ECE 2025 HW 7 SOLUTIONS
DUE WEEK OF 27 FEB

7.1 + 7.2 LTI + CAUSAL?

DEFINITIONS

- (i) p140 **LINEAR** IF: $a f(x_1) + b f(x_2) = f(a x_1 + b x_2)$
- (ii) p139 **TIME INVARIANT** IF: $y[n-n_0] = f(x[n-n_0])$ -or- delayed output = f(delayed input)
- (iii) p123 **CAUSAL** IF OUTPUT DOES NOT DEPEND ON FUTURE VALUES

(i) LINEAR?

7.1a i

$$\alpha(x_1[n] + 3x_1[n-1] + x[n-2]) + \beta(x_2[n] + 3x_2[n-1] + x_2[n-2])$$

$$\stackrel{?}{=} (\alpha x_1[n] + \beta x_2[n]) + 3(\alpha x_1[n-1] + \beta x_2[n-1]) + (\alpha x_1[n-2] + \beta x_2[n-2])$$

$$\stackrel{?}{=} \alpha(x[n] + 3x_1[n-1] + x_1[n-2]) + \beta(x_2[n] + 3x_2[n-1] + x_2[n-2])$$

✓ TRUE

7.2a i

$$\alpha x[n+1]^2 + \beta x_2[n+2]^2 \stackrel{?}{=} (\alpha x_1[n+1] + \beta x_2[n+1])^2$$

$$= \alpha^2 x_1[n+1]^2 + \alpha\beta x_1[n+1]x_2[n+1] + \beta^2 x_2[n+1]$$

Not Identical

New CROSS-PRODUCT TERM (CAN CAUSE NEW FREQUENCIES...)

NOT IDENTICAL

~~LINEAR~~ ✓ NON-LINEAR

(ii) TIME INVARIANT?

SUBSTITUTE for n: $m - m_0$

7.1a ii

$$y[m-m_0] = x[m-m_0] + 3x[m-m_0-1] + x[m-m_0-2]$$

$$= f(x[m-m_0])$$

✓ TRUE

7.2a ii

$$y[m-m_0] = (x[m-m_0])^2$$

$$= f(x[m-m_0])$$

TRUE ✓

(iii) CAUSAL?

7.1a iii

$$y[now] = x[now] + 3x[now-1] + x[now-2]$$

now — now; history — history

✓ TRUE - Does not depend on future

7.2a iii

$$y[now] = (x[now+1])^2$$

now — future

~~CAUSAL~~ ✓ NON-CAUSAL - Depends on future

7.1

7.2

FIND $y_1[n]$ FOR $x_1[n] = 2\cos\left(\frac{3}{4}\pi n\right)$

7.1a — LINEAR

$y_1[n] = x[n] + 3x[n-1] + x[n-2]$

$y_1[n] = \left(e^{-j\frac{3}{4}\pi[n-0]} + e^{+j\frac{3}{4}\pi[n-0]}\right) + 3\left(e^{-j\frac{3}{4}\pi[n-1]} + e^{+j\frac{3}{4}\pi[n-1]}\right) + \left(e^{-j\frac{3}{4}\pi[n-2]} + e^{+j\frac{3}{4}\pi[n-2]}\right)$

$= e^{-j\frac{3}{4}\pi n}\left[e^{+j\frac{3}{4}\pi 0} + 3e^{+j\frac{3}{4}\pi 1} + e^{+j\frac{3}{4}\pi 2}\right] + e^{+j\frac{3}{4}\pi n}\left[e^{-j\frac{3}{4}0\pi} + 3e^{-j\frac{3}{4}\pi 1} + e^{-j\frac{3}{4}\pi 2}\right]$

$= e^{-j\frac{3}{4}\pi n}\left(1.5858\angle{+3/4\pi\,[rad]}\right) + e^{+j\frac{3}{4}\pi n}\left(1.5858\angle{-3/4\pi\,[rad]}\right)$

(TIME VARYING: $e^{\pm j\frac{3}{4}\pi n}$; CONSTANT: $1.5858\angle{\pm 3/4\pi\,[rad]}$; CONJUGATES)

$= e^{-j\hat{\omega}n} A^* + e^{+j\hat{\omega}n} A$

$= 2 \cdot A \cdot \cos(\hat{\omega}n)$

$= 3.1716 \cos\left(\frac{3}{4}\pi n - \frac{3}{4}\pi\right)$

SAME FREQUENCY AS INPUT ($x_1[n]$) SIGNAL

7.2b — NON-LINEAR

$y_1[n] = \left(x[n+1]\right)^2$

$y_1[n] = 2\cos\left(\frac{3}{4}\pi[n+1]\right)$

$= e^{-j\frac{3}{4}\pi(n+1)} + e^{+j\frac{3}{4}\pi(n+1)}$

(CROSS MULTIPLY exp() TERMS
COLLECT TERMS
INVERSE EULER → COSINE)

or

(TRIG IDENTITY
$2\cos(\alpha)\cos(\beta) = \cos(\alpha+\beta) + \cos(\alpha-\beta)$)

$y_1[n] = 2\cos\left(\frac{3}{4}\pi(n+1) + \frac{3}{4}\pi(n+1)\right) + 2\cos(0)$

$= 2\cos\left(\frac{3}{2}\pi(n+1)\right) + 2$

$= 2\cos\left(\frac{1}{2}\pi(n+1)\right) + 2$

0 rad/sec = "DC"

$\frac{\pi}{2}$ rad/sample

$\frac{3\pi}{4}$ rad/sample → GONE (INITIAL FREQ)

NEW FREQUENCIES: RESULT OF NON-LINEAR SYSTEM

7.1

7.2

FIND $y_2[n]$ WHEN $x_2[n] = 4 + 4\cos\left(\frac{3}{4}\pi(n-1)\right)$

7.1e

$1\cdot4 + 3\cdot4 + 1\cdot4 = 5\cdot4 = 20$

$$y_2[n] = 5\cdot4 + 4\cos\left(\tfrac{3}{4}\pi(n-1)\right)$$
$$= 5\cdot4 + 2\cdot\left(y_1[n-1]\right)$$
$$= 5\cdot4 + 2\cdot\left(3.1716\cos\left(\tfrac{3}{4}\pi(n-1) - \tfrac{3}{4}\pi\right)\right)$$
$$= 5\cdot4 + 6.3431\cos\left(\tfrac{3}{4}\pi n - \tfrac{3}{4}\pi - \tfrac{3}{4}\pi\right)$$
$$= 5\cdot4 + 6.3431\cos\left(\tfrac{3}{4}\pi n - \tfrac{3}{2}\pi\right)$$

SAME FREQUENCIES AS INPUT SIGNAL

NON-LINEAR – PLUG + CHUG

LINEAR – SCALING + SUPER-POSITION

7.2c

$$y_2[n] = \left[4 + 4\cos\left(\tfrac{3}{4}\pi(n-1+1=n)\right)\right]^2$$

NON-LINEAR – SO WE CANNOT USE SCALING OR SUPERPOSITION

SO, PLUG + CHUG

$$= 4^2\left[1 + \cos\left(\tfrac{3}{4}\pi n\right)\right]^2$$
$$= 16\left[1 + 2\cos\left(\tfrac{3}{4}\pi n\right) + \cos\left(\tfrac{3}{4}\pi n\right)^2\right]$$
$$= 8\left[2 + 4\cos\left(\tfrac{3}{4}\pi n\right) + \cos\left(\tfrac{3}{2}\pi n\right) + 1\right]$$

FROM TRIG IDENTITY

$$= 24 + 32\cos\left(\tfrac{3}{4}\pi n\right) + 8\cos\left(\tfrac{3}{2}\pi n\right)$$
$$= 24 + 32\cos\left(\tfrac{3}{4}\pi n\right) + 8\cos\left(\tfrac{1}{2}\pi n\right)$$

FREQUENCIES IN INPUT SIGNAL — A NEW FREQUENCY

7.1b

$$y[n]\Big|_{x[n]=\delta[n]} = [1, 3, 1] = h[n]$$

$h[n<0]=0$
$h[0]=1 = b_0$
$h[1]=3 = b_1$
$h[2]=1 = b_2$
$h[n>2]=0 = b_{k>2}$

$$H(\hat{\omega}) = \sum_{k=0}^{\text{FILTER LENGTH}-1} b_k e^{-j\hat{\omega}k} = \sum_{k=0}^{\text{FILTER LENGTH}-1} h[k] e^{-j\hat{\omega}k}$$ FOR LTI FIR system

$$= 1 e^{-j\hat{\omega}0} + 3 e^{-j\hat{\omega}1} + 1 e^{-j\hat{\omega}2}$$

$$= e^{-j\hat{\omega}}\left[e^{+j\hat{\omega}1} + 3\cancel{e^{-j\hat{\omega}0}}\,1 + e^{-j\hat{\omega}1}\right]$$

