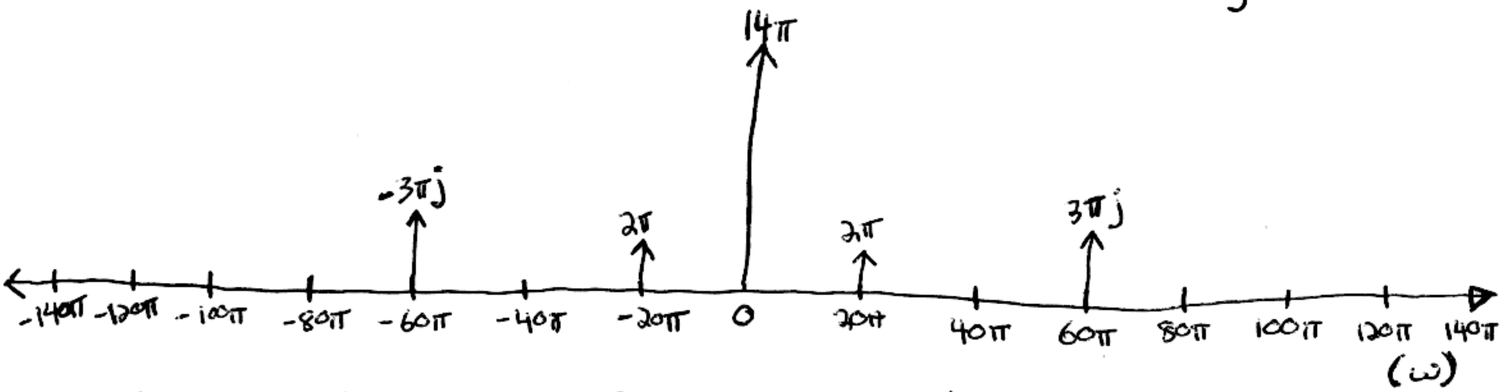
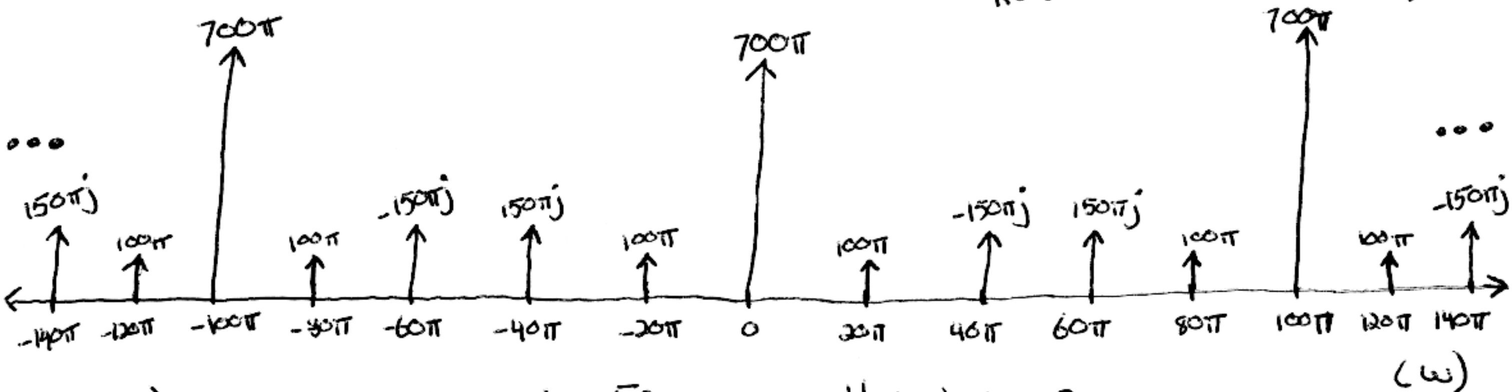
12.1 a) $X(j\omega) = 2\pi(7)\,\delta(\omega) + \pi(2)\,\delta(\omega \mp 20\pi) + \pi(3)\underbrace{e^{\pm j\pi/2}}_{\pm j}\delta(\omega \mp 60\pi)$



b) Highest non-zero frequency component of $X(j\omega)$ lies at $\omega_b = 60\pi$. Thus Nyquist rate $= 2\omega_b = 120\pi \ \frac{rad}{s}$ (60 Hz)
Therefore maximum value of $T_s = \frac{2\pi}{120\pi} = \frac{1}{60}$ sec.

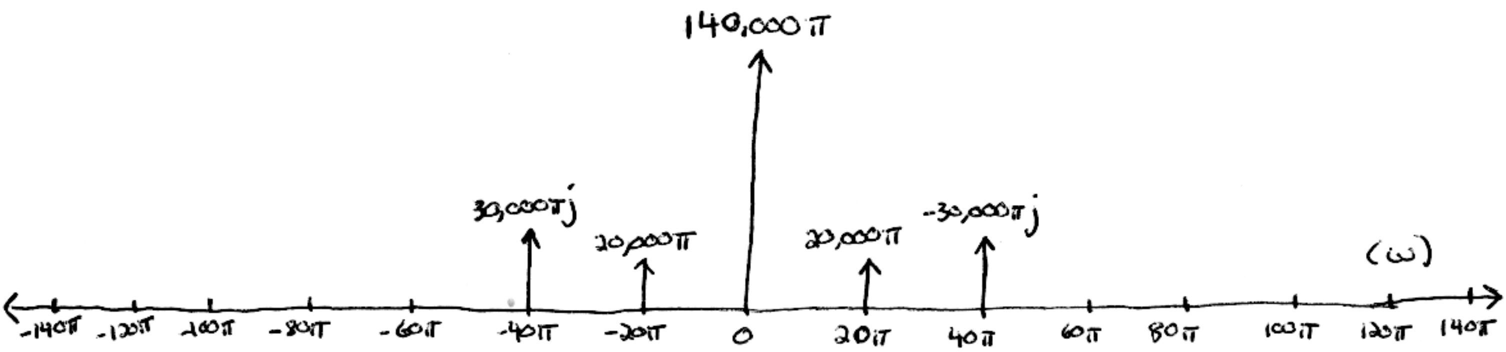
c) If $\omega_s = 100\pi$ then $P(j\omega) = \frac{2\pi}{T_s}\sum_{k=-\infty}^{\infty}\delta(\omega - k\omega_s) = 100\pi\sum_{k=-\infty}^{\infty}\delta(\omega - 100\pi k)$

and $X_s(j\omega) = \frac{1}{2\pi}X(j\omega) * P(j\omega) = 50\sum_{k=-\infty}^{\infty} X(j(\omega - 100\pi k))$



