

EE-2025

Spring-00

LECTURE #3

Phasor Addition Theorem

14-Jan-99

Web-CT Info

- Check the Bulletin Board for msgs
 - MAKE YOUR OWN POSTINGS
- Lectures are being posted
 - PDF format (4 per page)
- Quiz Dates:
 - Quiz #1 on 4-Feb (Friday)
 - Quiz #2 on 3-March
 - Quiz #3 on 7-April

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Homework Info

- Prob Set #1 due **in RECITATION**
 - **At the beginning of class**
- On-Line HW #1 ends Monday nite
 - Last attempt is scored
- HW will be posted by Wednesdays
 - Due during the following Week
- Solutions will be posted
 - Hopefully on Thurs nite or Friday.

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Homework Formatting

- Cover page with
 - Name
 - Lab section, ie, L05, L21, etc.
 - Recitation Prof's name
 - See example on Web-CT
- Write on ONE side only
 - Use Engineer's paper or plain paper
- **STAPLE**

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Lab Info

- NT passwd = **SSN or old password**
- Lab **FAQs** are being posted
- Lab #1 Report
 - Due week of 24-Jan
 - Turn in during your lab time
 - Write-up sections 2 and 3
 - Include INSTRUCTOR VERIFICATION
- Lab #2 will be posted next week

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Lab Info: next week only

- Week of Jan 17th
 - Labs will be staffed
 - Catch up on Lab #1 Warm-up
 - Ask MATLAB questions
- MATLAB Help: T,Wed,Thur 6PM VL-456
- Learn Lab TA's format requirements

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READING ASSIGNMENTS

- This Lecture:
 - Chapter 2, pp. 31-43
- Other Reading:
 - Appendix A: Complex Numbers
 - Appendix B: MATLAB
 - Next Lecture: start Chapter 3

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Z DRILL (Complex Arith)

The screenshot displays a MATLAB Z-DRILL interface. The top section is titled "HERE is the QUESTION" and contains input fields for two complex numbers, z_1 and z_2 , in rectangular form. The inputs are: z_1 with $r_1 = 1.5$ and $ang_1 = -0.75\pi$; and z_2 with $r_2 = 2$ and $ang_2 = 0.125\pi$. Below the inputs are dropdown menus for "z_1 in RECTANGULAR" and "z_2 in RECTANGULAR", and a "Plot z1 and z2" button. The middle section is titled "COMMANDS" and includes a "Select OPERATION:" dropdown set to "z1 + z2 (Add)", a "Select LEVEL:" dropdown set to "Pro", and buttons for "New Quiz" and "Exit Z Quiz". The right section is titled "YOUR GUESS" and contains input fields for $r = 1$, $theta = -\pi/4$, $x = 0.7071$, and $y = -0.7071$, along with buttons for "Check Answer" and "See Vector Sum". The bottom section shows two vector plots in the complex plane. The left plot shows the two input vectors z_1 (red) and z_2 (green) originating from the origin. The right plot shows the resulting vector $z_1 + z_2$ (red) and the vector $z_1 + z_2$ (green) originating from the origin.

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LECTURE OBJECTIVES

- Phasors = Complex Amplitude
 - Complex Numbers **represent** Sinusoids

$$z(t) = Xe^{j\omega t} = (Ae^{j\phi})e^{j\omega t}$$

- Develop the ABSTRACTION:
 - Adding Sinusoids = Complex Addition
 - **PHASOR ADDITION THEOREM**

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AVOID Trigonometry

- Algebra, even complex, is **EASIER !!!**
- Can you recall $\cos(\theta_1 + \theta_2)$?
- Use the real part of $e^{j(\theta_1 + \theta_2)}$

$$\begin{aligned} e^{j(\theta_1 + \theta_2)} &= e^{j\theta_1} e^{j\theta_2} \\ &= (\cos \theta_1 + j \sin \theta_1)(\cos \theta_2 + j \sin \theta_2) \\ &= \cos \theta_1 \cos \theta_2 - \sin \theta_1 \sin \theta_2 + j(\dots) \end{aligned}$$

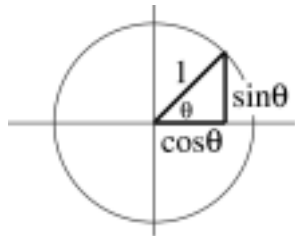
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Euler's FORMULA

