

**ECE 2025 Spring 2000**  
**Lab #12: The  $z$ ,  $n$ , and  $\hat{\omega}$  Domains**

Date: 24–27 April 00

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The lab report/verification will be done “on-line” with Web-CT. The write-up below should serve as an example of the types of questions you will have to answer in Web-CT, but there will be some variety.

⇒ An On-line Survey for Lab & Course Evaluation will also be held during this week’s lab time.

This is *the official* Lab #12 description; it is derived from Lab C.10 in Appendix C of the text.

The lab report for this lab is just the **Web-CT on-line** questions, which address a list of observations to be made when using the pez GUI.

*The on-line report should be completed during the lab time.*

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## 1 Introduction & Objective

The objective for this lab is to build an intuitive understanding of the relationship between the location of poles and zeros in the  $z$ -domain, the impulse response  $h[n]$  in the  $n$ -domain, and the frequency response  $H(e^{j\hat{\omega}})$  (the  $\hat{\omega}$ -domain). A graphical user interface (GUI) called pez was written in MATLAB for doing interactive explorations of the three domains.<sup>1</sup>

## 2 Warm-up

Invoke pez by simply typing pez at the MATLAB prompt, if you have the *DSP First Toolbox* installed. A control panel with a few buttons and a plot of the unit circle in the complex  $z$ -plane will pop up. You can use the control window to selectively place poles and zeros in the  $z$ -plane, and then observe (in a second window) how their placement affects the impulse and frequency responses. If the plots need manual updating, click on the **Redo Plots** button under the **<Quicksize...>** menu.

The **Real Time Drag Plots** button will put pez in a mode such that an individual pole/zero (pair) can be moved around and the corresponding  $H(e^{j\hat{\omega}})$  and  $h[n]$  plots will be updated as you drag the pole (or zero).

Since exact placement of poles and zeros with the mouse is difficult, an **Edit By Co-Ord** button is provided for numerical entry of the real and imaginary parts, or magnitude and angle (a separate edit window appears when you use this option). Before you can edit a pole or zero, however, you must first select it with the mouse. Removal of individual poles or zeros can also be performed by clicking on the **Delete Poles & Zeros** button (again, a separate window will appear). Note that all poles and/or zeros can be easily cleared by clicking on the **<Clear...>** menu, and then selecting Poles, Zeros, or All.

Play around with pez for a few minutes to gain some familiarity with the interface. Implement the following first-order system:

$$H(z) = \frac{1 - z^{-1}}{1 + 0.9z^{-1}}$$

by placing its poles and zeros at the correct location in the  $z$ -plane.

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<sup>1</sup>pez was written by Craig Ulmer. It is part of the *DSP-First Toolbox*, so you can run it with MATLAB on your own computer.



CD-ROM

PEZ

### 3 Laboratory: Relationships between $z$ , $n$ , and $\hat{\omega}$ domains

Work through the following exercises and keep track of your observations by filling in the worksheet at the end of this assignment (or answering the on-line questions). In general, you want to make note of the following quantities:



CD-ROM

Instructor  
Verification  
Sheet

- How does  $h[n]$  change with respect to its rate of decay? For example, when  $h[n] = a^n u[n]$ , the impulse response will fall off more rapidly when  $a$  is smaller.
- If  $h[n]$  exhibits an oscillation period, what is the period? Also, estimate the decay rate of the “envelope.”
- How does  $H(e^{j\hat{\omega}})$  change with respect to peak location and peak width?

Note: review the “3-Domain Movies” in Chapters 7 and 8 on the *DSP-First* CD-ROM for examples of these relationships.