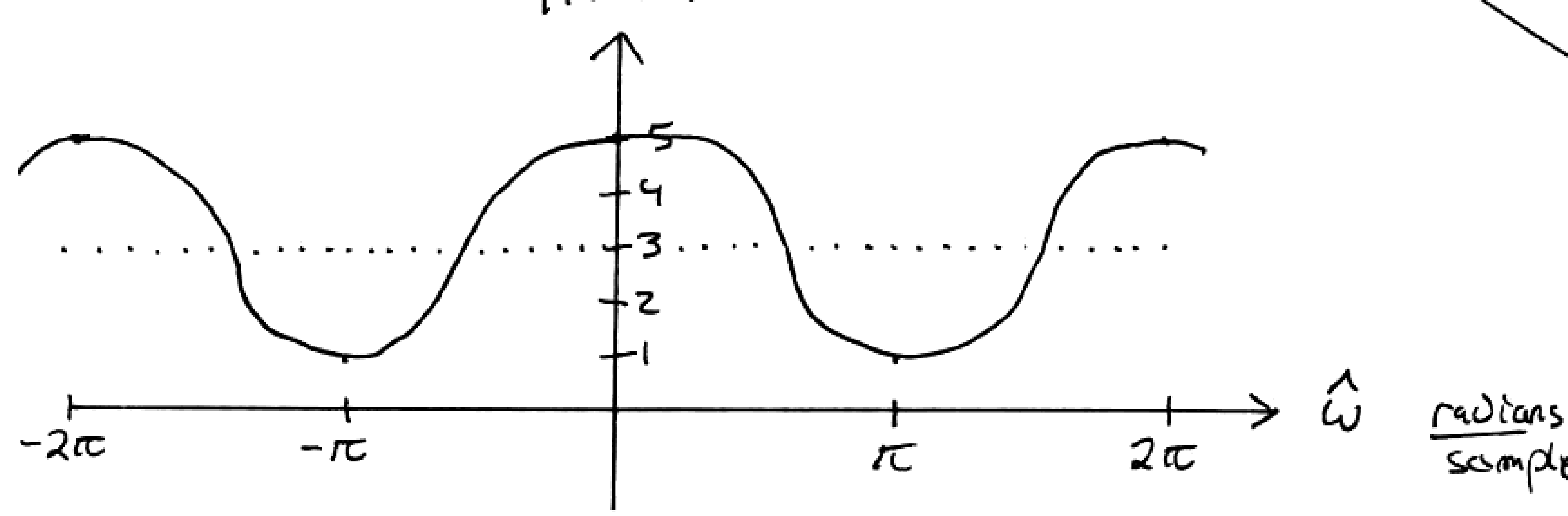
$$= e^{-j\hat{\omega}}\left[3 + e^{-j\hat{\omega}} + e^{+j\hat{\omega}}\right]$$

$$= \underbrace{e^{-j\hat{\omega}}}_{\text{PHASE}}\ \underbrace{\left[3 + 2\cos(\hat{\omega})\right]}_{\text{MAGNITUDE}}$$ APPLY INVERSE EULER (not "Oiler")

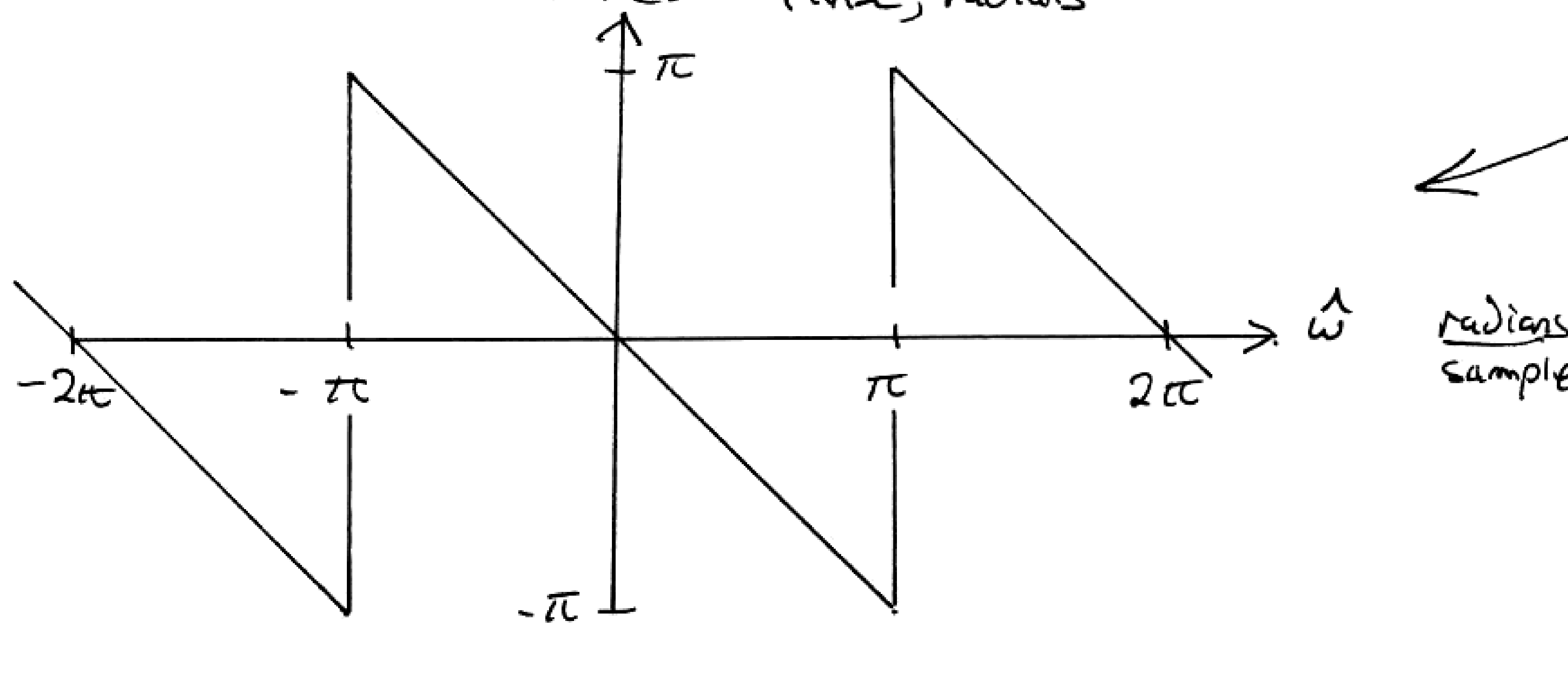
$$H(\hat{\omega}) = (3 + 2\cos(\hat{\omega}))\angle(-\hat{\omega}\ \text{radians}) = |H(\hat{\omega})|\ \angle H(\hat{\omega})$$

