EE-2025 Spring-2000

Lecture 7

Sampling & Aliasing

7-Feb-00

Information

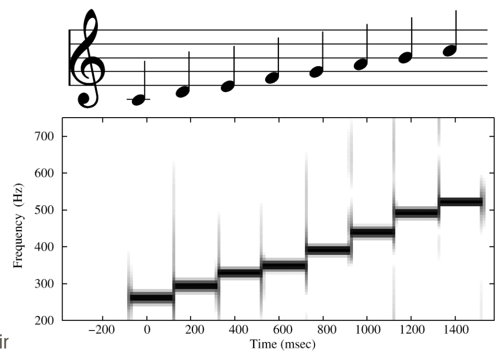
- Check the Bulletin Board for msgs
- Problem Set #4 due this week
- Lab #4 is Music Synthesis
  - Worth 150 Points
  - Formal lab Report
  - Listening Tests the next week.

READING ASSIGNMENTS

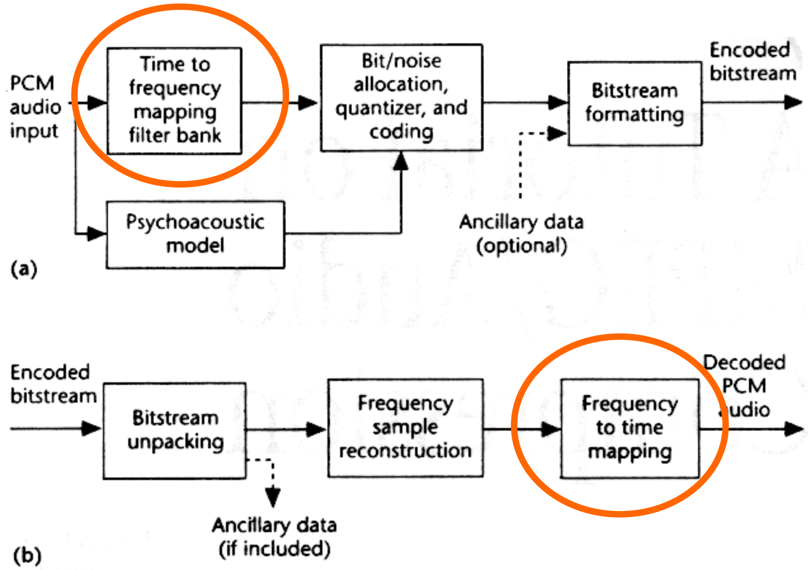
- This Lecture:
  - Chapter 4, pp. 83–94
- Other Reading:
  - Recitation: Chapter 4, pp. 90–100
    - Strobe Demo
  - Next Lecture: Chap. 4, pp. 100–111

CD-ROM DEMOS

- USE THE DEMOS
- Chapter 3: Spectrum
  - DEMOS of SPECTROGRAM
  - BEAT NOTES/AM
  - SPEECH
  - MUSIC
  - FM & Chirps



MP-3 Block Diagram



LECTURE OBJECTIVES

- SAMPLING can cause ALIASING
  - Sampling Theorem
  - Sampling Rate > 2(Highest Frequency)
- Spectrum for digital signals, x[n]
  - Normalized Frequency

$$\hat{\omega} = \omega T_s = \frac{2\pi f}{f_s} + 2\pi\ell$$

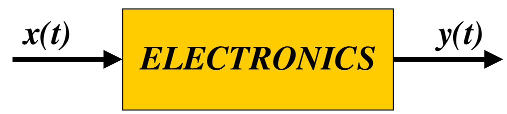
ALIASING

SYSTEMS Process Signals

- PROCESSING GOALS:
  - Change x(t) into y(t)
    - For example, more BASS
  - Improve x(t), e.g., image deblurring
  - Extract Information from x(t)

