EE-2025 Spring-00

LECTURE #3

Phasor Addition Theorem

14-Jan-99

Web-CT Info

- Check the Bulletin Board for msgs
  - MAKE YOUR OWN POSTINGS
- Lectures are being posted
  - PDF format (4 per page)
- Quiz Dates:
  - Quiz #1 on 4-Feb (Friday)
    - Quiz #2 on 3-March
    - Quiz #3 on 7-April

Homework Info

- Prob Set #1 due in RECITATION
  - At the beginning of class
- On-Line HW #1 ends Monday nite
  - Last attempt is scored
- HW will be posted by Wednesdays
  - Due during the following Week
- Solutions will be posted
  - Hopefully on Thurs nite or Friday.

Homework Formatting

- Cover page with
  - Name
  - Lab section, ie, L05, L21, etc.
  - Recitation Prof's name
  - See example on Web-CT
- Write on ONE side only
  - Use Engineer's paper or plain paper
- STAPLE

Lab Info

- NT passwd = SSN or old password
- Lab FAQs are being posted
- Lab #1 Report
  - Due week of 24-Jan
  - Turn in during your lab time
  - Write-up sections 2 and 3
  - Include INSTRUCTOR VERIFICATION
- Lab #2 will be posted next week

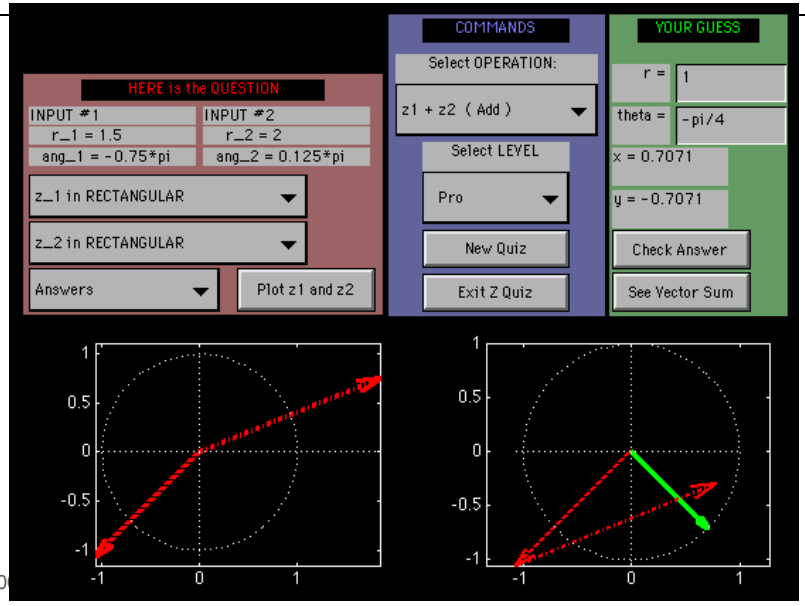
Lab Info: next week only

- Week of Jan 17th
  - Labs will be staffed
  - Catch up on Lab #1 Warm-up
  - Ask MATLAB questions
- MATLAB Help: T,Wed,Thur 6PM VL-456
- Learn Lab TA's format requirements

READING ASSIGNMENTS

- This Lecture:
  - Chapter 2, pp. 31-43
- Other Reading:
  - Appendix A: Complex Numbers
  - Appendix B: MATLAB
  - Next Lecture: start Chapter 3

Z DRILL (Complex Arith)



LECTURE OBJECTIVES

- Phasors = Complex Amplitude
  - Complex Numbers represent Sinusoids

$$z(t) = Xe^{j\omega t} = (Ae^{j\varphi})e^{j\omega t}$$

- Develop the ABSTRACTION:
  - Adding Sinusoids = Complex Addition
  - PHASOR ADDITION THEOREM

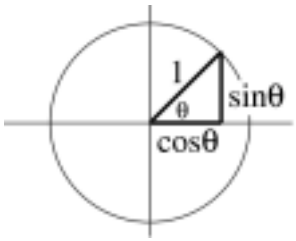
AVOID Trigonometry

- Algebra, even complex, is EASIER !!!
- Can you recall $\cos(\theta_1+\theta_2)$?
- Use the real part of $e^{j(\theta_1+\theta_2)}$

$$e^{j(\theta_1+\theta_2)} = e^{j\theta_1}e^{j\theta_2}$$
$$= (\cos\theta_1 + j\sin\theta_1)(\cos\theta_2 + j\sin\theta_2)$$
$$= \cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2 + j(\ldots)$$

