GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2000
Problem Set #10

Assigned: 24-Mar-00
Due Date: Week of 3-April-00

Quiz #3 will be on 7-April (Friday). Coverage will be Homeworks #8, #9 and #10; as well as Chapter 7 in *DSP First* plus the Continuous-Time Signals & Systems notes (Chapters 10–12).

Reading: Read Ch. 11 all, and Ch. 12 (notes), pp. 1200–1214, pp. 1218–1229 and pp. 1232–1234.

$\Longrightarrow$ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

PROBLEM 10.1*:

In each of the following cases, use known Fourier transform pairs together with some Fourier transform properties to complete the following Fourier transform pair relationships:

(a) $x(t) = 2\delta(t-2)e^{t} \quad \Longleftrightarrow \quad X(j\omega) =$

(b) $x(t) = \qquad\qquad \Longleftrightarrow \quad X(j\omega) = \delta(\omega-5) + \delta(\omega+5)$

(c) $x(t) = \qquad\qquad \Longleftrightarrow \quad X(j\omega) = 20\dfrac{\sin(200\omega)}{\omega}$

(d) $x(t) = u(t+3)u(3-t) \quad \Longleftrightarrow \quad X(j\omega) =$

PROBLEM 10.2*:

Use the *delay property* of Fourier transforms, $x(t-t_d) \Longleftrightarrow e^{-j\omega t_d}X(j\omega)$, to determine the Fourier transform of the following signals:

(a) $x(t) = \delta(t-5)\cos(t)$

(b) $x(t) = \delta(t-5) * \cos(t)$

(c) $x(t) = u(t) - u(t-5)$

(d) $x(t) = e^{-7t}u(t) - e^{-7t}u(t-3)$
Hint: rearrange into the following form: $x(t) = e^{-7t}u(t) - \beta e^{-7(t-3)}u(t-3)$

(e) $x(t) = [u(t) - u(t-5)]\cos(50\pi t)$
In this part, use *frequency shifting property:* $x(t)e^{j\omega_c t} \Longleftrightarrow X(j(\omega-\omega_c))$,

PROBLEM 10.3*:

A continuous-time linear time-invariant system has impulse response

$$h(t) = \delta(t-3) - e^{-7(t-3)}u(t-3).$$

(a) Determine the frequency response $H(j\omega)$ of the system. Simplify your answer so that one part becomes a rational function with powers of $(j\omega)$ in the numerator and denominator, and the other part can be associated with the delay in $h(t)$.

(b) Plot the magnitude squared, $|H(j\omega)|^2 = H(j\omega)H^*(j\omega)$, versus ω. Likewise, plot the phase $\angle H(j\omega)$ as a function of ω.
Hint: use MATLAB or a calculator for this.

(c) Use superposition to find the output due to an input that is the sum of three terms:

$$x(t) = 7 + 7\cos(7t + \tfrac{1}{2}\pi) + \delta(t-7).$$

Hint: Use the easiest method (impulse response or frequency response) to find the output due to each component of the input.

PROBLEM 10.4*:

Here is a Fourier transform that will be useful in the lab project for Labs #9 and #10.

(a) Define the finite duration signal $x(t)$ to be one half cycle of a sine wave:

$$x(t) = \begin{cases} \sin(\pi t) & \text{for } 0 \le t \le 1 \\ 0 & \text{elsewhere} \end{cases}$$

Make a plot of $x(t)$ over the range $-3 \le t \le 3$.

(b) Determine $X(j\omega)$, the Fourier transform of $x(t)$. One approach is to use Euler's formula to break the integral down into integrating two complex exponentials.

(c) Plot the magnitude and phase of $X(j\omega)$ from the previous part. You will probably have to use MATLAB to make these plots from the formula that you derive.

