EE-2025 Spring-2000

Lecture 4

Spectrum Representation

21-Jan-00

Web-CT Info

- Bulletin Board has all OFFICIAL msgs
- Lectures are being posted
  - PDF format (4 per page)
- Upcoming Events:
  - Quiz #1 on 4–Feb (Friday)
  - On–Line Lab Quiz at start of Lab #3
    - MATLAB Questions: SHORT & EASY

James THURBER

- It is better to know some of the questions than all of the answers

Lab Info

- NT passwd = SSN or old password
- MATLAB Help: Mon & Thur 6PM VL–456
- Lab #1 Report
  - Turn in during your lab time
  - Write–up sections 2 and 3
  - Include INSTRUCTOR VERIFICATION
- Lab #2 has been posted
  - For week of 24–Jan

Homework & Lab Info

- Written HW #2 due NEXT WEEK
  - In Recitation, at the beginning
- HW #1 Solutions posted
  - Thursday nite
- MATLAB Help:
  - Mon & Thur 6PM VL-456
  - Planned for rest of the semester

Homework Formatting

- Cover page with
  - Name
  - Lab section, ie, L05, L21, etc.
  - Recitation Prof's name
- Write on ONE side only
  - Prefer Engineering paper, or plain
    - NO LINES
- STAPLE

READING ASSIGNMENTS

- This Lecture:
  - Chapter 3, pp. 48–61
- Other Reading:
  - Appendix A: Complex Numbers
  - Appendix B: MATLAB
  - Next Lecture: Chapter 3

HISTORY

- What company's first successful product was a sine-wave generator?
  - Variable frequency
  - Lab Instrument

HEWLETT PACKARD Agilent

LECTURE OBJECTIVES

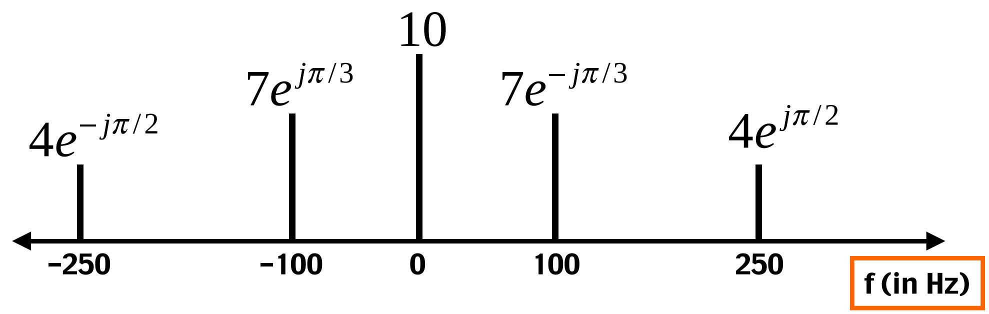
- Sinusoids with DIFFERENT Frequencies
  - SYNTHESIZE by Adding Sinusoids

$$x(t) = \sum_{k=1}^{N} A_k \cos(2\pi f_k t + \varphi_k)$$

- SPECTRUM Representation
  - Graphical Form shows DIFFERENT Freqs

FREQUENCY DIAGRAM

- Plot Complex Amplitude vs. Freq



Another FREQ. Diagram

Frequency is the vertical axis

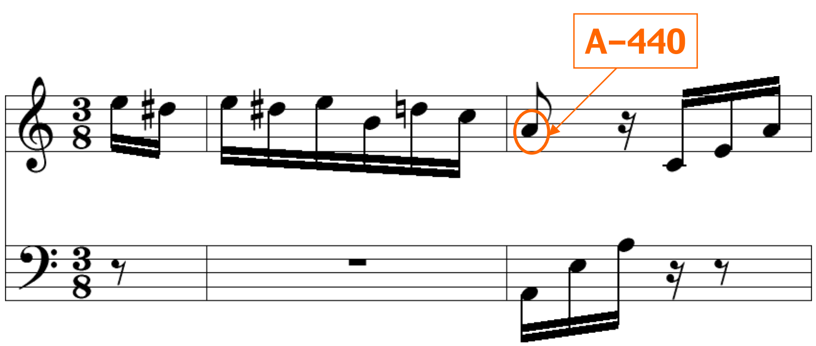


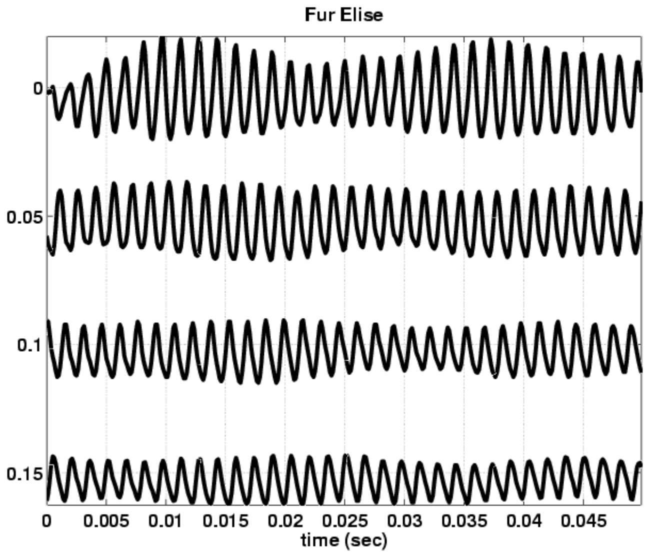
Figure 3.18 Sheet-music notation is a time–frequency diagram.

Time is the horizontal axis

MOTIVATION

- Synthesize Complicated Signals
  - Musical Notes
    - Piano uses 3 strings for many notes
    - Chords: play several notes simultaneously
  - Human Speech
    - Vowels have dominant frequencies
    - Application: computer generated speech
  - Can all signals be generated this way?

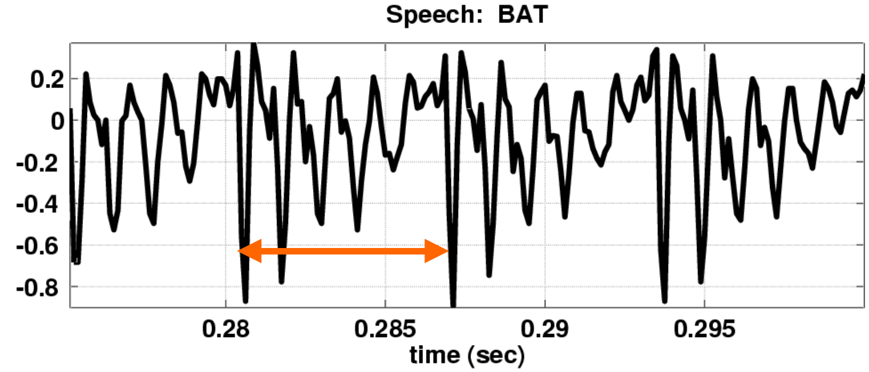
Fur Elise WAVEFORM



Beat Notes

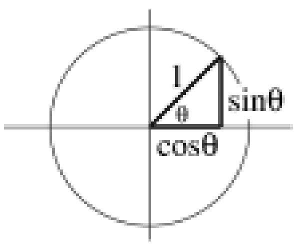
Speech Signal: BAT

- Nearly Periodic in Vowel Region
  - Period is (Approximately) T = 0.0065 sec



Euler's FORMULA

- Complex Exponential
  - Real part is cosine
  - Imaginary part is sine
  - Magnitude is one



$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

$$\boxed{e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)}$$

INVERSE Euler's Formula

- Solve for cosine (or sine)

$$e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$$

$$e^{-j\omega t} = \cos(-\omega t) + j\sin(-\omega t)$$

$$e^{-j\omega t} = \cos(\omega t) - j\sin(\omega t)$$

$$e^{j\omega t} + e^{-j\omega t} = 2\cos(\omega t)$$

$$\boxed{\cos(\omega t) = \tfrac{1}{2}(e^{j\omega t} + e^{-j\omega t})}$$

INVERSE Euler's Formula

- Solve for **cosine** (or sine)

$$\cos(\omega t) = \tfrac{1}{2}\left(e^{j\omega t} + e^{-j\omega t}\right)$$

$$\sin(\omega t) = \tfrac{1}{2j}\left(e^{j\omega t} - e^{-j\omega t}\right)$$

SPECTRUM Interpretation

- Cosine = sum of 2 complex exponentials:

$$A\cos(7t) = \tfrac{A}{2}e^{j7t} + \tfrac{A}{2}e^{-j7t}$$

  - One has a positive frequency
  - The other has **negative** freq.
  - Amplitude of each is half as big

SPECTRUM of SINE

- Sine = sum of 2 complex exponentials:

$$A\sin(7t) = \tfrac{A}{2j}e^{j7t} - \tfrac{A}{2j}e^{-j7t}$$

$$= \tfrac{1}{2}Ae^{-j0.5\pi}e^{j7t} + \tfrac{1}{2}Ae^{j0.5\pi}e^{-j7t}$$

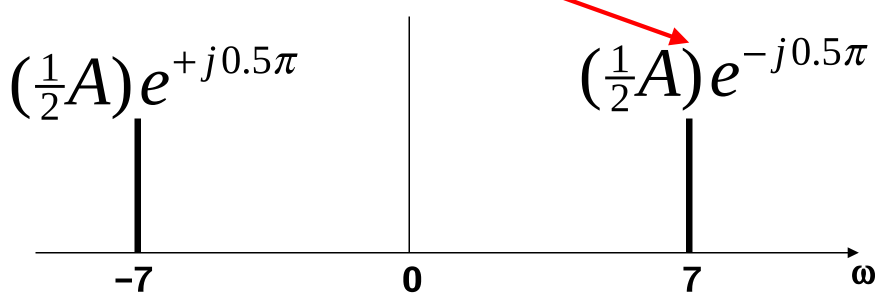
$$\tfrac{-1}{j} = j = e^{j0.5\pi}$$

  - Positive freq. has phase = -0.5π
  - Negative freq. has phase = $+0.5\pi$

GRAPHICAL SPECTRUM

EXAMPLE of SINE

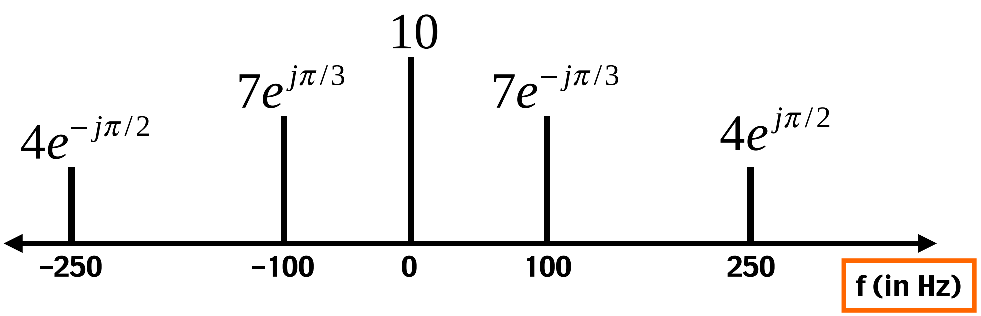
$$A\sin(7t) = \tfrac{1}{2}Ae^{-j0.5\pi}e^{j7t} + \tfrac{1}{2}Ae^{j0.5\pi}e^{-j7t}$$



AMPLITUDE, PHASE & FREQUENCY are shown

SPECTRUM ---> SINUSOID

- Add the spectrum components:



What is the formula for the signal x(t)?

Gather (A,ω,ϕ) information

- Frequencies:
  - −250 Hz
  - −100 Hz
  - 0 Hz
  - 100 Hz
  - 250 Hz
- Amplitude & Phase
  - 4 $-\pi/2$
  - 7 $+\pi/3$
  - 10 0
  - 7 $-\pi/3$
  - 4 $+\pi/2$

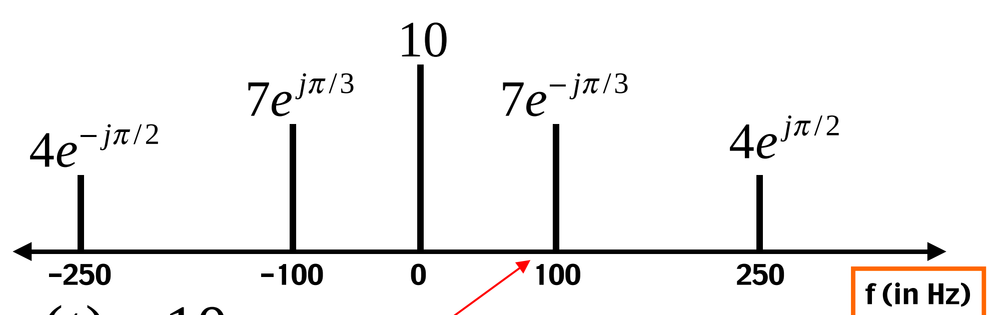
Note the conjugate phase
Zero freq always has zero phase (for real $x(t)$)
DC is another name for zero-freq component

Add Spectrum Components-1

- Frequencies:
  - −250 Hz
  - −100 Hz
  - 0 Hz
  - 100 Hz
  - 250 Hz
- Amplitude & Phase
  - 4 $-\pi/2$
  - 7 $+\pi/3$
  - 10 0
  - 7 $-\pi/3$
  - 4 $+\pi/2$

$$x(t) = 10 + 7e^{-j\pi/3}e^{j2\pi(100)t} + 7e^{j\pi/3}e^{-j2\pi(100)t} + 4e^{j\pi/2}e^{j2\pi(250)t} + 4e^{-j\pi/2}e^{-j2\pi(250)t}$$

Add Spectrum Components-2



$$x(t) = 10 + 7e^{-j\pi/3}e^{j2\pi(100)t} + 7e^{j\pi/3}e^{-j2\pi(100)t} + 4e^{j\pi/2}e^{j2\pi(250)t} + 4e^{-j\pi/2}e^{-j2\pi(250)t}$$

Simplify Components

$$x(t) = 10 + 7e^{-j\pi/3}e^{j2\pi(100)t} + 7e^{j\pi/3}e^{-j2\pi(100)t} + 4e^{j\pi/2}e^{j2\pi(250)t} + 4e^{-j\pi/2}e^{-j2\pi(250)t}$$

Use Euler's Formula to get REAL sinusoids:

$$A\cos(\omega t + \varphi) = \tfrac{A}{2}e^{j\varphi}e^{j\omega t} + \tfrac{A}{2}e^{-j\varphi}e^{-j\omega t}$$

FINAL ANSWER

$$x(t) = 10 + 14\cos(2\pi(100)t - \pi/3) + 8\cos(2\pi(250)t + \pi/2)$$

So, we get the general form:

$$x(t) = \sum_{k=1}^{N} A_k \cos(2\pi f_k t + \varphi_k)$$

Summary: GENERAL FORM

$$x(t) = A_0 + \sum_{k=1}^{N} A_k \cos(2\pi f_k t + \varphi_k)$$

$$x(t) = X_0 + \sum_{k=1}^{N} \Re e\left\{X_k e^{j2\pi f_k t}\right\}$$

$$X_k = A_k e^{j\varphi_k}$$

$$\text{Frequency} = f_k$$

$$\Re e\{z\} = \tfrac{1}{2}z + \tfrac{1}{2}z^*$$

$$x(t) = X_0 + \sum_{k=1}^{N} \left\{\tfrac{1}{2}X_k e^{j2\pi f_k t} + \tfrac{1}{2}X_k^* e^{-j2\pi f_k t}\right\}$$

NEGATIVE FREQUENCY

- Is negative frequency real?
- Doppler Radar provides an example
  - Police radar measures speed by using the Doppler shift principle
  - Let's assume 400Hz <---> 60 mph
  - +400Hz means towards the radar
  - −400Hz means away (opposite direction)
  - Think of a train whistle

Example: Synthetic Vowel

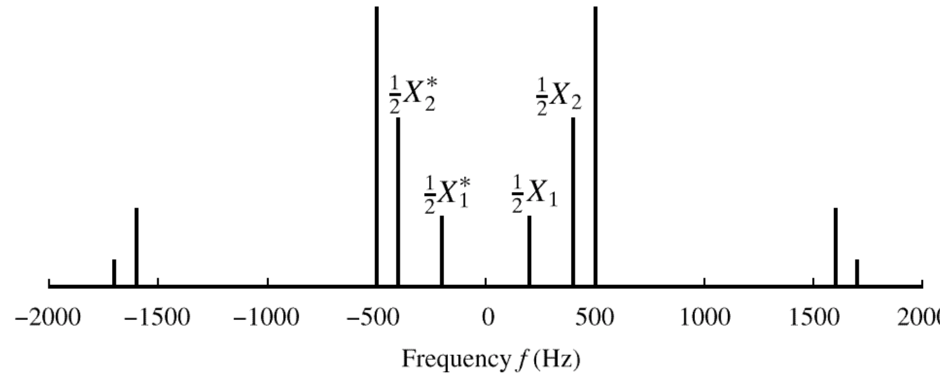
- Sum of 5 Frequency Components

f_k (Hz)	X_k	Mag	Phase (rad)
200	$(771 + j12202)$	12,226	1.508
400	$(-8865 + j28048)$	29,416	1.876
500	$(48001 - j8995)$	48,836	−0.185
1600	$(1657 - j13520)$	13,621	−1.449
1700	$4723 + j0$	4723	0

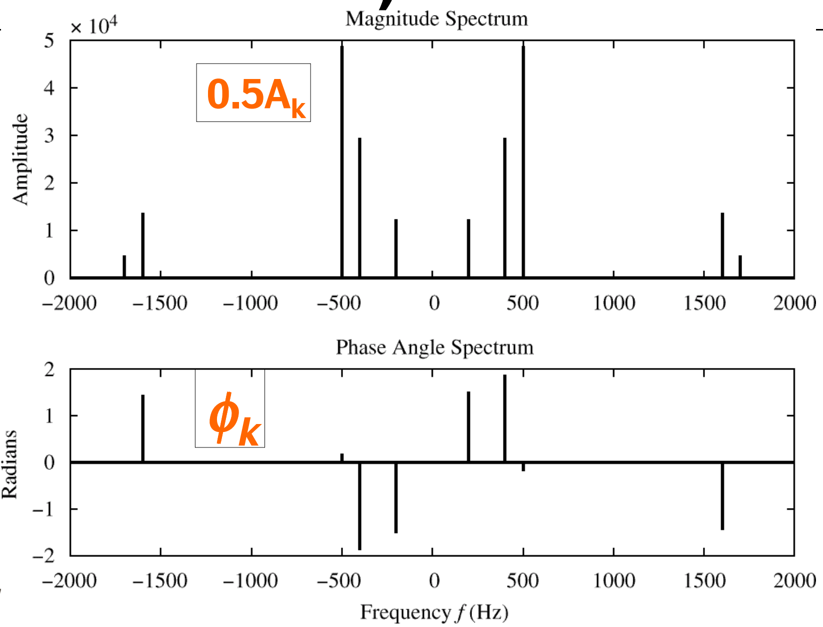
Table 3.1: Complex amplitudes for harmonic signal that approximates the vowel sound "ah".

SPECTRUM of VOWEL

- Note: Spectrum has $0.5X_k$ (except X_{DC})
- Conjugates in negative frequency



SPECTRUM of VOWEL (Polar Format)



Vowel Waveform (sum of all 5 components)

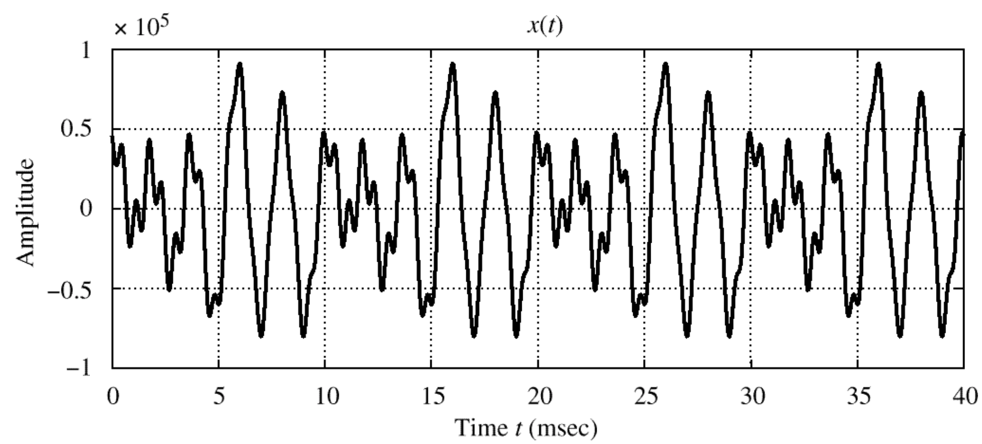


Figure 3.11 Sum of all of the terms in (3.3.4). Note that the period is 10 msec, which equals $1/f_0$.