- **Complex Exponential**
 - Real part is cosine
 - Imaginary part is sine
 - Magnitude is one



$$e^{j\theta} = \cos(\theta) + j \sin(\theta)$$

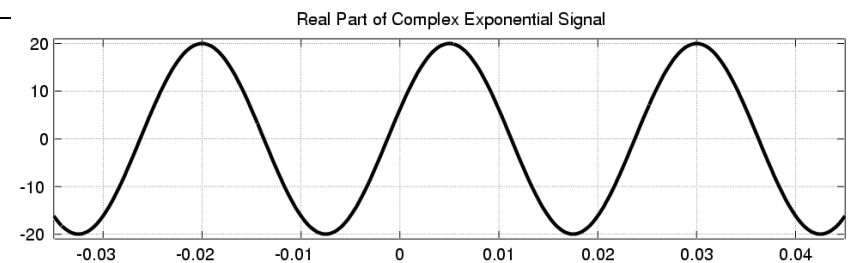
$$e^{j\omega t} = \cos(\omega t) + j \sin(\omega t)$$

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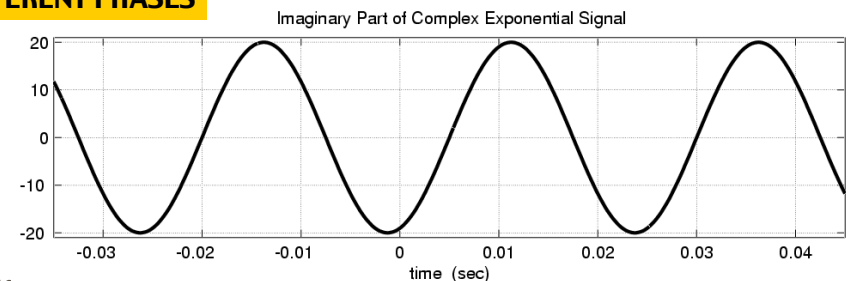
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Real & Imaginary Part Plots



DIFFERENT PHASES



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COMPLEX EXPONENTIAL

$$e^{j\omega t} = \cos(\omega t) + j \sin(\omega t)$$

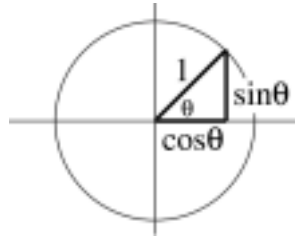
Rotating Vector

Angle changes vs. time

$\theta = \omega t$

ex: $\omega = 10\pi$

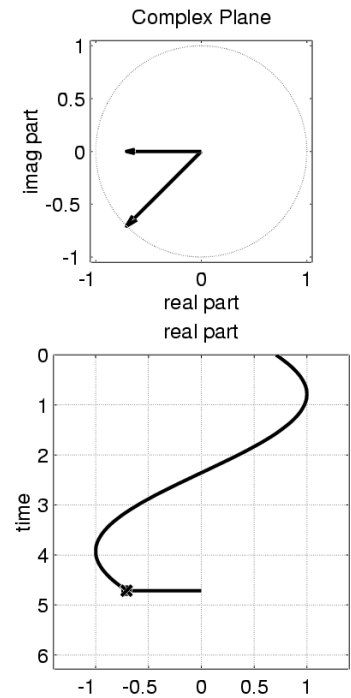
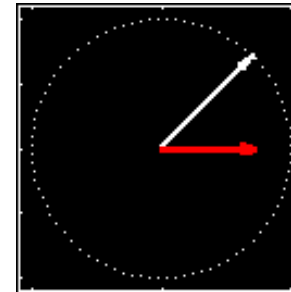
Rotates 0.1π in 0.01 secs



$$e^{j\theta} = \cos(\theta) + j \sin(\theta)$$

Rotating Phasor

See Demo on CD-ROM
Chapter 2



Cos = REAL PART

Real Part of Euler's

$$\cos(\omega t) = \Re\{e^{j\omega t}\}$$

General Sinusoid

$$x(t) = A \cos(\omega t + \varphi)$$

So,

$$\begin{aligned} A \cos(\omega t + \varphi) &= \Re\{Ae^{j(\omega t + \varphi)}\} \\ &= \Re\{Ae^{j\varphi} e^{j\omega t}\} \end{aligned}$$