System IMPLEMENTATION

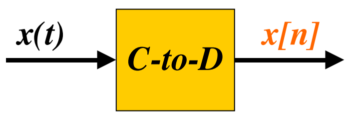
- ANALOG/ELECTRONIC:
  - Circuits: resistors, capacitors, op-amps



- DIGITAL/MICROPROCESSOR
  - Convert x(t) to numbers stored in memory

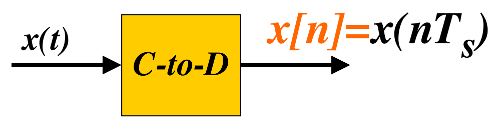
SAMPLING x(t)

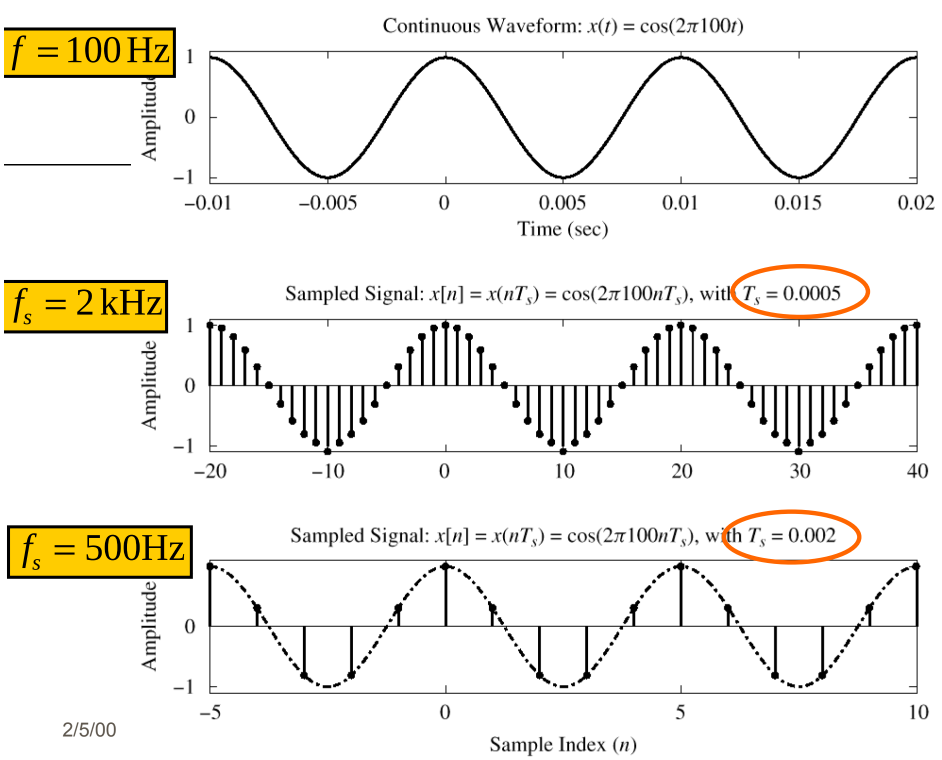
- **SAMPLING PROCESS**
  - Convert x(t) to **numbers** x[n]
  - "n" is an integer; x[n] is a sequence
  - "n" is the storage address in memory
- **UNIFORM SAMPLING at $t = nT_s$**
  - IDEAL: $x[n] = x(nT_s)$



SAMPLING RATE, f_s

- **SAMPLING RATE (f_s)**
  - $1/T_s$ = NUMBER of SAMPLES PER SECOND
  - T_s = 125 microsec --> f_s = 8000 samples/sec
    - UNITS ARE HERTZ: 8000 Hz
- **UNIFORM SAMPLING at $t = nT_s = n/f_s$**
  - IDEAL: $x[n] = x(nT_s) = x(n/f_s)$





SAMPLING THEOREM

- **HOW OFTEN ?**
  - DEPENDS on FREQUENCY of SINUSOID
  - ANSWERED by SHANNON/NYQUIST
  - ALSO DEPENDS on "RECONSTRUCTION"

> *Shannon Sampling Theorem*
> A continuous-time signal $x(t)$ with frequencies no higher than f_{max} can be reconstructed exactly from its samples $x[n] = x(nT_s)$, if the samples are taken at a rate $f_s = 1/T_s$ that is greater than $2f_{max}$.