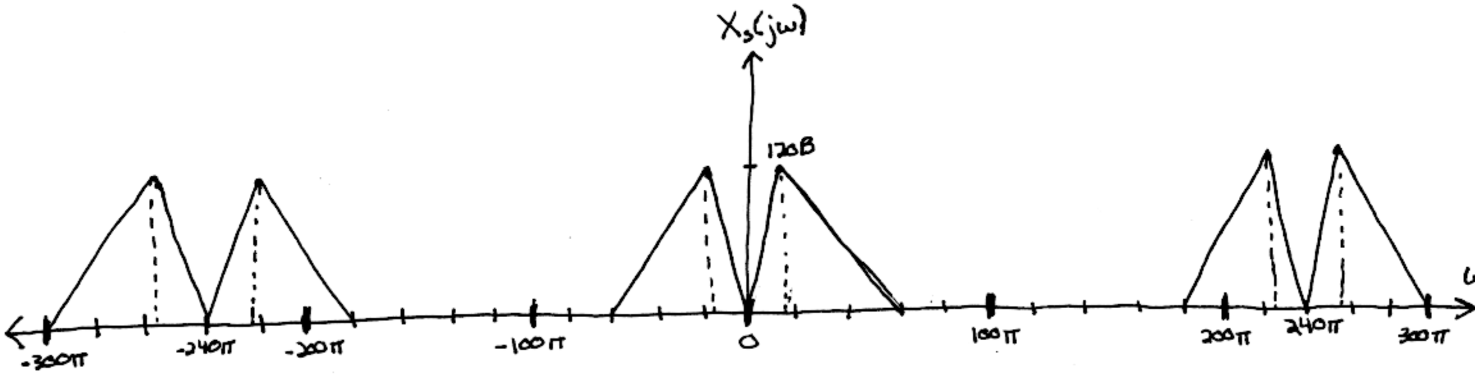
d) impluses at $|\omega| \leq 50\pi$ are multiplied by 200, all others vanish. (multiplied by 0)

$X_r(j\omega) = 2\pi(70{,}000)\,\delta(\omega) + \pi(20{,}000)\,\delta(\omega \mp 20\pi) + \pi(30{,}000)\,e^{\mp j\pi/2}\,\delta(\omega \mp 40\pi)$

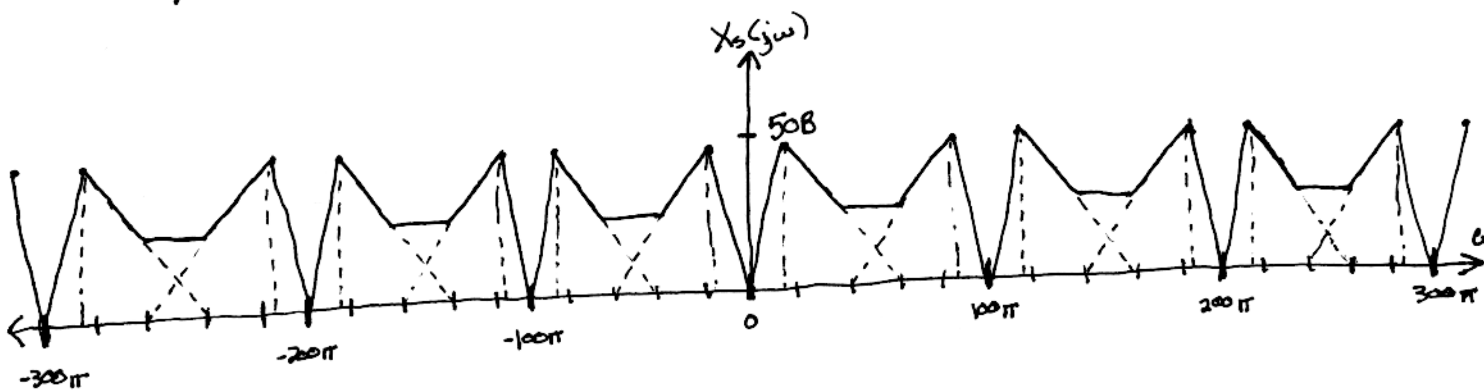


12.2 a) Maximum non-zero frequency component at $\omega_b = 60\pi$, hence we should choose $\omega_s \geq 2\omega_b = 120\pi$ (Nyquist rate). Choosing twice this rate yields $\omega_s = 240\pi$ and

$$X_s(j\omega) = 120\sum_{k=-\infty}^{\infty} X_a(j(\omega - 240\pi k)).$$

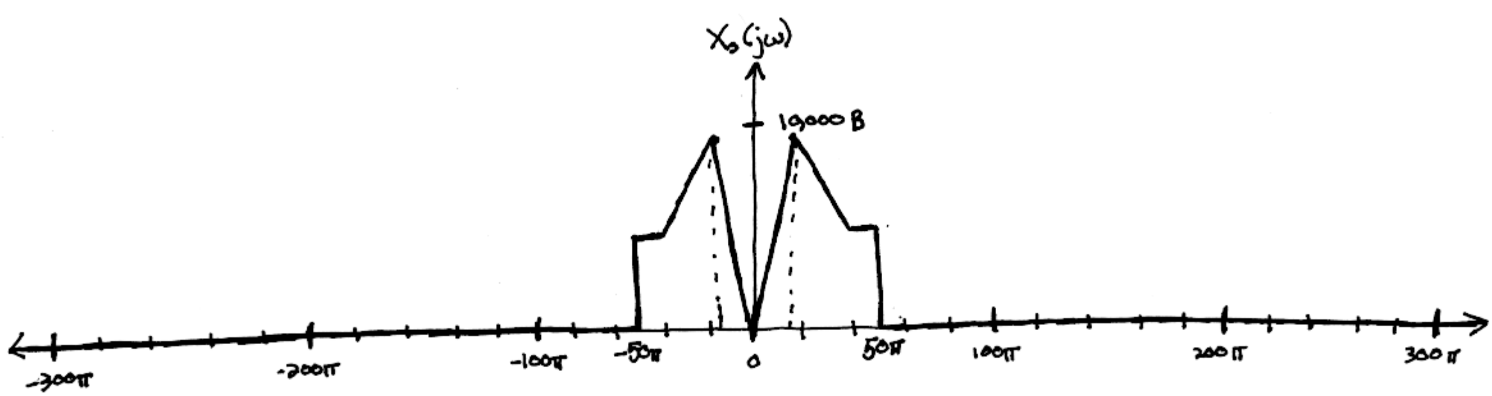


b) If $\omega_s = 100\pi$ then $X_s(j\omega) = 50\sum_{k=-\infty}^{\infty} X_a(j(\omega - 100\pi k))$



