EE-2025 Spring-00

LECTURE #2

Complex Exponentials

10-Jan-00

INFORMATION

- MATLAB: Mon,T,Wed in VL-456 (6, 7?)
- LABS start THIS week (TODAY)
  - Attend correct section (in CoC-309)
  - Computer acct: gtxxxx, password: SSN
  - Verification must be signed during Lab
- RECITATIONS
  - Attend your assigned time

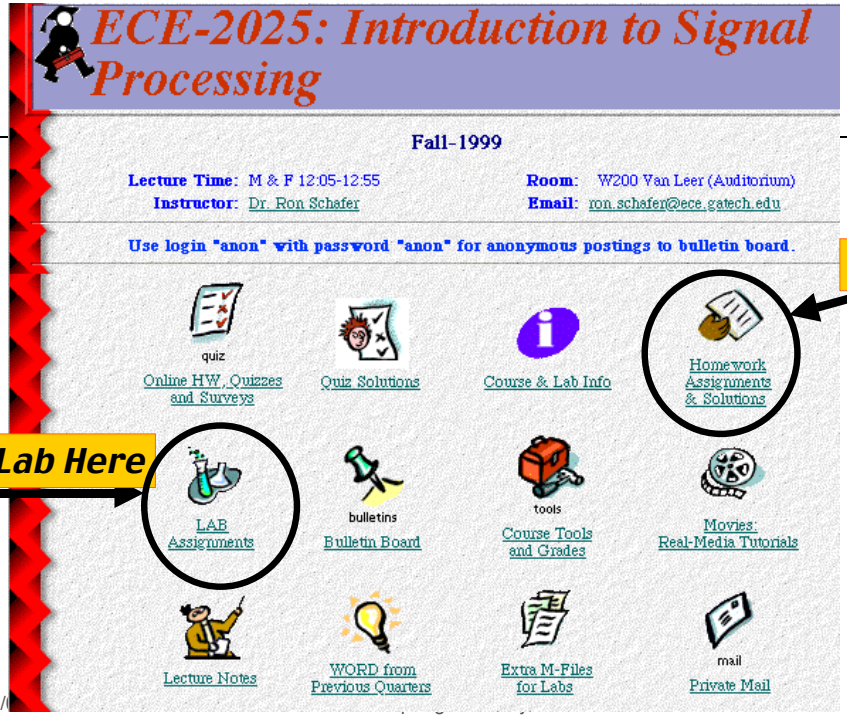
HOMEWORK #1

- Written Part-conventional

  Hand in STAPLED papers—unfolded with cover page (example on WebCT)
- On-Line HW in Web-CT
  - Under the "On-Line HW, Quizzes.." link
  - BEFORE Midnight on Monday (17-Jan)
- Several Easy Problems (ie, Drill)
  - Take it up to 5 times
  - Last score counts

REMINDERS

- Web-CT Password:
  - SSN, or student number; change it soon
- Hard copy of Instructor Verification Sheet
  - Get PDF file of Lab#1 from WebCT
  - Lab #1 is different from the book
- HW #1 due next week (in Recitation)
  - Get PDF file from WebCT



READING ASSIGNMENTS

- This Lecture:
  - Chapter 2, pp. 17–32
- Appendix A: Complex Numbers
- Appendix B: MATLAB
- Next Lecture: finish Chap. 2, pp. 31–43

LECTURE OBJECTIVES

- Define Sinusoid from a plot
- Relate TIME–SHIFT to PHASE
- Introduce an ABSTRACTION:
  - Complex Numbers represent Sinusoids
  - Complex Exponential Signal

$$z(t) = Ze^{j\omega t}$$

SINUSOIDAL SIGNAL

$$A\cos(\omega t+\varphi)$$

- FREQUENCY ω
  - Radians/sec
  - Hertz (cycles/sec)

$$\omega = (2\pi) f$$

- AMPLITUDE A
  - Magnitude
- PHASE φ
- PERIOD (in sec)