7.1c

$|H(\hat{\omega})|$ MAGNITUDE



$\angle H(\hat{\omega})$ PHASE, radians



PROBLEM 7.2*:

A second discrete-time system is defined by the input/output relation

$$y[n] = (x[n+1])^2. \tag{1}$$

(b) For the system of Equation (1), determine the output $y_1[n]$ when the input is

$$x_1[n] = 2\cos(0.75\pi n) = e^{j0.75\pi n} + e^{-j0.75\pi n}$$

The output can be computed as follows:

$$\begin{aligned} y_1[n] &= (x_1[n+1])^2 \\ &= \left(e^{j0.75\pi(n+1)} + e^{-j0.75\pi(n+1)}\right)^2 \\ &= e^{j1.5\pi(n+1)} + 2 + e^{-j1.5\pi(n+1)} \end{aligned}$$

Note that the frequencies $\hat{\omega} = 1.5\pi$ and $\hat{\omega} = -1.5\pi$ do not fall in the range $-\pi < \hat{\omega} \leq \pi$, so they have aliases within that range at $\hat{\omega} = -0.5\pi$ and $\hat{\omega} = 0.5\pi$, and the equation can be rewritten as follows:

$$\begin{aligned} y_1[n] &= e^{-j0.5\pi(n+1)} + 2 + e^{j0.5\pi(n+1)} \\ &= 2 + 2\cos(\pi(n+1)/2) \end{aligned}$$

(c) For the system of Equation (1), determine the output $y_2[n]$ when the input is

$$x_2[n] = 4 + 4\cos(0.75\pi(n-1)).$$

As in Part (b), $x_2[n]$ can be rewritten as follows:

$$x_2[n] = 4 + 2e^{j0.75\pi(n-1)} + 2e^{-j0.75\pi(n-1)}$$

From this, we can compute $y_2[n]$ as follows:

$$\begin{aligned} y_2[n] &= \left(4 + 2e^{j0.75\pi(n-1+1)} + 2e^{-j0.75\pi(n-1+1)}\right)^2 \\ &= 24 + 16e^{j0.75\pi n} + 16e^{-j0.75\pi n} + 4e^{j1.5\pi n} + 4e^{-j1.5\pi n} \\ &= 24 + 16e^{j0.75\pi n} + 16e^{-j0.75\pi n} + 4e^{-j0.5\pi n} + 4e^{j0.5\pi n} \\ &= 24 + 32\cos(0.75\pi n) + 8\cos(0.5\pi n) \end{aligned}$$

(d) Notice that in parts (b) and (c), the frequency is **doubled** by the squaring operation, so the input frequency of $\hat{\omega} = 0.75\pi$ becomes $\hat{\omega} = 1.5\pi$ in the output. However, we can add/subtract 2π from the output frequency to move $\hat{\omega} = 1.5\pi$ to $\hat{\omega} = -0.5\pi$. This is equivalent to what happens in aliasing of an A/D converter.

Another way to say this is to claim that since the squaring operation doubles the frequency, the sampling rate is no longer high enough. In any event, we can say that we have an alias of $\hat{\omega} = 1.5\pi$ at $\hat{\omega} = -0.5\pi$.

7.3

a) SEE PLOTS OF $\tilde{D}(\hat{\omega},6)$ and $|\tilde{D}(\hat{\omega},6)|$

ZERO CROSSINGS ARE MINS IN $|\tilde{D}(\hat{\omega},6)|$

b) PERIOD FOR NUMERATOR $\{\sin(3\hat{\omega})\}$ IS: $3\hat{\omega} = 2\pi$ or $\hat{\omega} = \frac{2}{3}\pi$

PERIOD FOR DENOMINATOR $\{\sin(\frac{1}{2}\hat{\omega})\}$ IS: $\frac{1}{2}\hat{\omega} = 2\pi$ or $\hat{\omega} = 4\pi$

BECAUSE $\frac{2}{3}\pi$ FITS AN INTEGER NUMBER OF TIMES INTO 4π

USE 4π = PERIOD

c) $\max(\tilde{D}(\hat{\omega},6))$ OCCURS WHEN THE DENOMINATOR $\to 0$

OR WHEN $\sin(\frac{1}{2}\hat{\omega}) = 0$

$\frac{1}{2}\hat{\omega} = k\pi$

$\hat{\omega} = k \cdot 2\pi$ $\qquad \hat{\omega} = [-6\pi, -4\pi, -2\pi, 0, 2\pi, 4\pi \ldots]$

d) $x(t) = 3 + 5\cos(4000\pi t) + 6\cos(6000\pi t - \pi/2)$

f		$\hat{f}$		$\hat{\omega}$	COMPLEX AMPLITUDE
3 kHz	=	$f_s \frac{1}{4}$	=	$\frac{\pi}{2}$	$6\angle -\frac{\pi}{2}$
2 kHz	=	$f_s \frac{1}{6}$	=	$\frac{\pi}{3}$	$5\angle 0$
0 Hz	=	$f_s 0$	=	0	$3\angle 0$