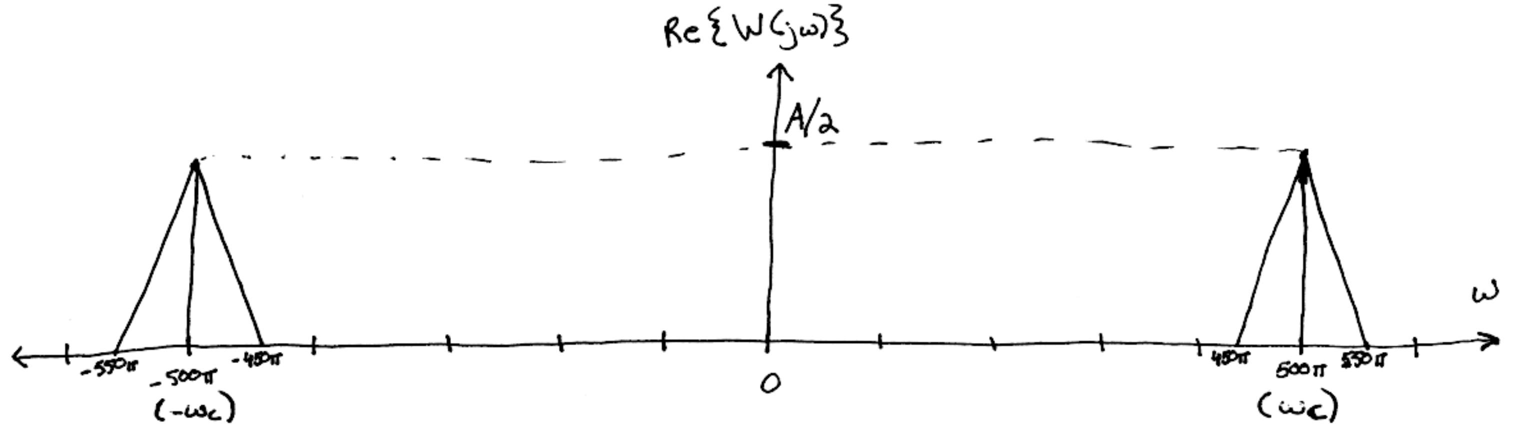
c) $H_r(j\omega) = 200$ for $|\omega| \leq \pi/T_s = \frac{1}{2}\omega_s = 50\pi$

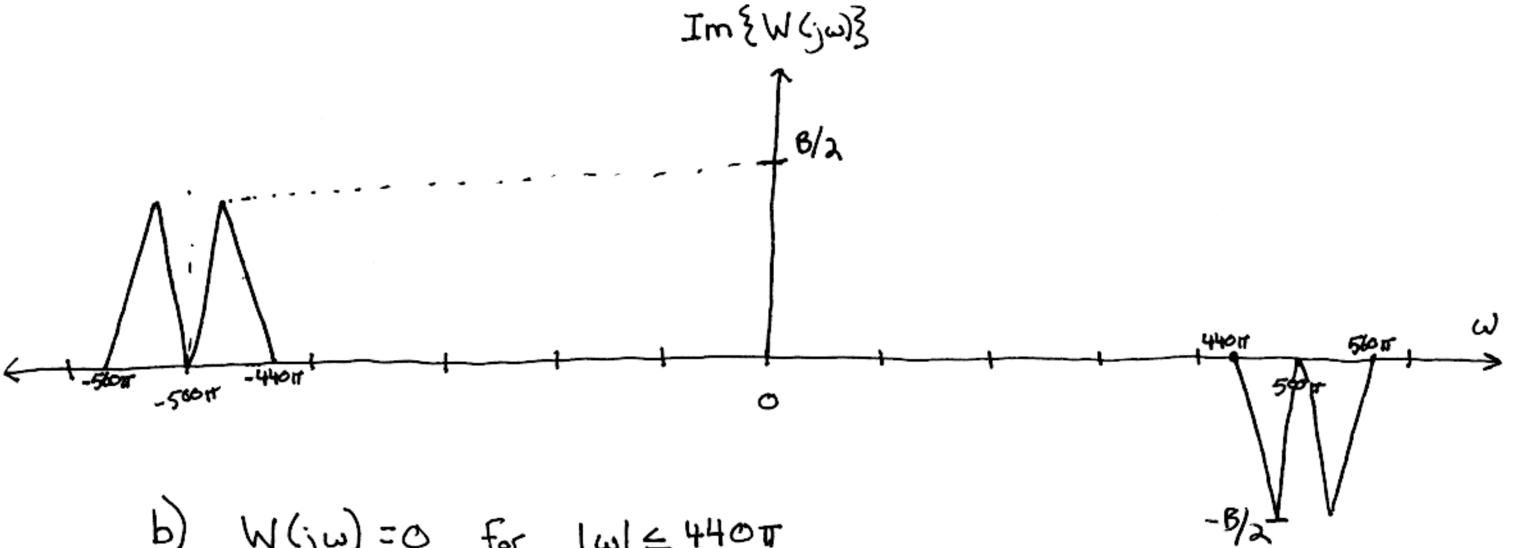
$H_r(j\omega) = 0$ for $|\omega| > 50\pi$



12.3 a) $w(t) = x_1(t)\cos\omega_c t + x_2(t)\sin\omega_c t$

$$W(j\omega) = \frac{1}{2\pi}X_1(j\omega) * \{\pi\,\delta(\omega-\omega_c) + \pi\,\delta(\omega+\omega_c)\}$$
$$+ \frac{1}{2\pi}X_2(j\omega) * \{-j\pi\,\delta(\omega-\omega_c) + j\pi\,\delta(\omega+\omega_c)\}$$
$$= \boxed{\frac{1}{2}\{X_1(j(\omega-\omega_c)) + X_1(j(\omega+\omega_c))\} - \frac{1}{2}j\{X_2(j(\omega-\omega_c)) - X_2(j(\omega+\omega_c))\}}$$