$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$

PLOT a COSINE SIGNAL

- Given the Formula

$$5\cos(0.3\pi t+1.2\pi)$$

- Make a plot
- Formula defines A, ω, and ϕ

$$A=5$$
$$\omega=0.3\pi$$
$$\varphi=1.2\pi$$

PLOTTING COSINE SIGNAL from the FORMULA

$$5\cos(0.3\pi t+1.2\pi)$$

- Determine period:

$$T=2\pi/\omega=2\pi/0.3\pi=20/3$$

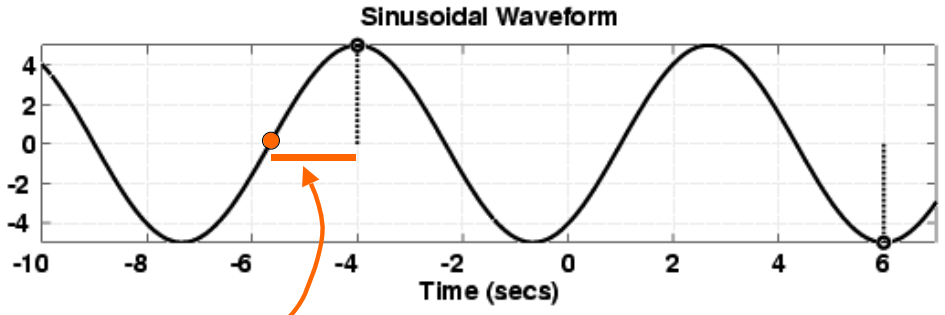
- Determine a peak location by solving

$$(\omega t+\varphi)=0$$

- Peak at t = −4
- Zero crossing is T/4 before or after

ANSWER for the PLOT

$$5\cos(0.3\pi t+1.2\pi)$$



$$T/4=(20/3)/4=5/3$$

TIME-SHIFT

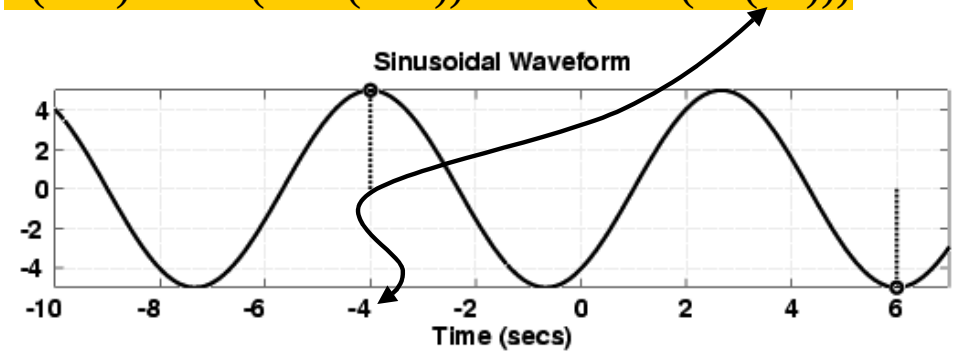
- In a mathematical formula replace t with $t-t_m$
- For example, $x(t-t_m) = \cos(\omega(t-t_m))$
- Then the t=0 point moves to $t=t_m$

$$x(t-t_m)=A\cos(\omega(t-t_m))$$

- Peak value of $\cos(\omega(t-t_m))$ is at $t=t_m$

TIME-SHIFTED SINUSOID

$x(t+4)=5\cos(0.3\pi(t+4))=5\cos(0.3\pi(t-(-4)))$



PHASE <--> TIME-SHIFT

- Equate the formulas:

$$A\cos(\omega(t-t_m)) = A\cos(\omega t+\varphi)$$

- and we obtain: $-\omega t_m = \varphi$
- or, $t_m = \dfrac{-\varphi}{\omega}$

EX: Time-Shift from Phase

- Frequency : $\omega = 30\pi$
- Phase: $\phi = -0.2\pi$
- What is the time shift?
  - Also called the "time delay"
  - $t_m = -\phi/\omega = -(-0.2\pi)/30\pi$
  - $t_m = 1/150$ sec.
  - Note: T = 1/15 sec. (period)

PHASE is AMBIGUOUS

- The cosine signal is periodic
  - Period is 2π

$$A\cos(\omega t+\varphi+2\pi) = A\cos(\omega t+\varphi)$$

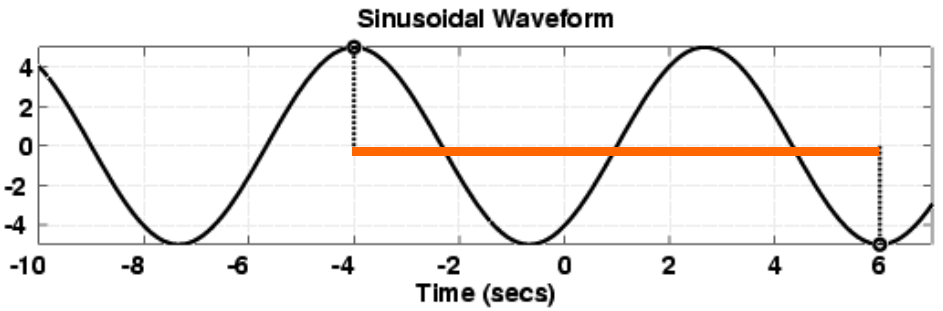
  - Thus adding any multiple of 2π leaves x(t) unchanged
  - if $t_m = \frac{-\varphi}{\omega}$, then

