GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2000
Problem Set #11

Assigned: 31-Mar-00
Due Date: Week of 10-April-00

Quiz #3 will be on 7-April (Friday). Coverage will be Homeworks #8, #9 and #10; as well as Chapter 7 in *DSP First* plus the Continuous-Time Signals & Systems notes (Chapters 10–12).

The will be a quiz review on Thursday (6-April) at 7pm in the ECE Auditorium.

Reading: Finish reading Chapter 12 and begin reading Chapter 13.

$\Longrightarrow$ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

The **THREE STARRED** problems will have to be turned in for grading. Problem 11.2 will count 30 points. A solution will be posted to the web.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

PROBLEM 11.1*:

In each of the following cases, use the table of known Fourier transform pairs together with the table of Fourier transform properties to complete the following Fourier transform pair relationships:

(a) $x(t) = \qquad\qquad\qquad \Longleftrightarrow \quad X(j\omega) = j20\pi\delta(\omega - 50\pi) - j20\pi\delta(\omega + 50\pi)$

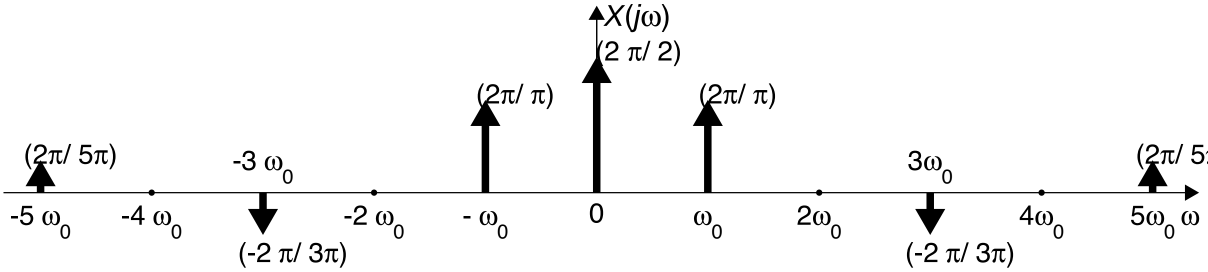
(b) $x(t) = \sin(4\pi t)\sin(50\pi t) \quad \Longleftrightarrow \quad X(j\omega) =$

(c) $x(t) = \dfrac{\sin 4\pi t}{\pi t}\sin(50\pi t) \quad \Longleftrightarrow \quad X(j\omega) =$

PROBLEM 11.2*:

This problem will be worth 30 points.

The periodic input to a LTI system $H(j\omega)$ has Fourier transform $X(j\omega)$ as defined below:



where the dark arrows denote impulses.

There are six possible filters: each one is described by one of the equations or graphs below. In each case determine the output signal $y(t)$. Justify your answer by giving a derivation, or by explaining how you used the Fourier transform filtering property to get $y(t)$. Give a simple formula for $y(t)$.

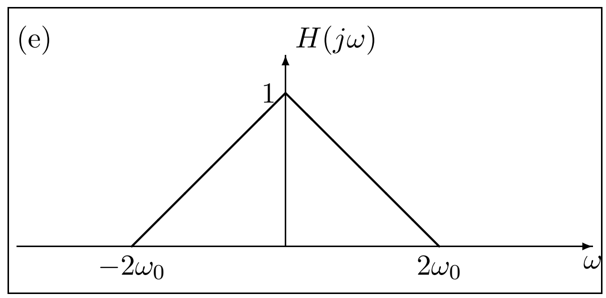
In most cases the output will be a sum of sinusoids, but it is permissible to write $y(t)$ in terms of $x(t)$, e.g., one of the answers can be written in the form $y(t) = Ax(t - t_d)$ and another one is $y(t) = x(t) - c$. Of course, you have to find the parameters, t_d, A and c.