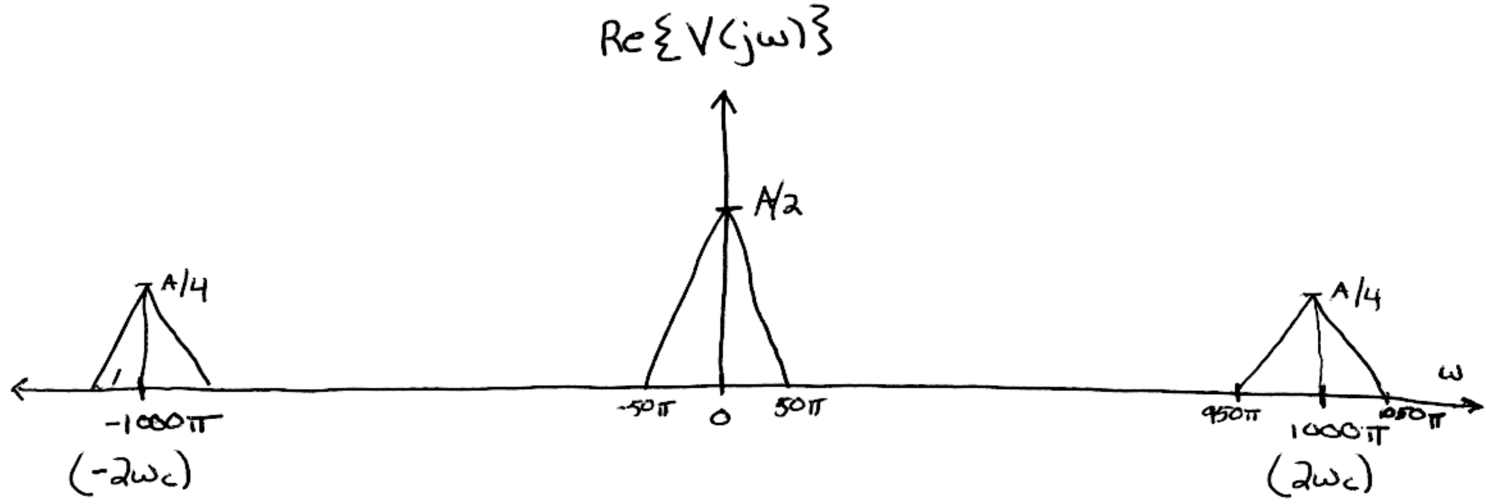


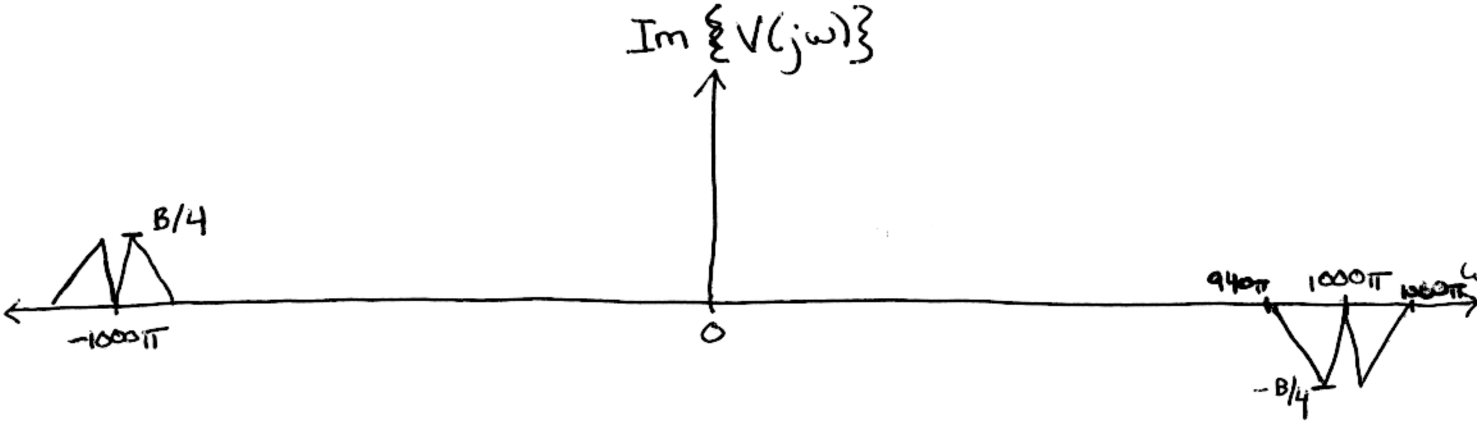


b) $W(j\omega) = 0$ for $|\omega| \leq 440\pi$
and $|\omega| \geq 560\pi$

c) $v(t) = w(t)\cos\omega_c t$

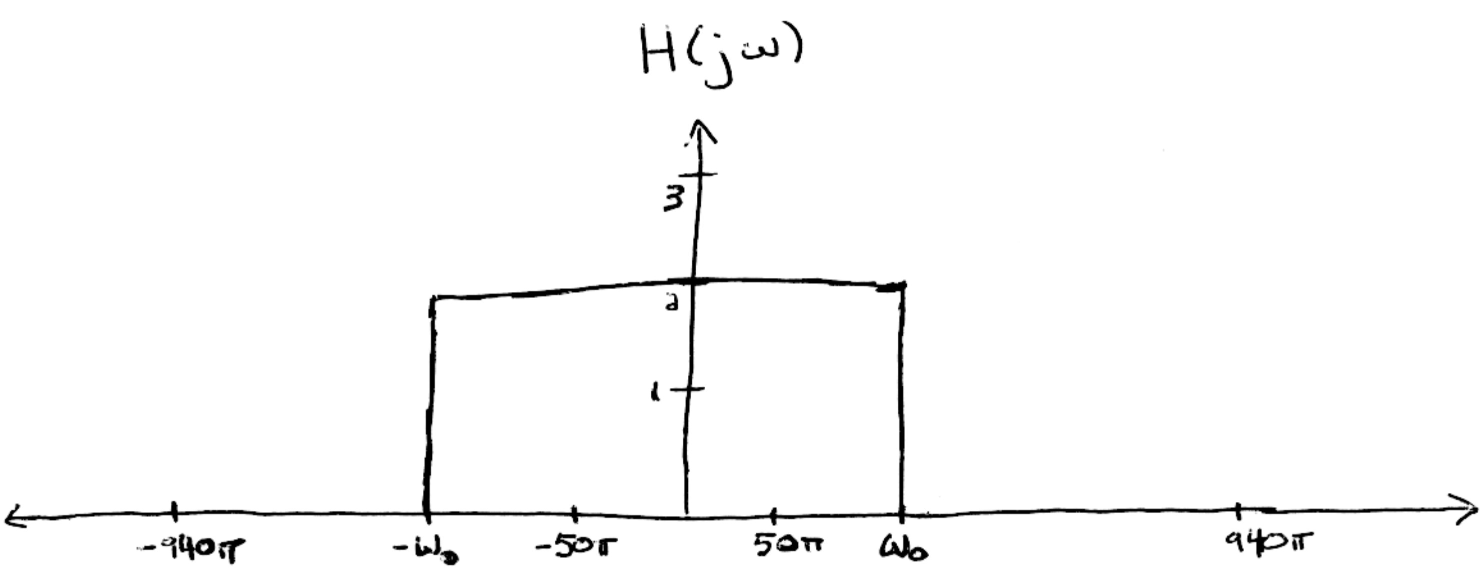
$$V(j\omega) = \frac{1}{2\pi}W(j\omega) * \{\pi\,\delta(\omega-\omega_c) + \pi\,\delta(\omega+\omega_c)$$
$$= \frac{1}{2}\{W(j(\omega-\omega_c)) + W(j(\omega+\omega_c))$$
$$= \boxed{\frac{1}{4}\{X_1(j(\omega-2\omega_c)) + 2X_1(j\omega) + X_1(j(\omega+2\omega_c)\} - \frac{1}{4}j\{X_2(j(\omega-2\omega_c)) - X_2(j(\omega+2\omega_c))\}}$$





d) $$H(j\omega) = \begin{cases} 2, & |\omega| \leq \omega_0 \\ 0, & |\omega| > \omega_0 \end{cases}$$

where ω_0 can be selected as any frequency between 50π and 940π



12.4 a)