$H(\hat{\omega} = 0 = DC) = 1$

$H(\hat{\omega} = \frac{\pi}{3}) = 0 \cdot e^{-j\frac{5}{2}\cdot\frac{\pi}{3}} = 0$

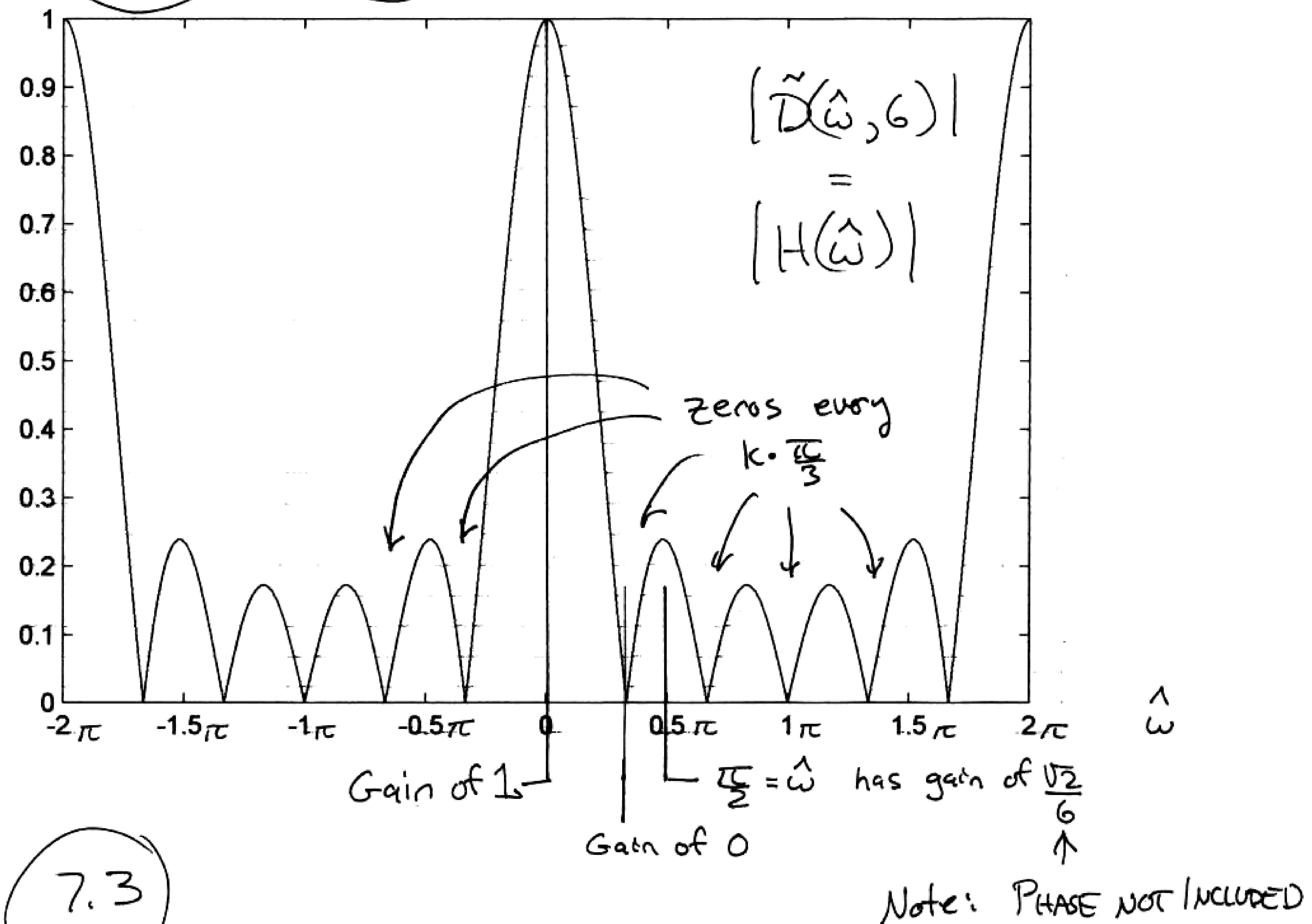
$H(\hat{\omega} = \frac{\pi}{2}) = -\frac{\sqrt{2}}{6} e^{-j\frac{5}{2}\cdot\frac{\pi}{2}} = \frac{\sqrt{2}}{6} e^{-j\frac{\pi}{4}} = \frac{\sqrt{2}}{6}\angle -\frac{\pi}{4}$

If we low pass filter to allow only $\left(\begin{matrix}\hat{\omega} < \pi \\ f < 6\text{kHz}\end{matrix}\right)$ rad/samp in $y(t)$:

$y(t) = 1 \cdot 3 + 0 \cdot 5\cos(4000\pi t) + (\frac{\sqrt{2}}{6}\angle -\frac{\pi}{4})\cos(6000\pi t - \frac{\pi}{2}) \cdot 6$

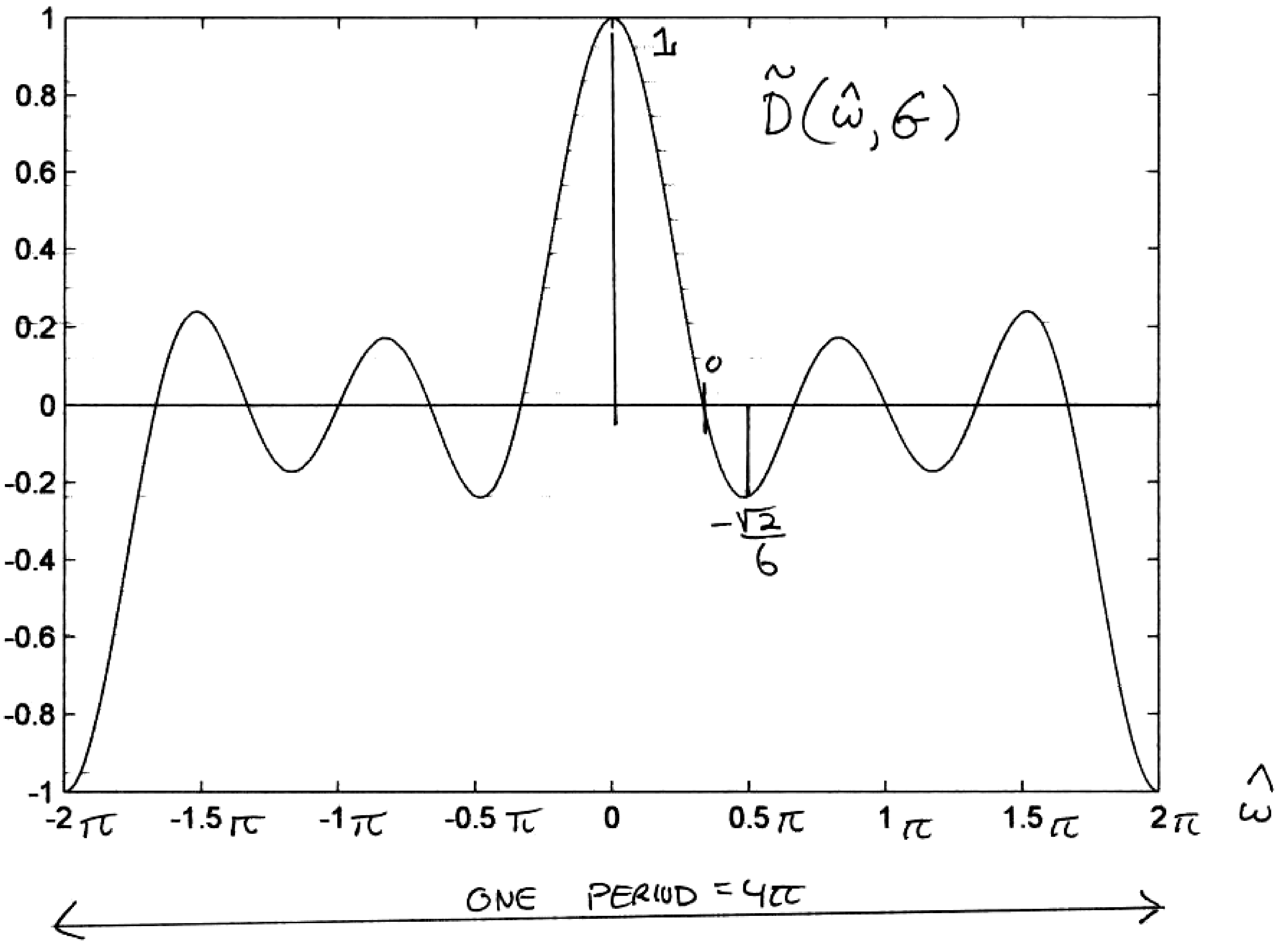
$= 3 + \sqrt{2}\cos(6000\pi t - \frac{3}{4}\pi)$

7.46 and 7.3



$|\tilde{D}(\hat{\omega}, 6)| = |H(\hat{\omega})|$
zeros every $k \cdot \frac{\pi}{3}$
Gain of 1
Gain of 0
$\frac{\pi}{2} = \hat{\omega}$ has gain of $\frac{\sqrt{2}}{6}$
Note: PHASE NOT INCLUDED
$\hat{\omega}$

7.3



$\tilde{D}(\hat{\omega}, 6)$
1
0
$-\frac{\sqrt{2}}{6}$
$\hat{\omega}$
ONE PERIOD = 4π

(7.4)

a) $h[n] = \left[\frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}\right]$

USE EQUATIONS ON P 158 AND P 179 OR DERIVE...

$$H(\hat{\omega}) = \sum_{k=0}^{5} h[k] e^{j\hat{\omega}k} \quad \text{(eqn 6.1.4)}$$

$$= e^{-j\frac{5}{2}\hat{\omega}} \frac{1}{6} \frac{\sin(3\hat{\omega})}{\sin(\hat{\omega}/2)} \quad \text{(eqn 6.74)}$$

b) SEE ATTACHED PLOT (ALSO USED FOR PROBLEM 7.3)

c) ONCE PAST INITIAL TRANSITION EFFECTS ("END-EFFECTS") THE OUTPUT FREQUENCY LOOKS LIKE A SCALED + SHIFTED COPY OF THE INPUT FREQUENCY.