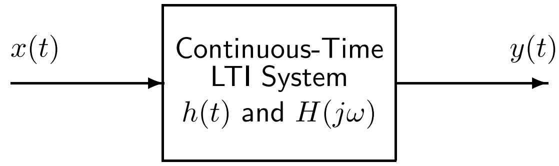
(d) Define a new signal

$$q(t) = \begin{cases} \cos(100\pi t) & \text{for } |t| \le 0.005 \\ 0 & \text{elsewhere} \end{cases}$$

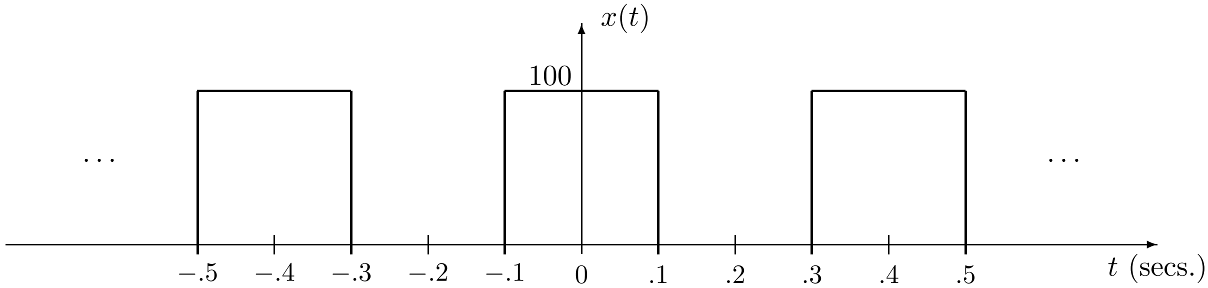
Use the scaling and shifting properties of the Fourier transform to write the formula for $Q(j\omega)$.
Hint: if you write $q(t)$ in terms of $x(t)$ as $q(t) = x(\alpha t - \beta)$, then you can apply the scaling and shifting properties. Which order should you use? Scale first and then shift, or vice versa?

(e) Prove that the $Q(j\omega)$ from the previous part is a purely real function; no imaginary part.

PROBLEM 10.5*:

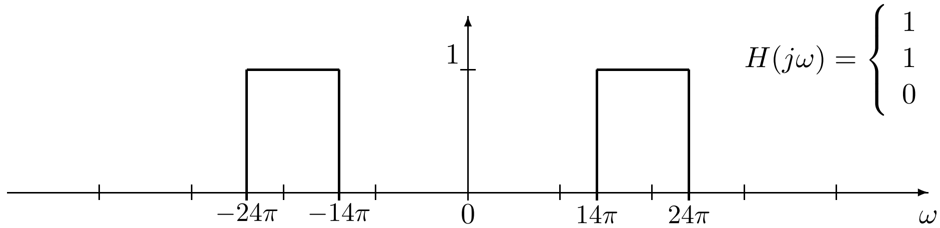


The input to the above LTI system is the periodic square wave $x(t)$ depicted below: **(Assume this same input for all parts below.)**



(a) Determine the Fourier series coefficients of the input signal $x(t)$. Utilize the known form for the square wave.

(b) Assume that the frequency response of the system is the ideal bandpass filter plotted below. On this same graph make a plot of the spectrum of the input signal $x(t)$. The spectrum is a plot versus frequency, so plot all the spectrum components that fit in the range: $-40\pi \le \omega \le 40\pi$.



$$H(j\omega) = \begin{cases} 1 & |\omega - 19\pi| \le 5\pi \\ 1 & |\omega + 19\pi| \le 5\pi \\ 0 & \text{elsewhere} \end{cases}$$

(c) Determine the spectrum of the output signal from the filter of part (b). Also, determine the Fourier series coefficients of the output signal.

(d) Also, give an equation for the output of the system, $y(t)$, that is valid for $-\infty < t < \infty$. **Your answer should be expressed in terms of only real quantities.**

(e) Now assume that the frequency response of the filter is changed to be

$$H(j\omega) = \begin{cases} 1 & \text{for } |\omega| > 3\pi \\ 0 & \text{for } |\omega| \le 3\pi \end{cases}$$

Determine output signal $y(t)$ from this new filter, and make a plot of $y(t)$ versus t.