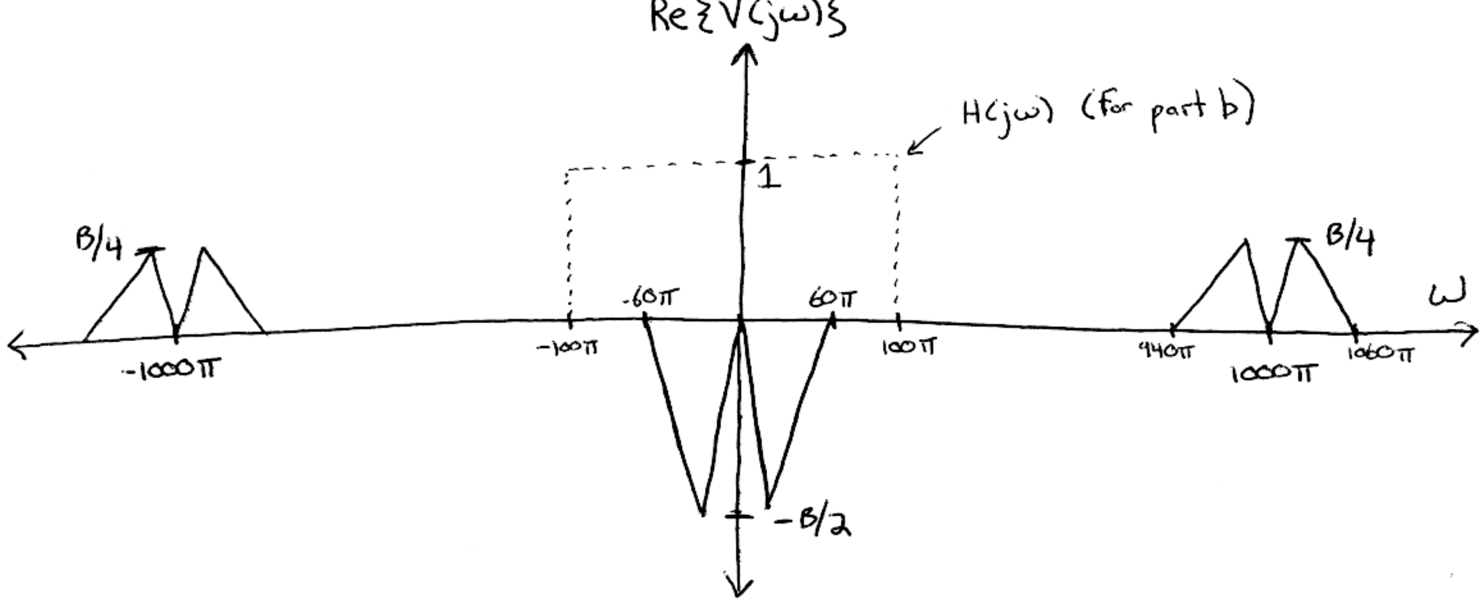
$$v(t) = w(t)\cos(\omega_c t + \pi/2)$$

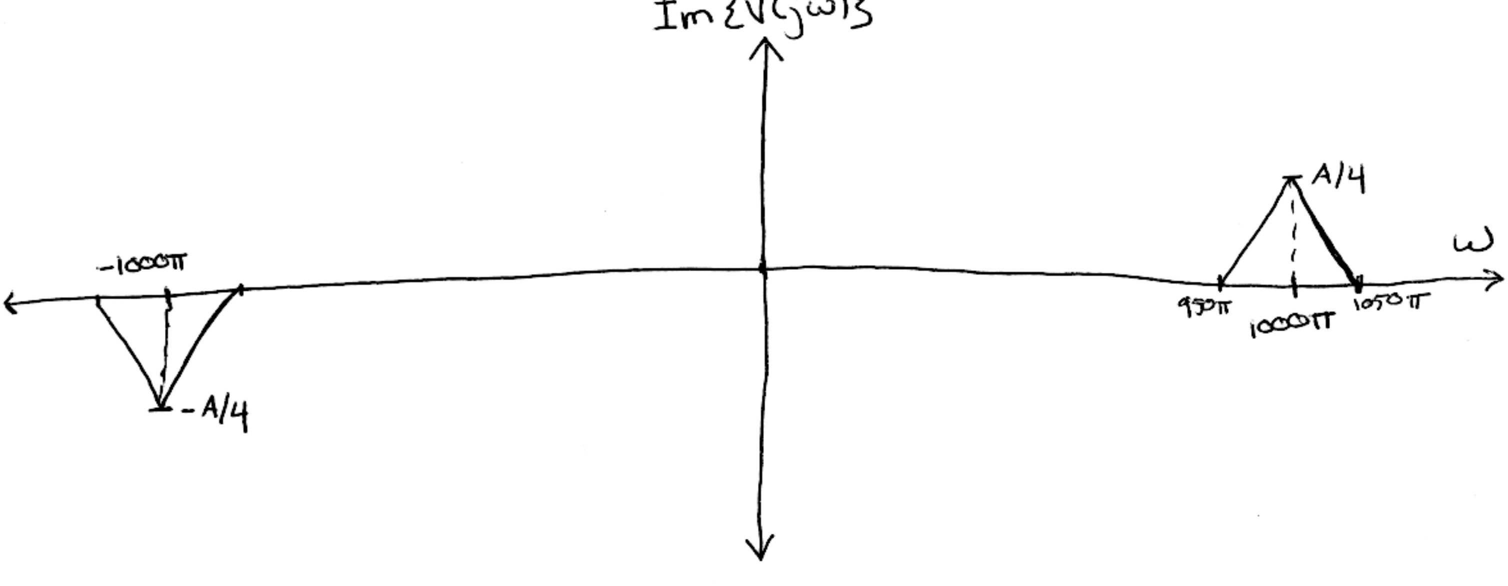
$$V(j\omega) = \frac{1}{2\pi} \underbrace{W(j\omega)}_{\text{(from problem 12.3)}} * \left\{ \pi e^{j\pi/2}\delta(\omega-\omega_c) + \pi e^{-j\pi/2}\delta(\omega+\omega_c) \right\}$$