BECAUSE $\hat{\omega} = \frac{\pi}{2}$, THERE ARE 4 SAMPLES PER 1 PERIOD

MATLAB INDEX → 1 2 3 4 5 6 7 8 9 10 11

in ← $x[n]$ = [0 6 0 −6 0 6 0 −6 0 6 0 −6 0 ...] (PERIOD = 4 SAMPLES)

h

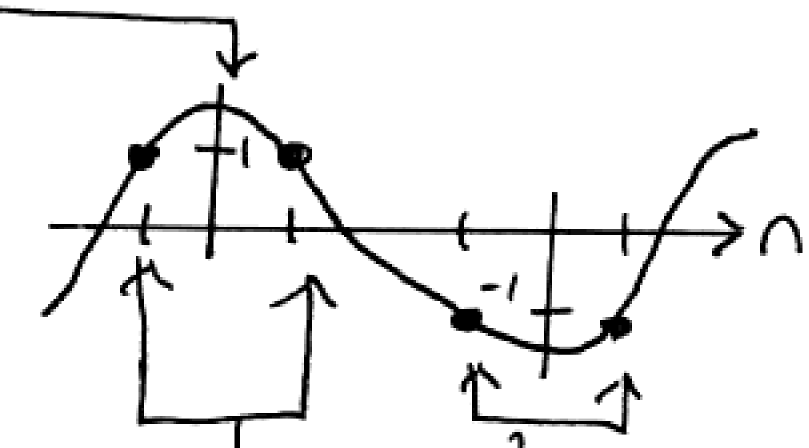
out → $y[n]$ = [0 1 1 0 0 1 1 −1 −1 1 1 −1 −1 ...]

(TRANSIENT RESPONSE: 0 1 1 0 0; STEADY STATE SINUSOID: 1 1 −1 −1 ...)

$$y[n \geq 6] = \sqrt{2} \cos\left(\frac{1}{2}\pi n - \frac{3}{4}\pi\right)$$

IF $A\cos(45^\circ) = 1$ THEN $A = \sqrt{2}$

I GUESSED + CONFIRMED



Symmetric
SO, MUST BE $0^\circ \pm 45^\circ$ or $180^\circ \pm 45^\circ$

− OR −

SOLVE
$y[7] = A\cos(\frac{1}{2}\pi 7 - \phi) = 1$
$y[8] = A\cos(\frac{1}{2}\pi 8 - \phi) = -1$
$y[9] = A\cos(\frac{1}{2}\pi 9 - \phi) = -1$
$y[10] = A\cos(\frac{1}{2}\pi 10 - \phi) = 1$ FOR A AND ϕ
etc...

d) SEE PLOT OF $H(\hat{\omega})$: ZERO FOR ANY $\hat{\omega} = k \cdot \frac{1}{3}\pi$ $k \in$ INTEGER

− OR −

WHEN IS (|NUMERATOR OF $H(\hat{\omega})$| = $\sin(3\hat{\omega})$) = ZERO?

WHEN $\sin(\hat{\omega}3) = 0$

$$\hat{\omega}3 = k\pi$$

$$\hat{\omega} = k\frac{\pi}{3}$$

RUNNING AVERAGE FILTER DERIVATION

PROBLEMS 7.3 + 7.4

SEMI-INFINITE GEOMETRIC SERIES

$$(\quad a^1 + a^2 + a^3 + a^4 + \ldots) = \sum_{k=1}^{\infty} a^k = \sum_{k=0}^{\infty} a^{k+1} = a\sum_{k=0}^{\infty} a^k$$

$$+ (a^0 \quad) = \longrightarrow + \quad a^0 = 1$$

$$(a^0 + a^1 + a^2 + a^3 + a^4 + \ldots) = \sum_{k=0}^{\infty} a^k = 1 + a\sum_{k=0}^{\infty} a^k$$

$$\sum_{k=0}^{\infty} a^k - a\sum_{k=0}^{\infty} a^k = 1$$

$$(1-a)\sum_{k=0}^{\infty} a^k = 1$$

$$\boxed{\sum_{k=0}^{\infty} a^k = \frac{1}{1-a} \quad \text{FOR } |a| < 1}$$

SEMI-INFINITE

FINITE GEOMETRIC SERIES

$0 \longrightarrow \infty$

$$\sum_{k=0}^{\infty} a^k = \frac{1}{1-a}$$

$M \longrightarrow \infty$

$$\sum_{\substack{k=M \\ k-M=0}}^{\infty} a^{k-M} = a^{-M}\sum_{k=M}^{\infty} a^k = \frac{1}{1-a}$$

$$\sum_{k=M}^{\infty} a^k = \frac{a^M}{1-a}$$

$0 \quad M-1$

$$\sum_{k=0}^{M-1} a^k = \sum_{k=0}^{\infty} a^k - \sum_{k=M}^{\infty} a^k = \boxed{\frac{1-a^M}{1-a} = \sum_{k=0}^{M-1} a^k}$$