COMPLEX AMPLITUDE

General Sinusoid

$$x(t) = A \cos(\omega t + \varphi) = \Re\{Ae^{j\varphi} e^{j\omega t}\}$$

Sinusoid = REAL PART of $(Ae^{j\varphi})e^{j\omega t}$

$$x(t) = \Re\{Xe^{j\omega t}\} = \Re\{z(t)\}$$

Complex AMPLITUDE = X

$$z(t) = Xe^{j\omega t} \quad X = Ae^{j\varphi}$$

POP QUIZ: Complex Amp

- Find the **COMPLEX AMPLITUDE** for:

$$x(t) = \sqrt{3} \cos(77\pi t + 0.5\pi)$$

- Use **EULER'S FORMULA**:

$$\begin{aligned} x(t) &= \Re\left\{\sqrt{3}e^{j(77\pi t + 0.5\pi)}\right\} \\ &= \Re\left\{\sqrt{3}e^{j0.5\pi}e^{j77\pi t}\right\} \end{aligned}$$

$$X = \sqrt{3}e^{j0.5\pi}$$

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WANT to ADD SINUSOIDS

- ALL SINUSOIDS HAVE SAME FREQUENCY**
- HOW to GET {Amp,Phase} of RESULT?**

$$x_1(t) = 1.7 \cos(2\pi(10)t + 70\pi/180)$$

$$x_2(t) = 1.9 \cos(2\pi(10)t + 200\pi/180)$$

$$x_3(t) = x_1(t) + x_2(t)$$

$$= 1.532 \cos(2\pi(10)t + 141.79\pi/180)$$



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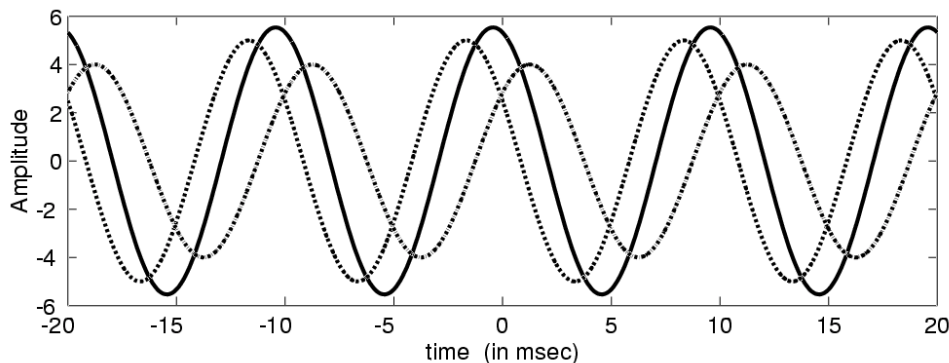
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ADD SINUSOIDS

- Sum Sinusoid has SAME Frequency**

Two Cosine Waves and Their Sum



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PHASOR ADDITION RULE

$$x(t) = \sum_{k=1}^N A_k \cos(\omega_0 t + \phi_k)$$

$$= A \cos(\omega_0 t + \phi)$$

Get the new complex amplitude by complex addition

$$\sum_{k=1}^N A_k e^{j\phi_k} = A e^{j\phi}$$

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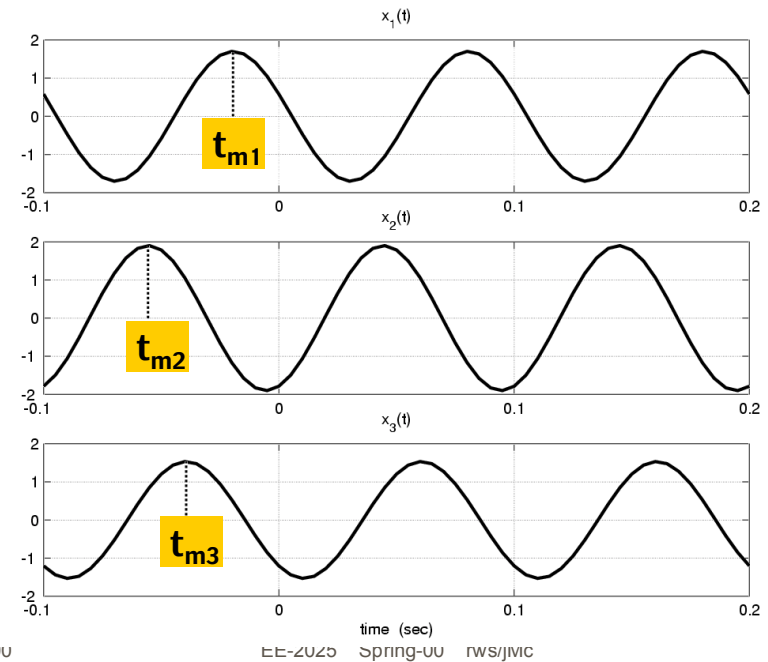
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Phasor Addition Proof

$$\begin{aligned} \sum_{k=1}^N A_k \cos(\omega_0 t + \phi_k) &= \sum_{k=1}^N \Re \{ A_k e^{j(\omega_0 t + \phi_k)} \} \\ &= \Re \left\{ \sum_{k=1}^N A_k e^{j\phi_k} e^{j\omega_0 t} \right\} \\ &= \Re \left\{ \left(\sum_{k=1}^N A_k e^{j\phi_k} \right) e^{j\omega_0 t} \right\} \\ &= \Re \{ (A e^{j\phi}) e^{j\omega_0 t} \} = A \cos(\omega_0 t + \phi) \end{aligned}$$