$$= \frac{1}{2}\left[\frac{1}{2}\left\{X_1(j(\omega-\omega_c)) + X_1(j(\omega+\omega_c))\right\} - \frac{1}{2}j\left\{X_2(j(\omega-\omega_c)) - X_2(j(\omega+\omega_c))\right\}\right]$$

$$* \left\{ j\,\delta(\omega-\omega_c) - j\,\delta(\omega+\omega_c) \right\}$$

$$= \boxed{\begin{aligned} &\frac{1}{4}j\left\{X_1(j(\omega-2\omega_c)) - X_1(j(\omega+2\omega_c))\right\} \leftarrow \text{imaginary part} \\ &+ \frac{1}{4}\left\{X_2(j(\omega-2\omega_c)) - 2X_2(j\omega) + X_2(j(\omega+2\omega_c))\right\} \leftarrow \text{real part} \end{aligned}}$$





b) $Y(j\omega) = V(j\omega)H(j\omega) = \underbrace{-\frac{1}{2}X_2(j\omega)}_{\text{Part of } V(j\omega) \text{ within passband}} \cdot 1 = -\frac{1}{2}X_2(j\omega)$

(see dotted line in graph from part a)

Hence, by linearity of the Fourier transform,

$$\boxed{y(t) = -\frac{1}{2}x_2(t)}$$

c) $v(t) = w(t-t_d)\cos(\omega_c t + \phi)$

$$V(j\omega) = \frac{1}{2\pi}\left\{e^{-j\omega t_d}W(j\omega)\right\} * \left\{\pi e^{j\phi}\delta(\omega-\omega_c) + \pi e^{-j\phi}\delta(\omega+\omega_c)\right\}$$

$$= \frac{1}{2}\left[e^{j\phi}e^{-j(\omega-\omega_c)t_d}W(j(\omega-\omega_c)) + e^{-j\phi}e^{-j(\omega+\omega_c)t_d}W(j(\omega+\omega_c))\right]$$

$$= \frac{1}{2}e^{-j\omega t_d}\left[e^{j(\omega_c t_d+\phi)}W(j(\omega-\omega_c)) + e^{-j(\omega_c t_d+\phi)}W(j(\omega+\omega_c))\right]$$

$\omega_c t_d + \phi = 500\pi(.021) + .5\pi = \underline{\underline{11\pi}}$ $\quad (e^{\pm j11\pi} = -1)$

$$= -\frac{1}{2}e^{-j\omega t_d}\left[W(j(\omega-\omega_c)) + W(j(\omega+\omega_c))\right]$$

$$= -\frac{1}{4}e^{-j\omega t_d}\left[\left\{\underbrace{X_1(j(\omega-2\omega_c))}_{\substack{\text{nonzero for}\\ 950\pi<\omega<1050\pi}} + \underbrace{2X_1(j\omega)}_{\substack{\text{nonzero for}\\ |\omega|<50\pi}} + \underbrace{X_1(j(\omega+2\omega_c))}_{\substack{\text{nonzero for}\\ -950\pi>\omega>-1050\pi}}\right\} - j\left\{\underbrace{X_2(j(\omega-2\omega_c))}_{\substack{\text{nonzero for}\\ 940\pi<\omega<1060\pi}} - \underbrace{X_2(j(\omega+2\omega_c))}_{\substack{\text{nonzero for}\\ -940\pi>\omega>-1060\pi}}\right\}\right]$$

$$Y(j\omega) = V(j\omega)H(j\omega) = \underbrace{-\frac{1}{2}e^{-j\omega t_d}X_1(j\omega)}_{\substack{\text{Only nonzero portion}\\ \text{of } V(j\omega) \text{ for } |\omega|\le 100\pi}} \cdot 1 = \boxed{e^{-j\omega t_d}\left(-\frac{1}{2}X_1(j\omega)\right)}$$

$$y(t) = -\frac{1}{2}x_1(t-t_d)$$

$$= \boxed{-\frac{1}{2}x_1(t-.021)}$$

(by delay property of Fourier Trans.)

12.5

a) To obtain exact reconstruction we must sample at the Nyquist frequency or higher. Since the bandwidth of $x(t)$ is given as 60π rad/s, the Nyquist frequency is $2 \cdot 60\pi = \boxed{120\pi}$

b) $y[n] = x[n] = x(nT_s)$ $\qquad T_s = \frac{2\pi}{\omega_s} = \frac{2\pi}{100\pi} = \frac{1}{50}$

$$= 7 + 2\cos(20\pi T_s n) + 3\cos(60\pi T_s n + \tfrac{\pi}{2})$$

$$= 7 + 2\cos(\tfrac{2\pi}{5} n) + 3\cos(\tfrac{6\pi}{5} n + \tfrac{\pi}{2})$$

This frequency passes through D/C converter unaliased since $\frac{2\pi}{5} < \pi$

Folding occurs for this term since $\frac{6\pi}{5} > \pi$

$$= 7 + 2\cos(\tfrac{2\pi}{5} n) + 3\cos\left(-(\tfrac{6\pi}{5} n + \tfrac{\pi}{2}) + 2\pi n\right)$$

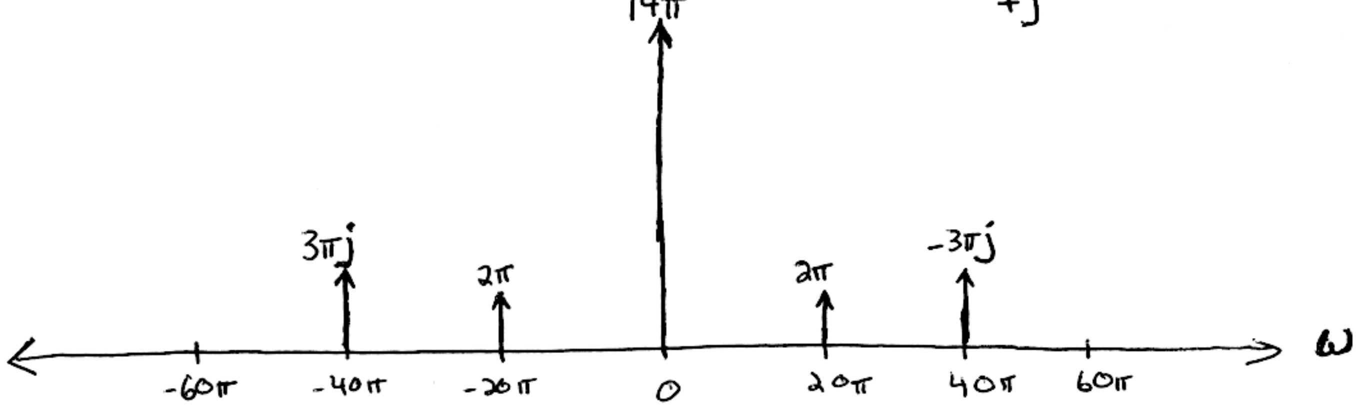
$$= 7 + 2\cos(\tfrac{2\pi}{5} n) + 3\cos(\tfrac{4\pi}{5} n - \tfrac{\pi}{2})$$