FINITE

$h[n] \to H(\hat{\omega})$

$$H(\hat{\omega}) = \sum_{k=0}^{L-1} h[n] e^{-j\hat{\omega}k} = \left\{ \frac{1}{L}\left(\frac{1-e^{-j\hat{\omega}L}}{1-e^{-j\hat{\omega}}}\right) = \frac{1}{L}\sum_{k=0}^{L-1} e^{j\hat{\omega}k} \right\} \text{ FOR L-ELEMENT RUNNING AVERAGE}$$

$$= \frac{1}{L}\,\frac{e^{-j\hat{\omega}\frac{L}{2}}}{e^{j\hat{\omega}2}}\,\frac{\left(e^{j\hat{\omega}\frac{L}{2}} - e^{-j\hat{\omega}\frac{L}{2}}\right)}{\left(e^{j\hat{\omega}/2} - e^{j\hat{\omega}/2}\right)} = \frac{e^{j\hat{\omega}(L-1)/2}}{L} \cdot \frac{\sin(\hat{\omega}L/2)}{\sin(\hat{\omega}/2)}$$

(7.5)

b) $H_1(\hat{\omega}) = e^{j\hat{\omega}0} + e^{-j\hat{\omega}2} = e^{-j\hat{\omega}}\left(e^{-j\hat{\omega}} + e^{j\hat{\omega}}\right)$ $\quad h_1 = [1, 0, 1]$

$= e^{-j\hat{\omega}}\, 2\cos(e\hat{\omega})$

$H_2(\hat{\omega}) = 7e^{-j\hat{\omega}5} + 7e^{-j\hat{\omega}6} = 7e^{-j\omega 5.5}\left(e^{-j\frac{\hat{\omega}}{2}} + e^{j\frac{\hat{\omega}}{2}}\right)$ $\quad h_2 = [0,0,0,0,0,7,7]$

$= 7e^{-j\omega 5.5}\, 2\cos\left(\frac{\hat{\omega}}{2}\right)$

a) $H_3(\hat{\omega}) = e^{-j\hat{\omega}} - e^{-j2\hat{\omega}} \Rightarrow h = [0, 1, -1]$

$= e^{-j\hat{\omega}\frac{3}{2}}\left(e^{j\omega\frac{1}{2}} - e^{-j\omega\frac{1}{2}}\right) = e^{-j\hat{\omega}\frac{3}{2}}\, 2j\sin(\hat{\omega}/2)$

c) $H_\Sigma = H_1 \cdot H_2 \cdot H_3 = \left(e^{j\hat{\omega}0} + e^{-j\hat{\omega}2}\right)\left(7e^{-j\hat{\omega}5} + 7e^{-j\hat{\omega}6}\right)\left(e^{-j\hat{\omega}} - e^{-j2\hat{\omega}}\right)$

= 8 TERMS
COLLECT CONJUGATES
INVERSE EULER → COSINE FORM

-OR-

d) IT'S PROBABLY FASTER TO CONVOLVE THEN $h_\Sigma \to H_\Sigma$ ($n \to \hat{\omega}$) (IN THIS CASE).

$h_\Sigma[n] = h_1[n] * h_2[n] * h_3[n]$

$= [1\ 0\ 1] * [0\ 0\ 0\ 0\ 0\ 7\ 7] * [0\ 1\ -1]$

$= [0\ 0\ 0\ 0\ 0\ 7\ 7\ 7\ 7] * [0\ 1\ -1]$

$= [0\ 0\ 0\ 0\ 0\ 0\ 7\ 0\ 0\ 0\ -7] \rightarrow y[n] = 7(x[n-6] - x[n-10])$

$H_\Sigma(\hat{\omega}) = 7e^{-j\hat{\omega}6} - 7e^{-j\hat{\omega}10}$

$= 7e^{-j\hat{\omega}8}\left(e^{j\hat{\omega}2} - e^{-j\hat{\omega}2}\right)$

$= 7e^{j\hat{\omega}8}\, 2j\sin(2\hat{\omega})$

$= 14e^{j\hat{\omega}8}\, e^{j\frac{\pi}{2}}\sin(2\hat{\omega})$

$= 14e^{j(\hat{\omega}8 + \frac{\pi}{2})}\sin(2\hat{\omega})$

$= \underbrace{e^{j(\hat{\omega}8 + \frac{\pi}{2})}}_{\text{PHASE}}\ \underbrace{14\sin(2\hat{\omega})}_{\text{MAGNITUDE}}$

Note: 6 SAMPLE DELAY BEFORE OUTPUT RESPONDS TO INPUT

THIS IS MANIFEST BY THE PHASE IN THE FREQUENCY RESPONSE

$\Phi \propto$ delay