### 4 Real Poles

- Use `pez` to place a pole at  $z = \frac{1}{2}$ . You may have to use the `Edit by Co-Ord` button to get the location exactly right. Use the plots for this case as the reference for answering the next four parts.
- Move the pole close to the origin (still on the real axis). You can do this by clicking on the pole and dragging it to the new location. Describe the changes in the impulse response  $h[n]$  and the frequency response  $H(e^{j\hat{\omega}})$ .
- You can also move poles and zeros under the influence of the `Real Time Drag Plots` option in `pez`. When this box is checked, the impulse response and frequency response plots are updated while you move the pole (or zero). Once this mode is set, click on the pole you want to move and start to drag it slowly. Watch for the update of the plots in the secondary window. After the real-time updating has started, you can release the mouse button and the pole (or zero) will follow the cursor. Click on the pole once more to stop moving it and to stop the updating. It is sometimes a little tricky to use. Also the display may be jerky unless you have a high-performance computer with fast graphics.  
Move the real pole slowly from  $z = \frac{1}{2}$  to  $z = 1$  and observe the changes in the impulse response  $h[n]$  and the frequency response  $H(e^{j\hat{\omega}})$ .
- Place the pole exactly on the unit circle (or maybe just inside at a radius of 0.99999999). Describe the changes in  $h[n]$  and  $H(e^{j\hat{\omega}})$ . What do you expect to see for  $H(e^{j\hat{\omega}})$ ?
- Move the pole outside the unit circle. Describe the changes in  $h[n]$ . Explain how the appearance of  $h[n]$  validates the statement that the system is not stable. In this case, the frequency response  $H(e^{j\hat{\omega}})$  is not legitimate because the system is no longer stable.
- In general, where should poles be placed to guarantee system stability? By stability we mean that the system’s output does not blow up.

### 5 Complex Poles and Zeros

If the denominator polynomial  $A(z)$  has a complex root, it will have a second root at the conjugate location when the polynomial coefficients are real. For example, if we place a root at  $z = \frac{1}{3} + j\frac{1}{2}$ , then we will also get one at  $z = \frac{1}{3} - j\frac{1}{2}$ .

- What property of the polynomial coefficients of  $A(z) = 1 - a_1z^{-1} - a_2z^{-2}$  will guarantee that the roots come in conjugate pairs?
- Clear all the poles and zeros from `pez`. Now place a pole with magnitude 0.85 at an angle of  $45^\circ$ ; and then two zeros at the origin. Note that `pez` automatically places a conjugate pole in the  $z$ -domain. The frequency response has a peak—record the frequency (location) of this peak.
- Change the angle of the pole: move the pole to  $90^\circ$ , then  $135^\circ$ . Describe the changes in  $|H(e^{j\hat{\omega}})|$ . Concentrate on the location of the peak.

Next, we will put complex zeros on the unit circle to see the effect on  $|H(e^{j\hat{\omega}})|$ .

- Clear all poles and zeros from `pez`. Now place zeros at the following locations:  $z_1 = -1$ ,  $z_2 = 0 - j$  and  $z_3 = 0 + j$  (remember that conjugate pairs such as  $z_2$  and  $z_3$  will be entered simultaneously). Judging from the impulse and frequency responses what type of filter have you just implemented?

## 6 Filter Design

In this section, we will use `pez` to place the poles and zeros of  $H(z)$  to make a filter with a desirable frequency response. *Filter design* is a process that selects the coefficients  $\{a_k\}$  and  $\{b_k\}$  to accomplish a given task.