corresponding analog freq. $= \frac{1}{T_s} \cdot \frac{2\pi}{5} = 20\pi$

corresponding analog freq. $= \frac{1}{T_s} \cdot \frac{4\pi}{5} = 40\pi$

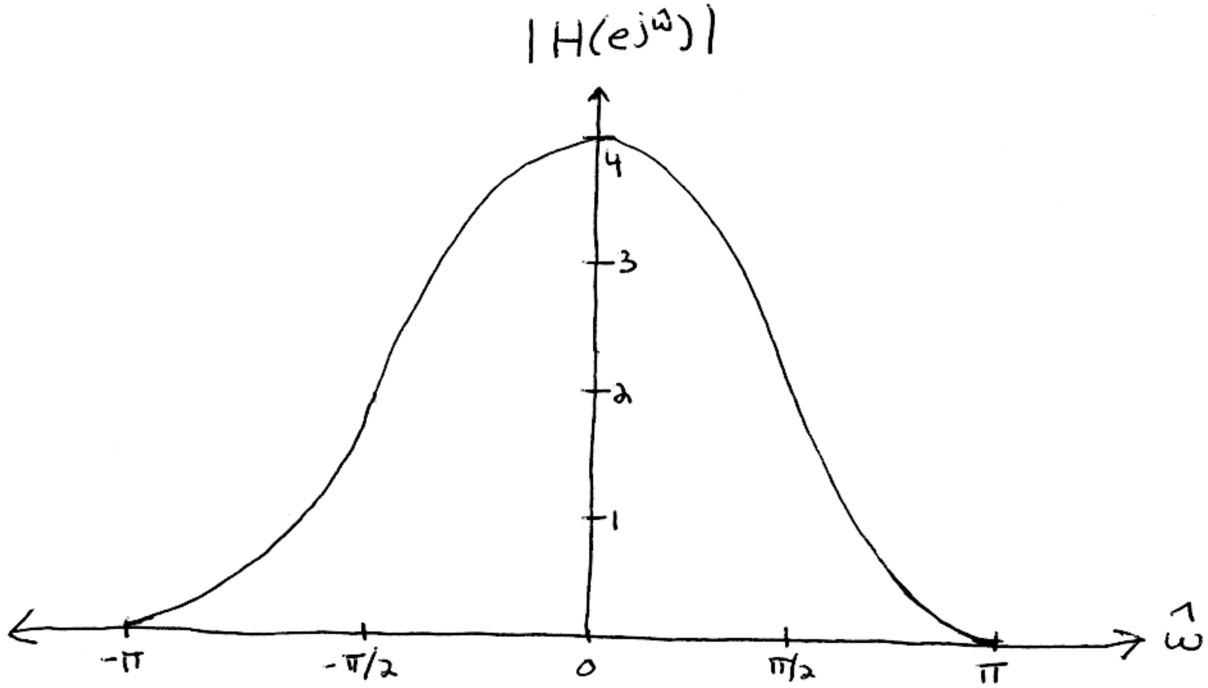
$$y(t) = \boxed{7 + 2\cos(20\pi t) + 3\cos(40\pi t - \tfrac{\pi}{2})}$$

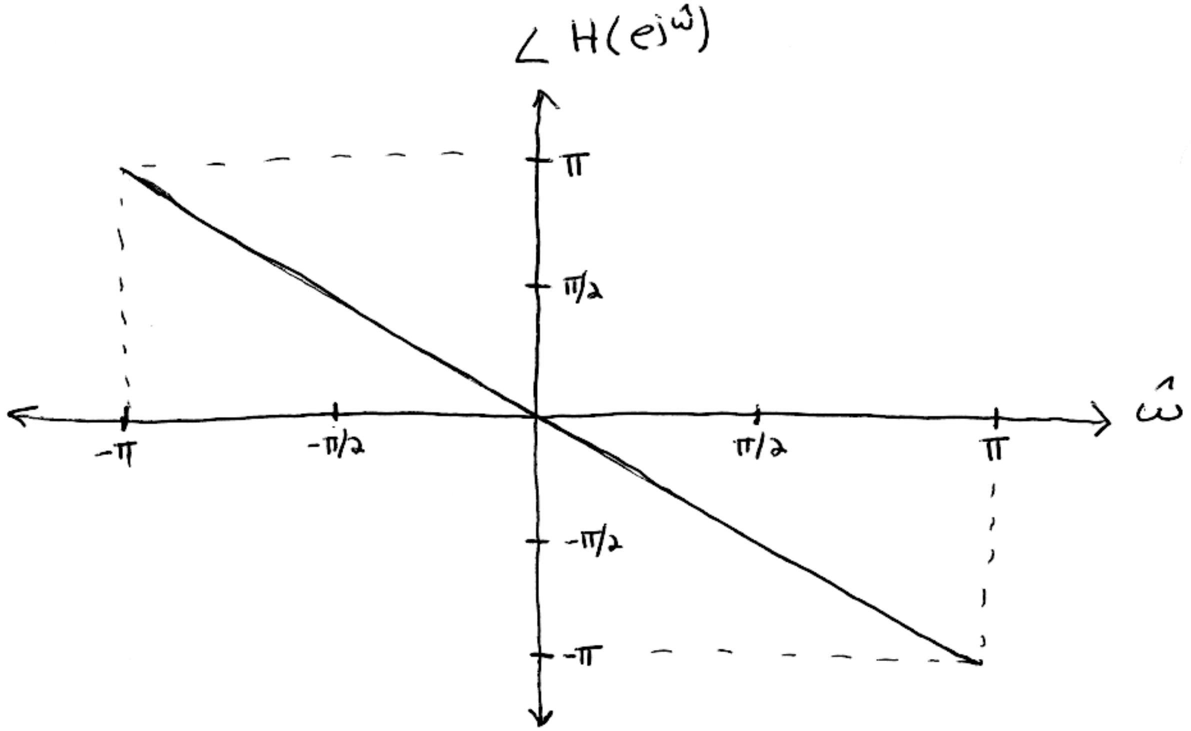
$$Y(j\omega) = 14\pi\,\delta(\omega) + 2\pi\,\delta(\omega \mp 20\pi) + 3\pi \underbrace{e^{\mp j\pi/2}}_{\mp j}\,\delta(\omega \mp 40\pi)$$