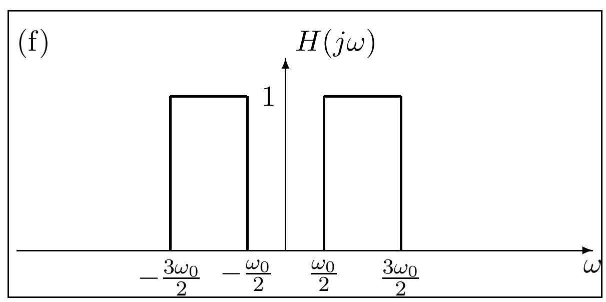
(a) $H(j\omega) = \begin{cases} 1 & |\omega| < \omega_0/2 \\ 0 & |\omega| > \omega_0/2 \end{cases}$

(b) $H(j\omega) = 500e^{-j2\omega/3}$

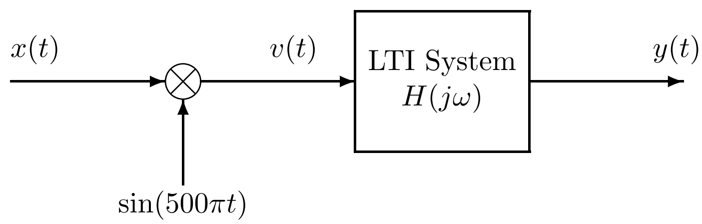
(c) $H(j\omega) = \begin{cases} e^{-j2\omega/3} & |\omega| < 3\omega_0/2 \\ 0 & |\omega| > 3\omega_0/2 \end{cases}$

(d) $H(j\omega) = \begin{cases} 0 & |\omega| < \omega_0/2 \\ 1 & |\omega| > \omega_0/2 \end{cases}$

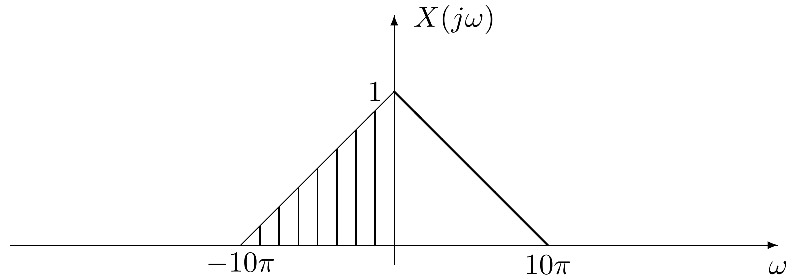




PROBLEM 11.3*:
Consider the following amplitude modulation system:



Assume that the input signal $x(t)$ has a bandlimited Fourier transform as depicted below



and the linear system has the frequency response of an ideal bandpass filter:

$$H(j\omega) = \begin{cases} 1 & 500\pi < |\omega| < 510\pi \\ 0 & \text{otherwise.} \end{cases}$$

(a) Plot the Fourier transform $H(j\omega)$ of the ideal BPF specified above.

(b) Plot the Fourier transform $V(j\omega)$ of the signal $v(t)$ at the output of the multiplier.

(c) Plot the Fourier transform $Y(j\omega)$ of the output signal $y(t)$ from the filter.

Note that the negative frequency portion of the Fourier transform $X(j\omega)$ is shaded. Mark the corresponding region or regions in your plots of $V(j\omega)$ and $Y(j\omega)$.

PROBLEM 11.4:

FOr a solution see Problem 12.4 in Fall-99.

A *periodic impulse train* with period T_0 is defined to be the signal

$$x(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_0).$$

(a) Plot this signal for $-3T_0 \le t \le 3T_0$.

(b) What is the fundamental frequency, ω_0 if $T_0 = 10$? **Use $T_0 = 10$ in all the following parts.**

(c) Determine the Fourier coefficients a_k in the Fourier series representation

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}.$$

(d) Plot the spectrum of this signal for $-4\omega_0 \le \omega \le 4\omega_0$.

(e) The periodic impulse train $x(t)$ is the input to a system with frequency response

$$H(j\omega) = \begin{cases} e^{-j\omega 4} & |\omega| < \omega_{co} \\ 0 & |\omega| > \omega_{co}. \end{cases}$$

Determine the output signal $y(t)$ if $\omega_{co} = \pi/T_0$.

(f) The periodic impulse train $x(t)$ is the input to a system with frequency response

$$H(j\omega) = \begin{cases} e^{-j\omega 4} & |\omega| < \omega_{co} \\ 0 & |\omega| > \omega_{co}. \end{cases}$$

Determine the output signal $y(t)$ if $\omega_{co} = 3\pi/T_0$.

PROBLEM 11.5:

In studying this material, you might want to work other old problems. Consult the solved problems under the EE-2201 link for more examples of filtering and modulation:

1. Spring & Fall-98, Winter & Spring 99, Problem Sets #3, #4 and #5.
2. Winter 98, Problem Sets #3, #4, #5 and #6.