2.5$:</u> Input frequencies: 500 to 0 Hz
Normalized frequencies: $2\pi(.5)$ to $2\pi(0)$
Output frequencies: <u>500 to 0 Hz</u>

(continued next page)

For $2.5 \le t \le 3.75$: Input frequencies: 0 to −500 Hz
Normalized frequencies: $2\pi(0)$ to $2\pi(-.5)$
Output frequencies: 0 to 500 Hz

For $3.75 \le t \le 5.0$: Input frequencies: −500 to −1000 Hz
Normalized frequencies: $2\pi(-.5)$ to $2\pi(-1)$
After aliasing: $2\pi(.5)$ to $2\pi(0)$
Output frequencies: 500 to 0 Hz



6.5 a) Note that $x_1[n] = 0$ for all n except $n=-1$ and $n=1$. Thus, given $y_1[n] = \sum_{k=0}^{M} b_k x_1[n-k]$, we may write

$$
\begin{aligned}
y_1[-1] &= & & b_0 x_1[-1] \\
y_1[0] &= & & b_1 x_1[-1] \\
y_1[1] &= b_0 x_1[1] &+& b_2 x_1[-1] \\
y_1[2] &= b_1 x_1[1] &+& b_3 x_1[-1] \\
y_1[3] &= b_2 x_1[1] &+& b_4 x_1[-1] \\
y_1[4] &= b_3 x_1[1] &+& b_5 x_1[-1] \\
&\vdots & & \vdots
\end{aligned}
$$

Substituting values for y_1 and x_1 gives

$$
\left\{
\begin{aligned}
0 &= b_0 & &\longrightarrow & b_0 &= \boxed{0} \\
1 &= b_1 & &\longrightarrow & b_1 &= \boxed{1} \\
2 &= b_0 + b_2 & &\longrightarrow & b_2 &= 2 - b_0 = 2 - 0 = \boxed{2} \\
4 &= b_1 + b_3 & &\longrightarrow & b_3 &= 4 - b_1 = 4 - 1 = \boxed{3} \\
2 &= b_2 + b_4 & &\longrightarrow & b_4 &= 2 - b_2 = 2 - 2 = 0 \\
3 &= b_3 + b_5 & &\longrightarrow & b_5 &= 3 - b_3 = 3 - 3 = 0 \\
0 &= b_4 + b_6 & &\longrightarrow & b_6 &= -b_4 = 0 \\
0 &= b_5 + b_7 & &\longrightarrow & b_7 &= -b_5 = 0 \\
&\vdots & & & &\vdots
\end{aligned}
\right.
$$

all other $b_k = 0$

$$\boxed{M = 3, \quad \{b_0, b_1, b_2, b_3\} = \{0, 1, 2, 3\}}$$

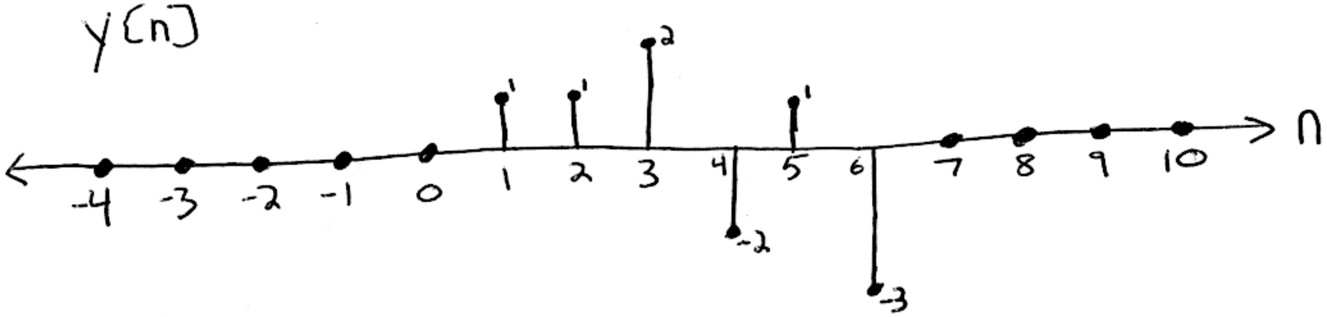
b) $x[n] = (\delta[n] + \delta[n-2]) - (\delta[n-1] + \delta[n-3])$

$= \quad x_1[n-1] \quad - \quad x_1[n-2]$

By LTI properties we therefore have

$y[n] = y_1[n-1] - y_1[n-2]$

n	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8
$y_1[n-1]$	0	0	0	0	0	1	2	4	2	3	0	0	0
$-y_1[n-2]$	0	0	0	0	0	0	-1	-2	-4	-2	-3	0	0
$y[n]$	0	0	0	0	0	1	1	2	-2	1	-3	0	0



c) $h[n] = b_0\delta[n] + b_1\delta[n-1] + b_2\delta[n-2] + b_3\delta[n-3]$

$= \boxed{\delta[n-1] + 2\delta[n-2] + 3\delta[n-3]}$

d) Filter is $\boxed{\text{causal}}$ because $h[n] = 0$ for all $n < 0$.