$$t_{m_2} = \frac{-(\varphi+2\pi)}{\omega} = \frac{-\varphi}{\omega} - \frac{2\pi}{\omega} = t_m - T$$

SINUSOID from a PLOT

- Measure the period, T
  - Between peaks or zero crossings
  - Compute frequency: $\omega = 2\pi/T$
- Measure time of peak: t_m
  - Compute phase: $\phi = -\omega\, t_m$
- Measure height of positive peak: A

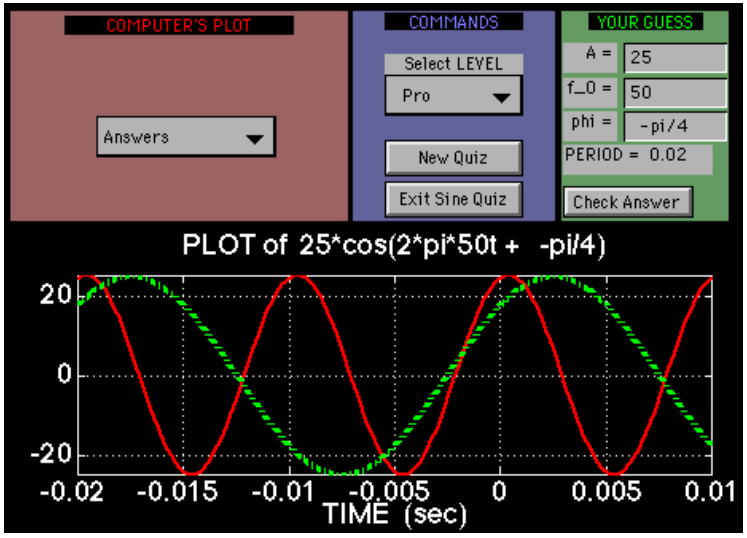
(A, ω, ϕ) from a PLOT



$T = 10/(1.5) = 20/3 \rightarrow \omega = 2\pi/T = 0.3\pi$

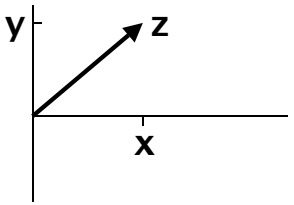
$t_m = -4 \rightarrow \phi = -(-4)(0.3\pi) = 1.2\pi$

SINE DRILL (MATLAB GUI)



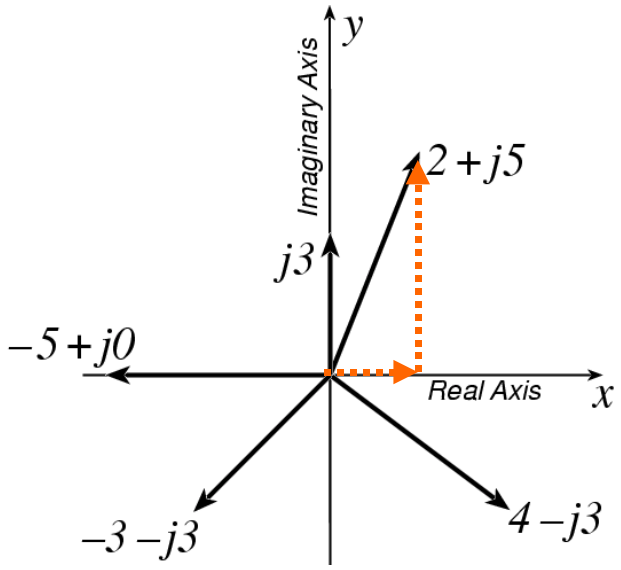
COMPLEX NUMBERS

- To solve: $z^2 = -1$
  - $z = j$
  - Math and Physics use $z = i$
- Complex number: $z = x + jy$