Euler's FORMULA

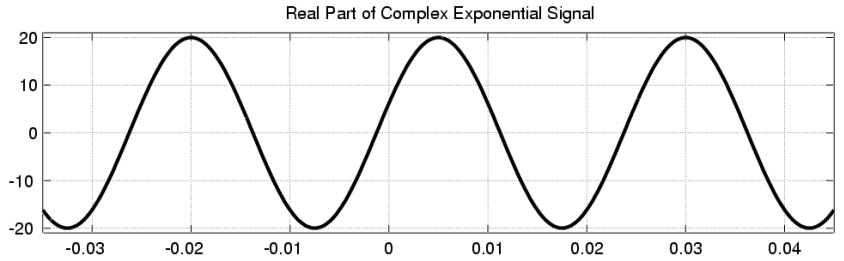
- Complex Exponential
  - Real part is cosine
  - Imaginary part is sine
  - Magnitude is one



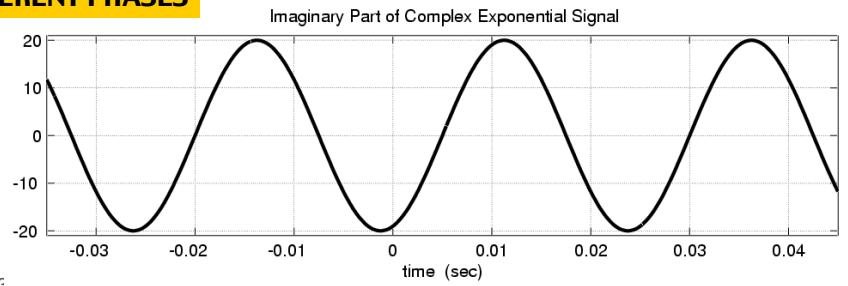
$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

$$e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$$

Real & Imaginary Part Plots



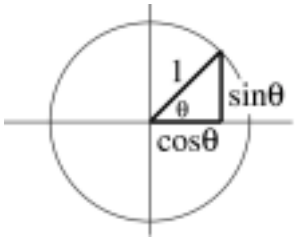
DIFFERENT PHASES



COMPLEX EXPONENTIAL

$$e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$$

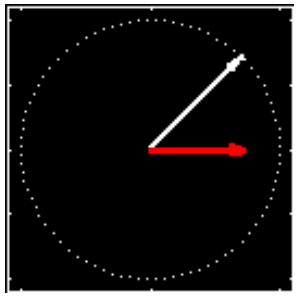
- **Rotating Vector**
  - **Angle changes vs. time**
  - **$\theta = \omega t$**
  - **ex:** $\omega = 10\pi$
  - **Rotates** 0.1π **in 0.01 secs**

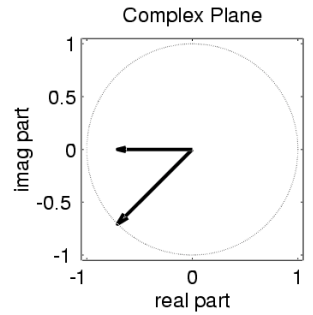


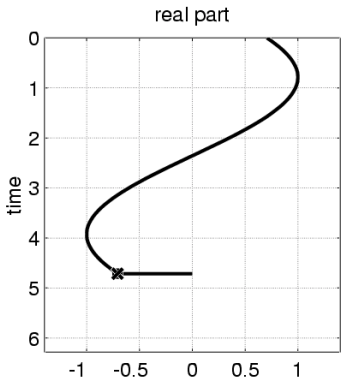
$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

Rotating Phasor

See Demo on CD-ROM Chapter 2







Cos = REAL PART

- **Real Part of Euler's**

$$\cos(\omega t) = \Re e\{e^{j\omega t}\}$$

- **General Sinusoid**

$$x(t) = A\cos(\omega t + \varphi)$$

- **So,**

$$A\cos(\omega t + \varphi) = \Re e\{Ae^{j(\omega t + \varphi)}\}$$
$$= \Re e\{Ae^{j\varphi}e^{j\omega t}\}$$

COMPLEX AMPLITUDE

- **General Sinusoid**

$$x(t) = A\cos(\omega t + \varphi) = \Re e\{Ae^{j\varphi}e^{j\omega t}\}$$

- **Sinusoid = REAL PART of $(Ae^{j\phi})e^{j\omega t}$**

$$x(t) = \Re e\{Xe^{j\omega t}\} = \Re e\{z(t)\}$$

- **Complex AMPLITUDE = X**

$$z(t) = Xe^{j\omega t} \qquad X = Ae^{j\varphi}$$

POP QUIZ: Complex Amp

- Find the COMPLEX AMPLITUDE for:

$$x(t) = \sqrt{3}\cos(77\pi t + 0.5\pi)$$

- Use EULER's FORMULA:

$$x(t) = \Re e\left\{\sqrt{3}e^{j(77\pi t + 0.5\pi)}\right\}$$

$$= \Re e\left\{\sqrt{3}e^{j0.5\pi}e^{j77\pi t}\right\}$$

$$X = \sqrt{3}e^{j0.5\pi}$$

WANT to ADD SINUSOIDS

- ALL SINUSOIDS HAVE SAME FREQUENCY
- HOW to GET {Amp,Phase} of RESULT ?

$$x_1(t) = 1.7\cos(2\pi(10)t + 70\pi/180)$$

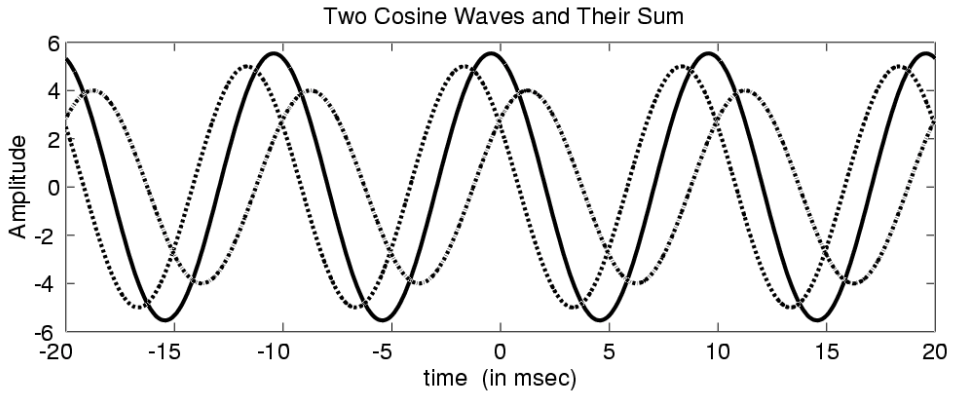
$$x_2(t) = 1.9\cos(2\pi(10)t + 200\pi/180)$$

$$x_3(t) = x_1(t) + x_2(t)$$

$$= 1.532\cos(2\pi(10)t + 141.79\pi/180)$$

ADD SINUSOIDS

- Sum Sinusoid has SAME Frequency



PHASOR ADDITION RULE

$$x(t) = \sum_{k=1}^{N} A_k\cos(\omega_0 t + \phi_k)$$

$$= A\cos(\omega_0 t + \phi)$$

Get the new complex amplitude by complex addition

$$\sum_{k=1}^{N} A_k e^{j\phi_k} = Ae^{j\phi}$$

Phasor Addition Proof

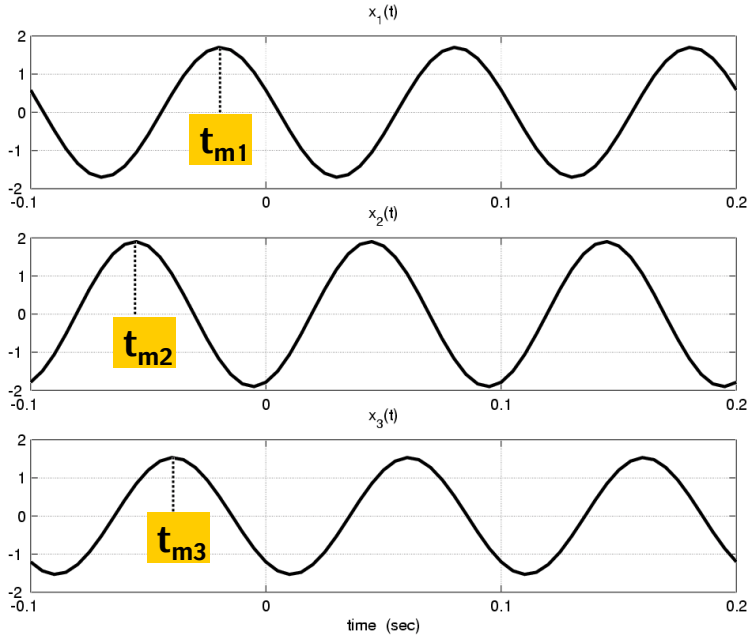
$$\sum_{k=1}^{N} A_k \cos(\omega_0 t + \phi_k) = \sum_{k=1}^{N} \Re e\left\{A_k e^{j(\omega_0 t+\phi_k)}\right\}$$

$$= \Re e\left\{\sum_{k=1}^{N} A_k e^{j\phi_k} e^{j\omega_0 t}\right\}$$

$$= \Re e\left\{\left(\sum_{k=1}^{N} A_k e^{j\phi_k}\right) e^{j\omega_0 t}\right\}$$

$$= \Re e\left\{\left(A e^{j\phi}\right) e^{j\omega_0 t}\right\} = A\cos(\omega_0 t + \phi)$$

ADD SINUSOIDS EXAMPLE

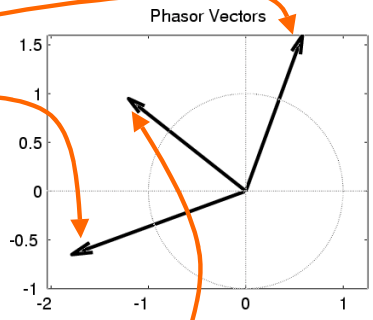


Convert Time-Shift to Phase

- Measure peak times:
  - t_{m1}=−0.0194, t_{m2}=−0.0556, t_{m3}=−0.0394
- Convert to phase (T=0.1)
  - $\phi_1 = -\omega t_{m1} = -2\pi(t_{m1}/T) = 70\pi/180$,
  - $\phi_2 = 200\pi/180$
- Amplitudes
  - A_1=1.7, A_2=1.9, A_3=1.532

Phasor Add: Numerical

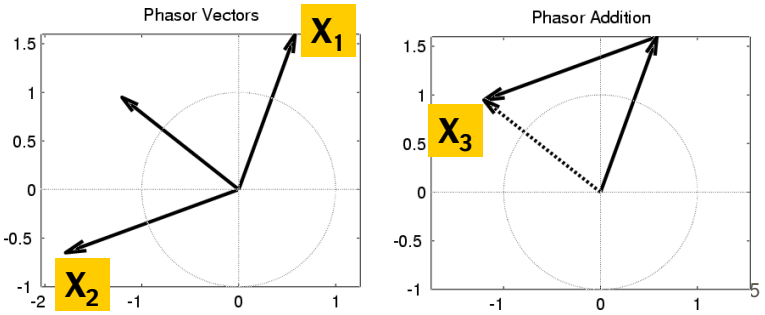
- Convert Polar to Cartesian
  - $X_1 = 0.5814 + j1.597$
  - $X_2 = -1.785 - j0.6498$
  - $X_3 = -1.204 + j0.9476$
- Convert back to Polar
  - $X_3 = 1.532$ at angle $141.79\pi/180$
  - This is the sum



ADD SINUSOIDS

$$x_1(t) = 1.7\cos(2\pi(10)t + 70\pi/180)$$
$$x_2(t) = 1.9\cos(2\pi(10)t + 200\pi/180)$$
$$x_3(t) = x_1(t) + x_2(t)$$
$$= 1.532\cos(2\pi(10)t + 141.79\pi/180)$$

VECTOR (PHASOR) ADD



POP QUIZ: Add Sinusoids

- **ADD THESE 2 SINUSOIDS:**

$$x_1(t) = \cos(77\pi t)$$
$$x_2(t) = \sqrt{3}\cos(77\pi t + 0.5\pi)$$

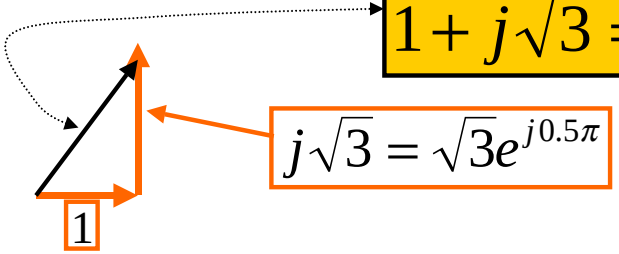
- **COMPLEX ADDITION:**

$$1e^{j0} + \sqrt{3}e^{j0.5\pi}$$

POP QUIZ (answer)

- **COMPLEX ADDITION:**

$$1 + j\sqrt{3} = 2e^{j\pi/3}$$



- **CONVERT back to cosine form:**

$$x_3(t) = 2\cos(77\pi t + \tfrac{\pi}{3})$$