ADD SINUSOIDS EXAMPLE



Convert Time-Shift to Phase

- Measure **peak times**:
 - $t_{m1} = -0.0194$, $t_{m2} = -0.0556$, $t_{m3} = -0.0394$
- Convert to **phase** ($T=0.1$)
 - $\phi_1 = -\omega t_{m1} = -2\pi(t_{m1}/T) = 70\pi/180$,
 - $\phi_2 = 200\pi/180$
- **Amplitudes**
 - $A_1 = 1.7$, $A_2 = 1.9$, $A_3 = 1.532$

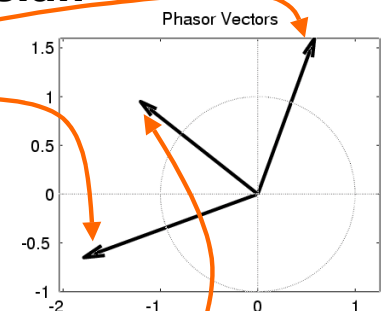
Phasor Add: Numerical

Convert Polar to Cartesian

- $X_1 = 0.5814 + j1.597$
- $X_2 = -1.785 - j0.6498$
- $X_3 = -1.204 + j0.9476$

Convert back to Polar

- $X_3 = 1.532$ at angle $141.79\pi/180$
- This is the sum



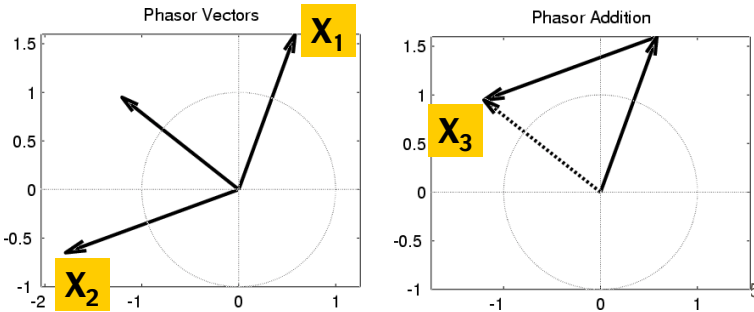
ADD SINUSOIDS

$$x_1(t) = 1.7 \cos(2\pi(10)t + 70\pi/180)$$

$$x_2(t) = 1.9 \cos(2\pi(10)t + 200\pi/180)$$

$$x_3(t) = x_1(t) + x_2(t)$$

$$= 1.532 \cos(2\pi(10)t + 141.79\pi/180)$$



**VECTOR
(PHASOR)
ADD**

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POP QUIZ: Add Sinusoids

■ ADD THESE 2 SINUSOIDS:

$$x_1(t) = \cos(77\pi t)$$

$$x_2(t) = \sqrt{3} \cos(77\pi t + 0.5\pi)$$

■ COMPLEX ADDITION:

$$1e^{j0} + \sqrt{3}e^{j0.5\pi}$$

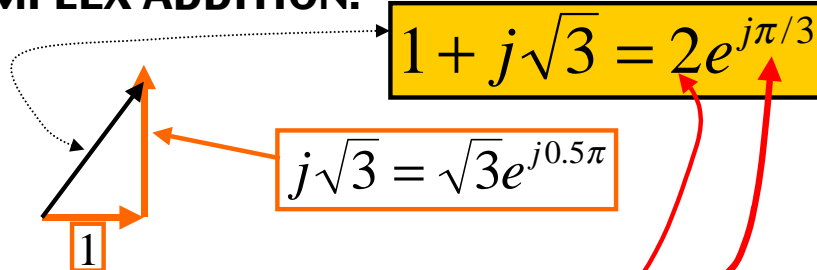
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POP QUIZ (answer)

■ COMPLEX ADDITION:



■ CONVERT back to cosine form:

$$x_3(t) = 2 \cos(77\pi t + \frac{\pi}{3})$$

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