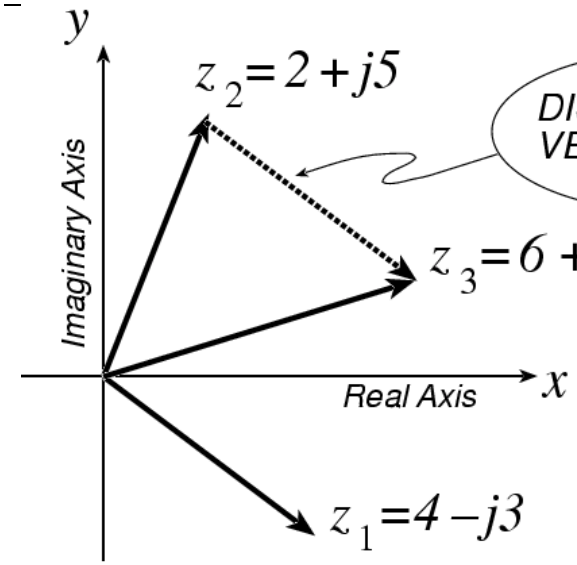


Cartesian coordinate system

PLOT COMPLEX NUMBERS



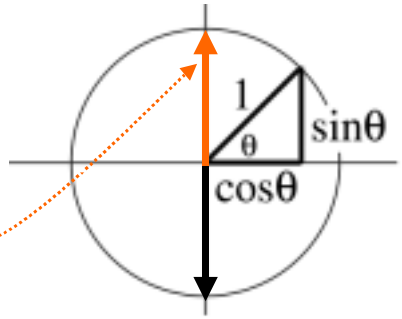
COMPLEX ADDITION = VECTOR Addition



$$z_3 = z_1 + z_2$$
$$= (4 - j3) + (2 + j5)$$
$$= (4 + 2) + j(-3 + 5)$$
$$= 6 + j2$$

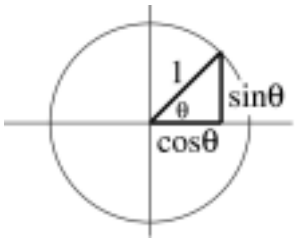
*** POLAR FORM ***

- Vector Form
  - Length =1
  - Angle = θ
- Common Values
  - j has angle of 0.5π
  - −1 has angle of π
  - −j has angle of 1.5π
  - or, its angle is −0.5π = 1.5π −2π



Euler's FORMULA

- Complex Exponential
  - Real part is cosine
  - Imaginary part is sine
  - Magnitude is one



$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

$$re^{j\theta} = r\cos(\theta) + jr\sin(\theta)$$

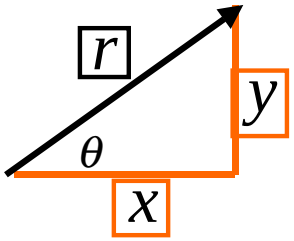
POLAR <--> RECTANGULAR

- Relate (x,y) to (r,θ)

$$z = x + jy = re^{j\theta}$$

$$r^2 = x^2 + y^2$$

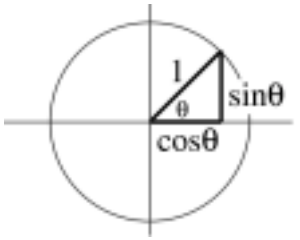
$$\theta = \mathrm{Tan}^{-1}\left(\frac{y}{x}\right)$$



COMPLEX EXPONENTIAL

$$e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$$

- Rotating Vector
  - Angle changes vs. time
  - θ=ω*t*
  - ex: ω=10π
  - Rotates 0.1π in 0.01 secs



$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

Cos = REAL PART

- Real Part of Euler's

$$\cos(\omega t) = \Re e\left\{e^{j\omega t}\right\}$$

- General Sinusoid

$$x(t) = A\cos(\omega t + \varphi)$$

- So,

$$A\cos(\omega t + \varphi) = \Re e\left\{Ae^{j(\omega t+\varphi)}\right\} = \Re e\left\{Ae^{j\varphi}e^{j\omega t}\right\}$$

COMPLEX AMPLITUDE

- General Sinusoid

$$x(t) = A\cos(\omega t + \varphi) = \Re e\left\{Ae^{j\varphi}e^{j\omega t}\right\}$$

- Sinusoid = REAL PART of $(Ae^{j\phi})e^{j\omega t}$

$$x(t) = \Re e\left\{Ze^{j\omega t}\right\} = \Re e\{z(t)\}$$

- Complex AMPLITUDE = Z

$$z(t) = Ze^{j\omega t} \qquad Z = Ae^{j\varphi}$$