c) $H(e^{j\hat{\omega}}) = 1e^{-j\hat{\omega}0} + 2e^{-j\hat{\omega}1} + 1e^{-j\hat{\omega}2}$

$$= e^{-j\hat{\omega}}(e^{j\hat{\omega}} + 2 + e^{-j\hat{\omega}}) = (2 + 2\cos\hat{\omega})e^{-j\hat{\omega}}$$





d) If $x(t) = e^{j\omega t}$, then $y(t) = H_{eff}(j\omega)\, e^{j\omega t}$

$$\downarrow$$

$$x[n] = e^{j\omega T_s n}$$

Assuming no aliasing:

$$y[n] = H(e^{j\omega T_s})\, e^{j\omega T_s n}$$

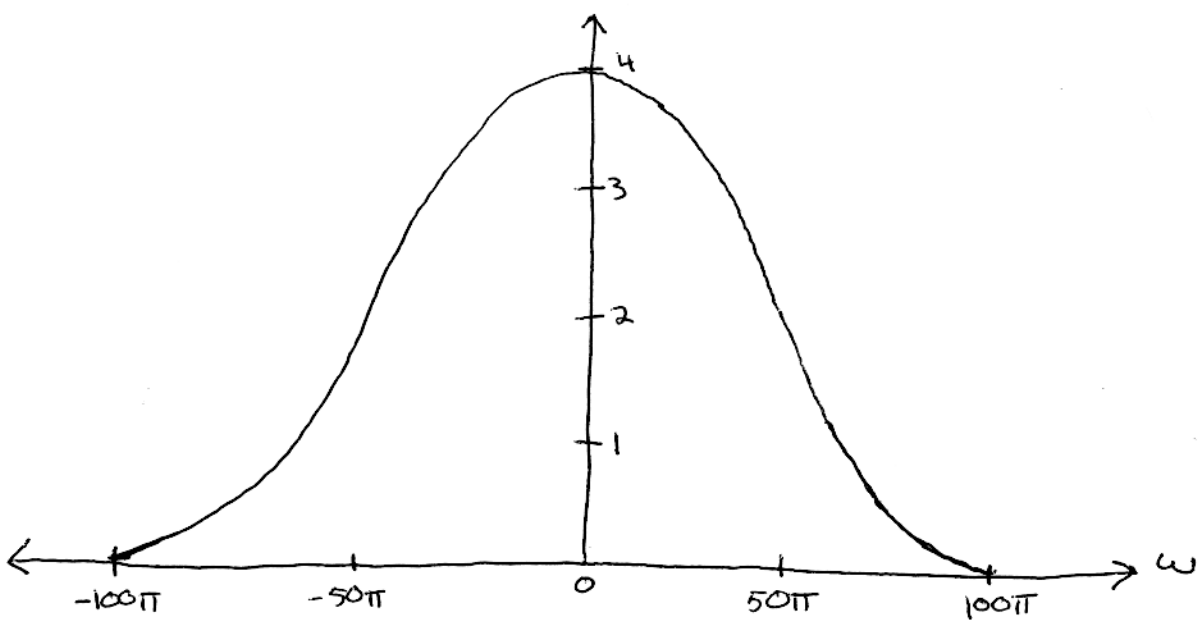
$$y(t) = H(e^{j\omega T_s})\, e^{j\omega t}$$

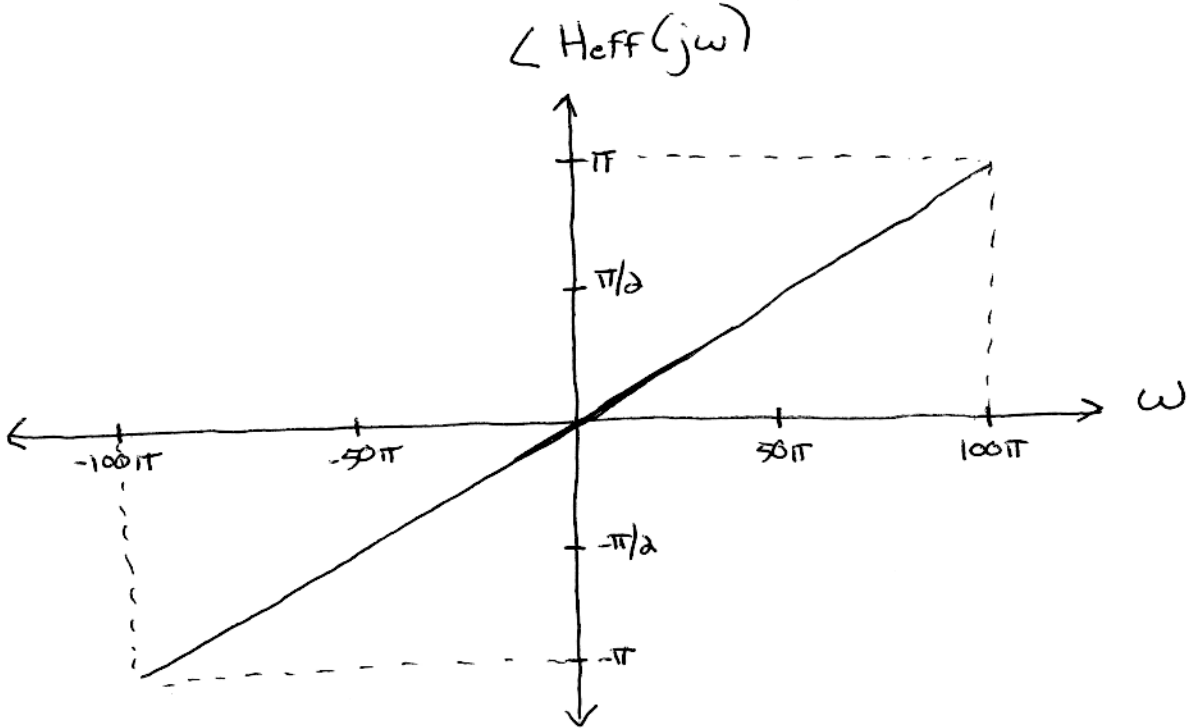
Hence $H_{eff}(j\omega) = H(e^{j\omega T_s})$

If $\omega_s = 200\pi$ rad/s then $T_s = \frac{2\pi}{200\pi} = \frac{1}{100}$ and so

$$H_{eff}(j\omega) = H(e^{j\omega/100}) = \left(2 + 2\cos\left(\frac{\omega}{100}\right)\right) e^{j\omega/100}$$

$|H_{eff}(j\omega)|$




∠ Heff(jω)
π
π/2
-π/2
-π
ω
-100π
-50π
50π
100π