GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2000
Problem Set #9

Assigned: 17-Mar-00
Due Date: Week of 27-Mar-00

Any grading questions on Quiz #2 must be resolved no later than 24-March; after that date the scores will not be changed.

Quiz #3 will be on 7-April (Friday). Coverage will be Homeworks #8, #9 and #10; as well as Chapter 7 in *DSP First* plus the Continuous-Time Signals & Systems notes (Chapters 10–12).

Reading: Read Chapter 10 in Notes.

$\Longrightarrow$ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

PROBLEM 9.1*:

Try your hand at expressing each of the following in a simpler form:

(a) $\delta(t-3) * [\delta(t) + 2e^{-t}\cos(5\pi t)u(t)] =$

(b) $[u(-t+3) - u(t)][\delta(t-1) + \delta(t-4)] =$

(c) $\dfrac{d}{dt}[\cos(5\pi t)u(t-1)] =$

(d) $\displaystyle\int_{-\infty}^{t} e^{-(\tau-1)}\delta(\tau-1)d\tau =$

Note: use properties of the impulse signal $\delta(t)$ and the unit-step signal $u(t)$ to perform the simplifications. For example, recall

$$\delta(t) = \frac{d}{dt}u(t) \qquad \text{where} \qquad u(t) = \begin{cases} 1 & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

Be careful to distinguish between multiplication and convolution; convolution is denoted by a "star", as in $u(t) * \delta(t-2)$.

PROBLEM 9.2*:

A linear time-invariant system has impulse response:

$$h(t) = \begin{cases} e^{-2t} & -1 \le t < 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Plot $h(\tau)$ and $h(t-\tau)$ as a functions of τ for $t = -5, 0$, and 5.

(b) Is the system causal? Justify your answer.

(c) Find the output $y(t)$ when the input is $x(t) = \delta(t+5)$.

(d) Use the convolution integral to determine the output $y(t)$ when the input is $x(t) = \begin{cases} 1 & 0 \le t < 5 \\ 0 & \text{otherwise.} \end{cases}$

PROBLEM 9.3*:

A continuous-time system is defined by the input/output relation

$$y(t) = \int_{-\infty}^{t+4} x(\tau)d\tau.$$

(a) Determine the impulse response, $h(t)$, of this system.

(b) Is this a stable system? Explain with a proof or counter-example.

(c) Is it a causal system? Explain with a proof or counter-example.

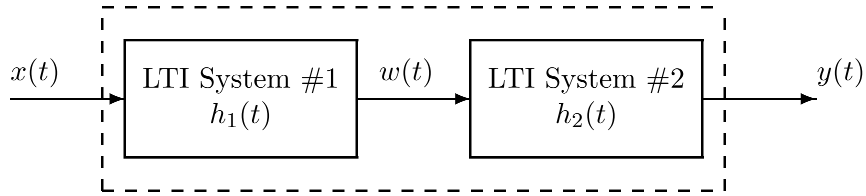
(d) Use the convolution integral to determine the output of the system when the input is the finite-length pulse:

$$x(t) = u(t+1) - u(t-1) = \begin{cases} 1 & -1 \le t < 1 \\ 0 & \text{elsewhere.} \end{cases}$$

Plot your answer.

(e) *In this part you can double check the previous part by using the step response of the system.* It happens to be true that the output from this system is $(t+4)u(t+4)$ when the input is $u(t)$. So, use the linearity and time-invariance properties of the system to determine the output when the input is $u(t+1) - u(t-1)$. State clearly how both linearity and time-invarince were used in this solution.

PROBLEM 9.4*:



(a) In this part, assme that the first system is described by the input/output relation

$$w(t) = \int_{-\infty}^{t} x(\tau)d\tau$$

and the second system has impulse response

$$h_2(t) = u(t-5)$$

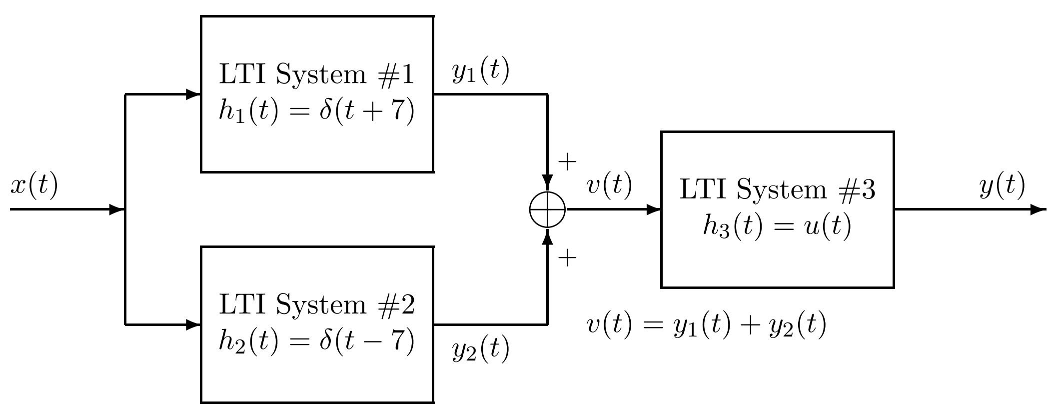
Find the impulse response of the overall system; i.e., find the output $y(t) = h(t)$ when the input is $x(t) = \delta(t)$.

(b) Now assume that the first system is described by the input/output relation

$$w(t) = \frac{dx(t)}{dt} + 3x(t)$$

and the overall system has an impulse response equal to an impulse, i.e., $h(t) = \delta(t)$. Show that the impulse response of the second system should be a one-sided exponential to make this true. Determine the formula for $h_2(t)$ in this case.
Note: this is the continuous-time version of deconvolution.

PROBLEM 9.5*:



(a) What is the impulse response of the overall LTI system (i.e., from $x(t)$ to $y(t)$)? Give your answer <u>both</u> as an equation and as a carefully labeled sketch.

(b) Is the overall system a causal system? Explain to receive credit.

(c) Is the overall system a stable system? Explain to receive credit.