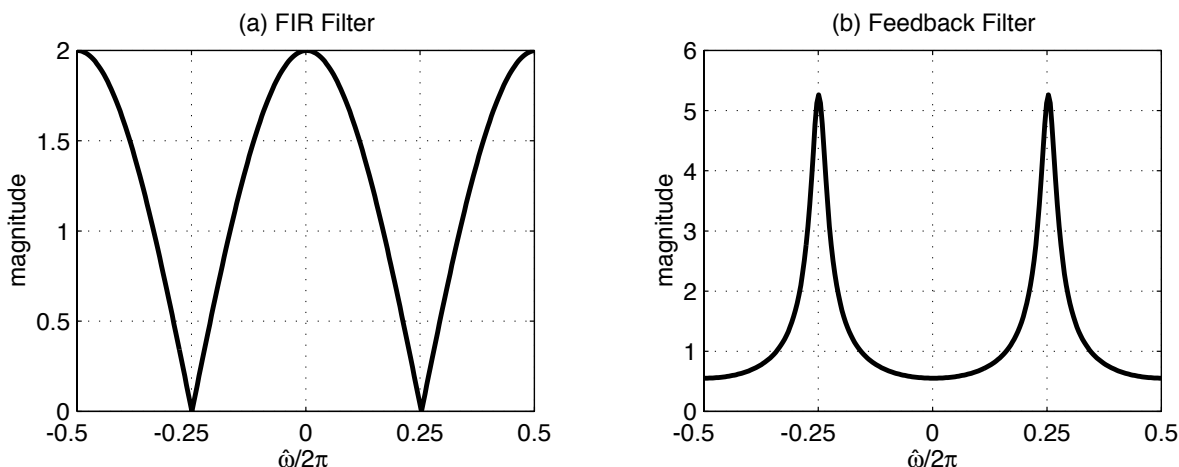


Figure 1: Magnitude response of two unknown filters. Frequency axis is normalized ( $\hat{\omega}/2\pi$ ). Use `pez` to help you find the filter coefficients that will match these frequency responses as closely as possible. (a) Second-order FIR filter. (b) Second-order IIR filter.

The task here is to create a filter that has a very narrow “notch.” This filter would be useful for removing one frequency component while leaving others undisturbed. The *notch filter* can be synthesized from the cascade of two simpler filters shown in Fig. 1.

- Start the process by using `pez` to design each of the filters given in Fig. 1. Both filters are second-order. Make sure that you enter the poles and zeros precisely. `pez` will do the conversion between between root locations and polynomial coefficients, but you could also do this with the MATLAB commands `roots` and `poly`. You can check your results by also calculating the filter coefficients by hand (see the next section on polynomials with complex coefficients). Record the coefficients of your filters in the table provided.

Note: Use `pez` or `freqz()` to verify that the frequency response of each filter is correct.

- (b) Now use `pez` to connect the filters together in a cascade. Place the poles and zeros, and then view the frequency response. Determine the filter coefficients for the overall cascaded filter  $H(z)$ .
- (c) Use `freqz()` to determine the frequency response of the cascade of the two filters that you “designed” in part (e). Plot the magnitude of the overall frequency response of the cascade system for  $-\pi < \hat{\omega} < \pi$ , and print a copy of the plot for your lab report.<sup>2</sup> Explain briefly why the frequency response magnitude has a notch, and explain why the gain at  $\hat{\omega} = 0$  and  $\hat{\omega} = \pi$  is the same.

## 7 Filter Coefficients from Roots (Optional)

Derive the filter coefficients for the denominator  $A(z) = 1 - a_1z^{-1} - a_2z^{-2}$  and numerator  $B(z) = b_0 + b_1z^{-1} + b_2z^{-2}$  when the poles are:

$$p_1 = 0.75e^{j\pi/4}, \quad p_2 = 0.75e^{-j\pi/4},$$

and the zeros are:

$$z_1 = 1, \quad z_2 = -1$$

Use the following relationship:

$$H(z) = \frac{B(z)}{A(z)} = G \frac{(1 - z_1z^{-1})(1 - z_2z^{-1})}{(1 - p_1z^{-1})(1 - p_2z^{-1})} \quad (1)$$

where  $z_1$  and  $z_2$  are the zeros, and  $p_1$  and  $p_2$  are the poles defined above. Determine  $G$  so that the maximum magnitude of  $H(e^{j\omega})$  is one. (Remember that MATLAB can multiply polynomials via its `conv` function.)

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<sup>2</sup>`pez` uses the normalized frequency scale:  $-\frac{1}{2} < \hat{\omega}/(2\pi) < \frac{1}{2}$ , which is the same as  $-\pi < \hat{\omega} < \pi$ .

**Lab #12**  
**ECE-2025 Spring-2000**  
**SAMPLE WORKSHEET**

This page is just a sample!

Name: \_\_\_\_\_ Date of Lab: \_\_\_\_\_

Evaluation: Completed on-line survey in Web-CT

Verified: \_\_\_\_\_ Date/Time: \_\_\_\_\_

Warm-up (Part 2): Implemented first-order system with pez:

Verified: \_\_\_\_\_ Date/Time: \_\_\_\_\_

Part	Observations
4(a)	$h[n]$ decays exponentially with no oscillations, $H(e^{j\hat{\omega}})$ has a hump at $\hat{\omega} = 0$
4(b)	
4(c)	
4(d)	
4(e)	
4(f)	
5(a)	
5(b)	
5(c)	
5(d)	

Part 6(a,b)	$a_k$	$b_k$
Filter 1		
Filter 2		
Cascade of 1 and 2		

Explanation of cascade frequency response: