EE-2025 Spring-2000

Lecture 5

Harmonics & Time-Varying Sinusoids

24-Jan-00

Web-CT Info

- Check the Bulletin Board for msgs
- Old Quizzes & Problems are linked
  - Quiz #1 on 4-Feb (Friday)
- Prob Set #2 due THIS WEEK
- On-Line Self Tests are available

Lab Info

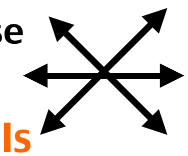
- Lab #1 Report
  - Turn in during your lab time
  - Write-up sections 2 and 3
  - Include INSTRUCTOR VERIFICATION
- Lab #2 this week
  - ERRORS ? ALWAYS Check Bulletin Board

READING ASSIGNMENTS

- This Lecture:
  - Chapter 3, pp. 57–61
  - Chapter 3, pp. 66–77
- Next Lecture: Notes
  - Fourier Series ANALYSIS
  - Replaces pp.62–65 in DSP First

Problem Solving Skills

- **Math Formula**
  - Sum of Cosines
  - Amp, Freq, Phase
- **Recorded Signals**
  - Speech
  - Music
  - No simple formula
- **Plot & Sketches**
  - S(t) versus t
  - Spectrum
- **MATLAB**
  - Numerical
  - Computation
  - Plotting list of numbers



LECTURE OBJECTIVES

- **Signals with HARMONIC Frequencies**
  - Add Sinusoids with $f_k = kf_0$

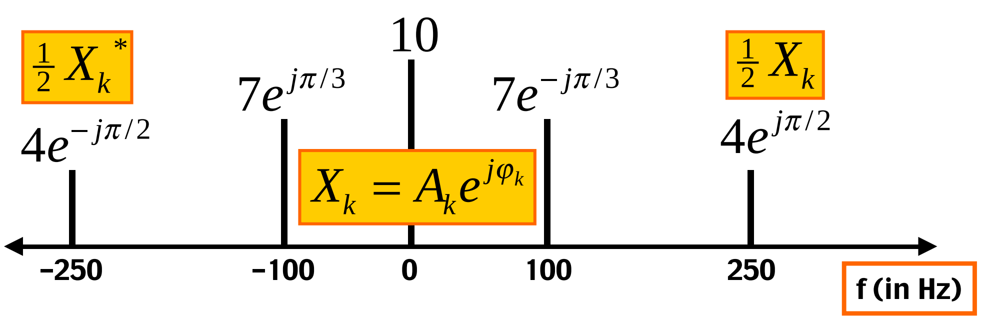
$$x(t) = A_0 + \sum_{k=1}^{N} A_k \cos(2\pi k f_0 t + \varphi_k)$$

- **FREQUENCY can change vs. TIME**
  - Chirps: $x(t) = \cos(\alpha t^2)$
  - Introduce Spectrogram Visualization

`(specgram.m)` `(plotspec.m)`

SPECTRUM DIAGRAM

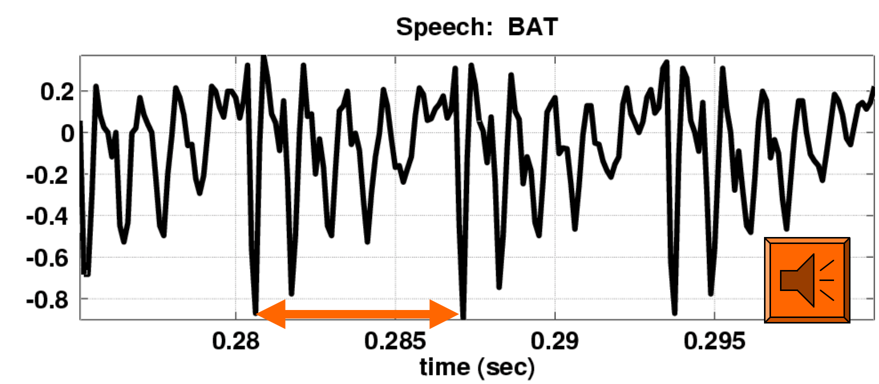
- **Recall Complex Amplitude vs. Freq**



$$x(t) = 10 + 14\cos(2\pi(100)t - \pi/3) + 8\cos(2\pi(250)t + \pi/2)$$

SPECTRUM for PERIODIC ?

- **Nearly Periodic in the Vowel Region**
  - Period is (Approximately) T = 0.0065 sec



PERIODIC SIGNALS

- Repeat every T secs
  - Definition

$$x(t) = x(t+T)$$

  - Example:

$$x(t) = \cos^2(3t) \qquad T = ? \qquad T = \tfrac{2\pi}{3} \qquad T = \tfrac{\pi}{3}$$

  - Speech can be “quasi-periodic”

Summary: GENERAL FORM

$$x(t) = A_0 + \sum_{k=1}^{N} A_k \cos(2\pi f_k t + \varphi_k)$$

$$X_0 = A_0 e^{j0}$$

$$x(t) = X_0 + \sum_{k=1}^{N} \Re e\left\{X_k e^{j2\pi f_k t}\right\}$$

$$X_k = A_k e^{j\varphi_k}$$

$$\text{Frequency} = f_k$$

$$\Re e\{z\} = \tfrac{1}{2}z + \tfrac{1}{2}z^*$$

$$x(t) = X_0 + \sum_{k=1}^{N} \left\{\tfrac{1}{2} X_k e^{j2\pi f_k t} + \tfrac{1}{2} X_k^* e^{-j2\pi f_k t}\right\}$$

Period of Complex Exponential

$$x(t) = e^{j\omega t}$$

$$x(t+T) = x(t)\,?$$

Definition: Period is T

$$e^{j\omega(t+T)} = e^{j\omega t}$$

$$e^{j2\pi k} = 1$$

$$\Rightarrow e^{j\omega T} = 1 \quad \Rightarrow \omega T = 2\pi k$$

$$\omega = \frac{2\pi k}{T} = \left(\frac{2\pi}{T}\right)k = \omega_0 k$$

k = integer

Harmonic Signal Spectrum

Therefore, we can only have: $f_k = k f_0$

$$x(t) = A_0 + \sum_{k=1}^{N} A_k \cos(2\pi k f_0 t + \varphi_k)$$

$$X_k = A_k e^{j\varphi_k}$$

$$x(t) = X_0 + \sum_{k=1}^{N} \tfrac{1}{2} X_k e^{j2\pi k f_0 t} + \sum_{k=1}^{N} \tfrac{1}{2} X_k^* e^{-j2\pi k f_0 t}$$

DEFINE FUNDAMENTAL

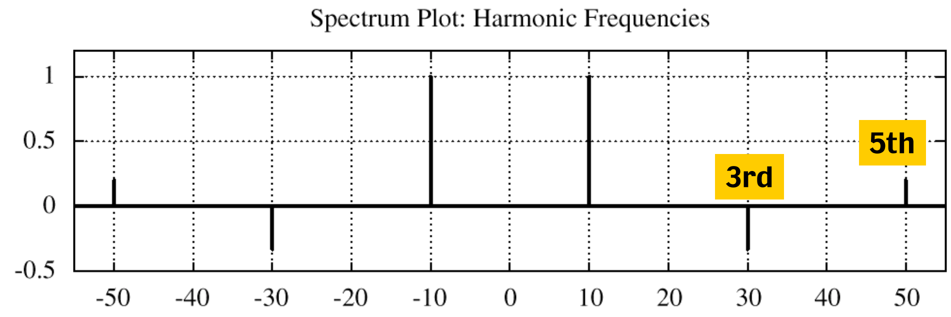
$$x(t) = A_0 + \sum_{k=1}^{N} A_k \cos(2\pi k f_0 t + \varphi_k)$$

$$f_k = k\, f_0 \qquad (\omega_0 = 2\pi\, f_0)$$

$$f_0 = \text{fundamental frequency}$$

$$T_0 = \text{fundamental Period} \qquad f_0 = \frac{1}{T_0}$$

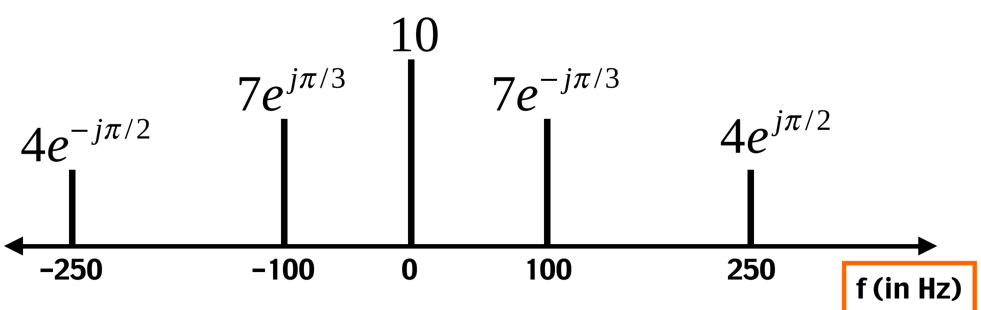
Harmonic Signal (3 Freqs)



What is the fundamental frequency? **10 Hz**

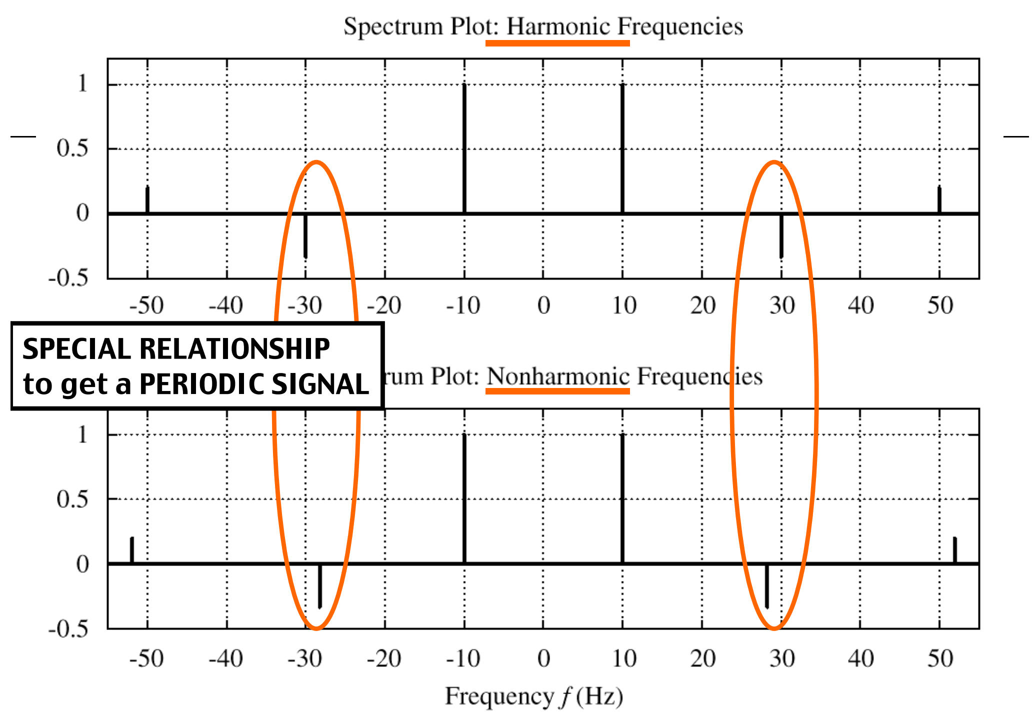
POP QUIZ: FUNDAMENTAL

- **Here's another spectrum:**

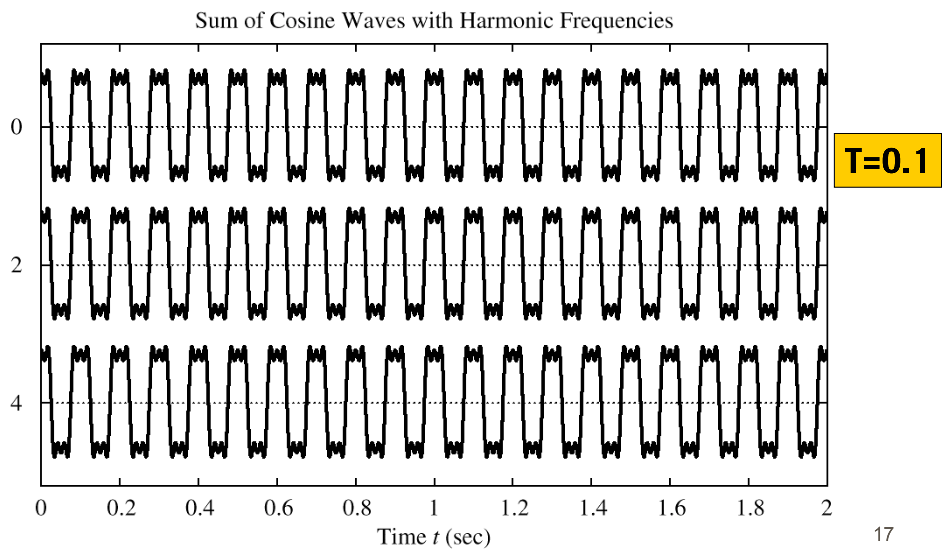


What is the fundamental frequency?

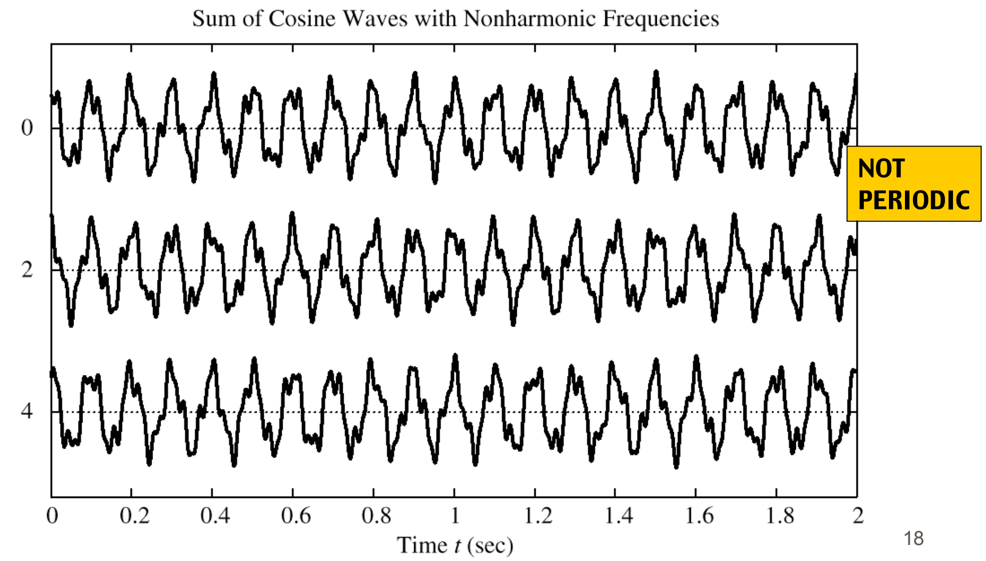
100 Hz ? **50 Hz ?**



Harmonic Signal (3 Freqs)

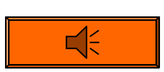


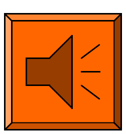
NON-Harmonic Signal



FREQUENCY ANALYSIS

- <u>**Now, a much HARDER problem**</u>
- **Given a recording of a song, have the computer write the music**





- **Can a machine extract frequencies?**
  - **COMPUTE the spectrum for x(t)**
    - **During short intervals**

Time-Varying FREQUENCIES Diagram

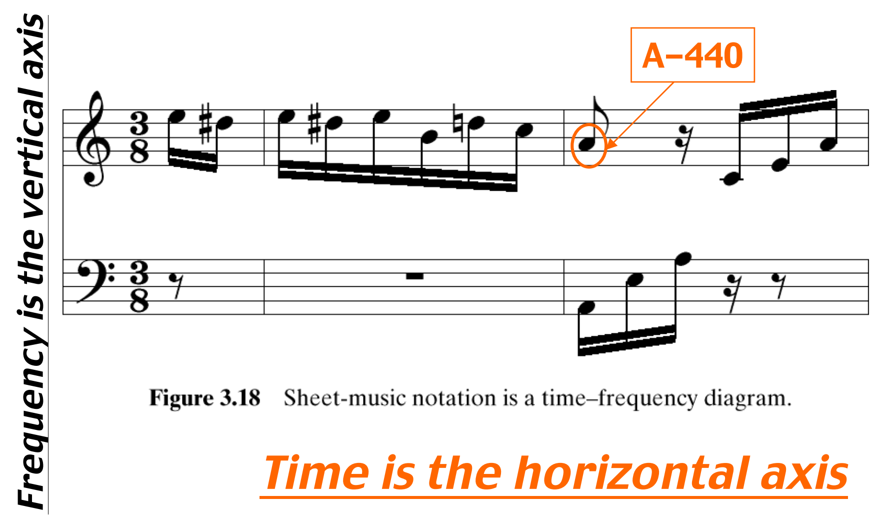
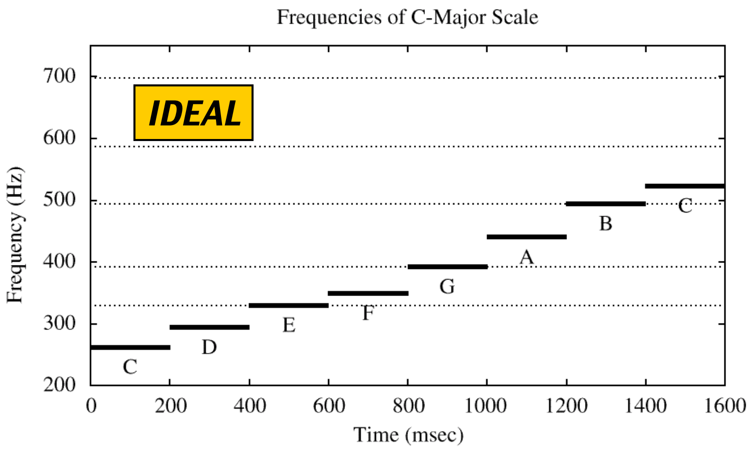


Figure 3.18 Sheet-music notation is a time–frequency diagram.

Time is the horizontal axis

SIMPLE TEST SIGNAL

- C–major SCALE: stepped frequencies
  - Frequency is constant for each note

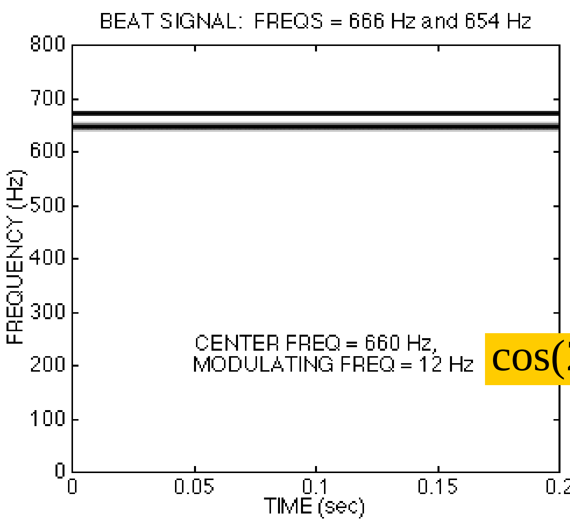


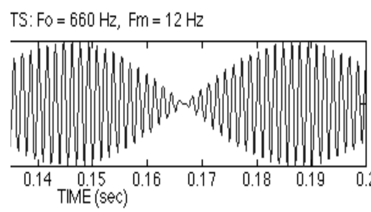
R-rated: ADULTS ONLY

- SPECTROGRAM Tool
  - MATLAB function is specgram.m
  - DSP First has spectgr.m (no plotting)
- ANALYSIS program
  - Takes x(t) as input
  - Produces spectrum values X_k
  - Breaks x(t) into SHORT TIME SEGMENTS
    - Then uses the FFT (Fast Fourier Transform)

SPECTROGRAM EXAMPLE

- Two Constant Frequencies: Beats





$$\cos(2\pi(660)t)\cos(2\pi(12)t)$$

AM Radio Signal

- Same as BEAT Notes

$$\cos(2\pi(660)t)\cos(2\pi(12)t)$$

BEATS: Fo = 660 Hz, Fm = 12 Hz

$$\tfrac{1}{2}\left(e^{j2\pi(660)t}+e^{-j2\pi(660)t}\right)\tfrac{1}{2}\left(e^{j2\pi(12)t}+e^{-j2\pi(12)t}\right)$$

$$\tfrac{1}{4}\left(e^{j2\pi(672)t}+e^{-j2\pi(672)t}+e^{j2\pi(648)t}+e^{-j2\pi(648)t}\right)$$

$$\tfrac{1}{2}\cos(2\pi(672)t)+\tfrac{1}{2}\cos(2\pi(648)t)$$

SPECTRUM of AM (Beat)

- 4 complex exponentials in AM:

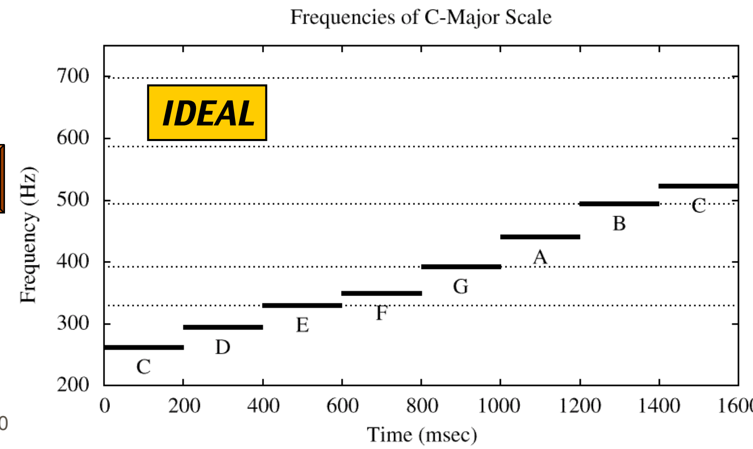
What is the fundamental frequency?

648 Hz ?

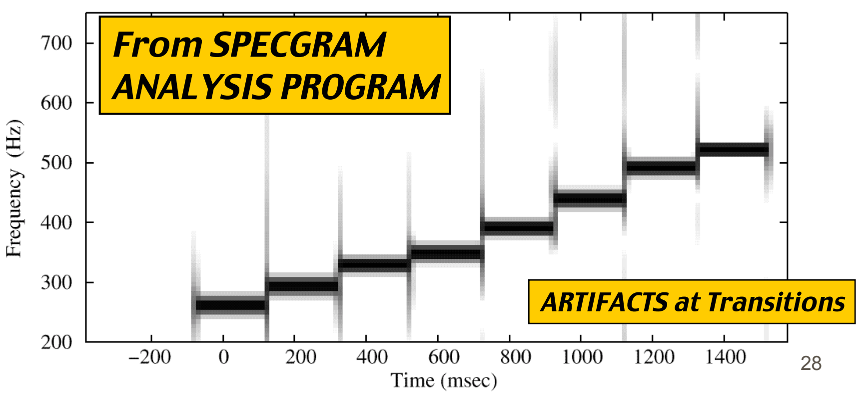
24 Hz ?

STEPPED FREQUENCIES

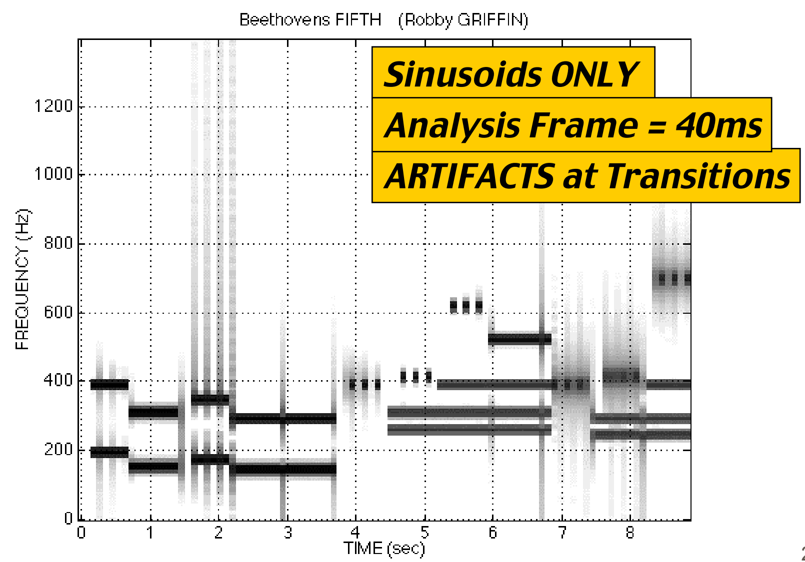
- C–major SCALE: successive sinusoids
  - Frequency is constant for each note



SPECTROGRAM of C-Scale



Spectrogram of LAB SONG



Time-Varying Frequency

- Frequency can change vs. time
  - Continuously, not stepped
- FREQUENCY MODULATION (FM)

$$x(t) = \cos(2\pi f_c t + v(t))$$

VOICE

- CHIRP SIGNALS
  - Linear Frequency Modulation (LFM)

New Signal: Linear FM

- Called Chirp Signals (LFM)
  - Quadratic phase

QUADRATIC

$$x(t) = A\cos(\alpha t^2 + 2\pi f_0 t + \varphi)$$

- Freq will change LINEARLY vs. time
  - Example of Frequency Modulation (FM)
  - Define "instantaneous frequency"

INSTANTANEOUS FREQ

- Definition

$$x(t) = A\cos(\psi(t))$$

$$\Rightarrow \omega_i(t) = \tfrac{d}{dt}\psi(t)$$

Derivative of the "Angle"

- For Sinusoid:

$$x(t) = A\cos(2\pi f_0 t + \varphi)$$

$$\psi(t) = 2\pi f_0 t + \varphi$$

$$\Rightarrow \omega_i(t) = \tfrac{d}{dt}\psi(t) = 2\pi f_0$$

Makes sense

INSTANTANEOUS FREQ of the Chirp

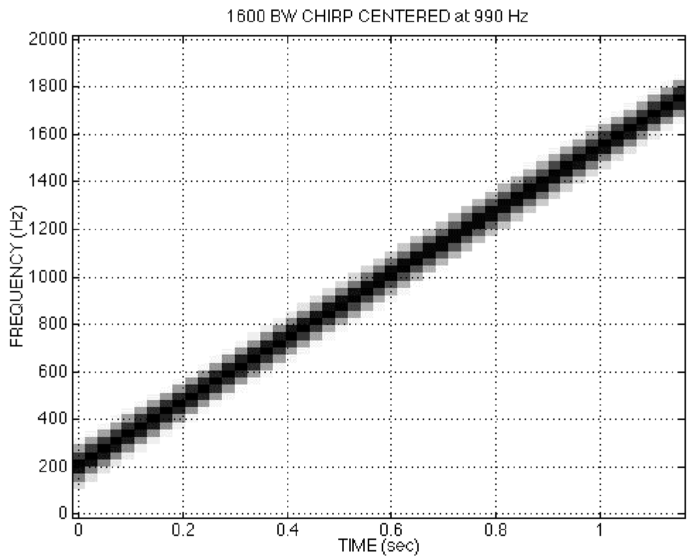
- Chirp Signals have Quadratic phase
- Freq will change LINEARLY vs. time

$$x(t) = A\cos(\alpha t^2 + \beta t + \varphi)$$

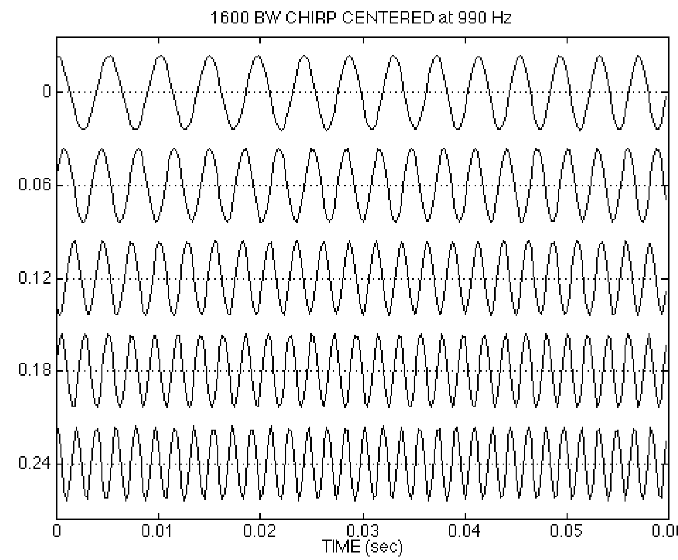
$$\Rightarrow \psi(t) = \alpha t^2 + \beta t + \varphi$$

$$\Rightarrow \omega_i(t) = \tfrac{d}{dt}\psi(t) = 2\alpha t + \beta$$

CHIRP SPECTROGRAM



CHIRP WAVEFORM



OTHER CHIRPS

- $\psi(t)$ can be anything:

$$x(t) = A\cos(\alpha\cos(\beta t) + \varphi)$$

$$\Rightarrow \omega_i(t) = \tfrac{d}{dt}\psi(t) = -\alpha\beta\sin(\beta t)$$

- $\psi(t)$ could be speech or music:
  - FM radio broadcast

SINE-WAVE FREQUENCY MODULATION (FM)