STORING DIGITAL SOUND

- $x[n]$ is a SAMPLED SINUSOID
  - A list of numbers stored in memory
- CD rate is 44,100 samples per second
  - 16-bit samples
  - Stereo uses 2 channels
- Number of bytes for 1 minute is
  - 2 X (16/8) X 60 X 44100 = 10.584 Mbytes

DISCRETE-TIME SINUSOID

- Change x(t) into x[n] **DERIVATION**

$$x(t) = A\cos(\omega t + \varphi)$$

$$x[n] = x(nT_s) = A\cos(\omega n T_s + \varphi)$$

$$x[n] = A\cos((\omega T_s)n + \varphi)$$

$$x[n] = A\cos(\hat{\omega} n + \varphi)$$

$$\hat{\omega} = \omega T_s$$

DEFINE DIGITAL FREQUENCY

DIGITAL FREQUENCY $\hat{\omega}$

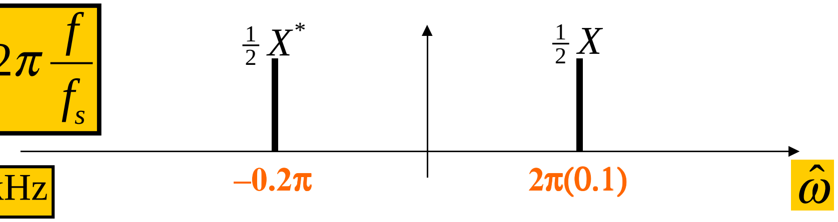
- $\hat{\omega}$ VARIES from 0 to 2π, as f varies from 0 to the sampling frequency
- DIGITAL FREQUENCY is <u>NORMALIZED</u>
- UNITS are radians, not rad/sec

$$\hat{\omega} = \omega T_s = \frac{2\pi f}{f_s}$$

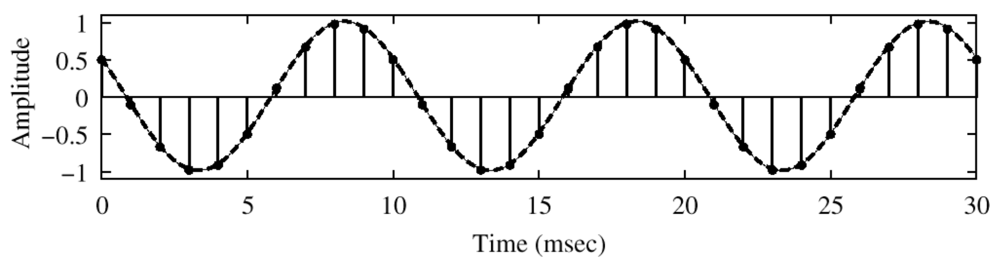
SPECTRUM (DIGITAL)

$$\hat{\omega} = 2\pi \frac{f}{f_s}$$

$f_s = 1\text{kHz}$



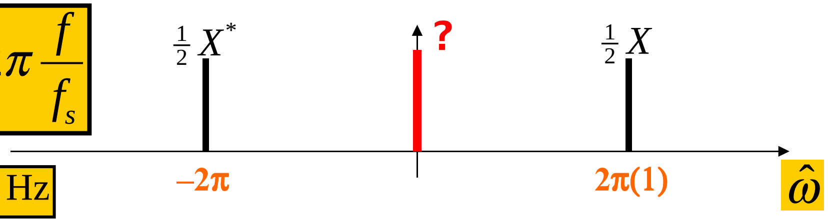
100-Hz Cosine Wave: Sampled with T_s = 1 msec (1000 Hz)

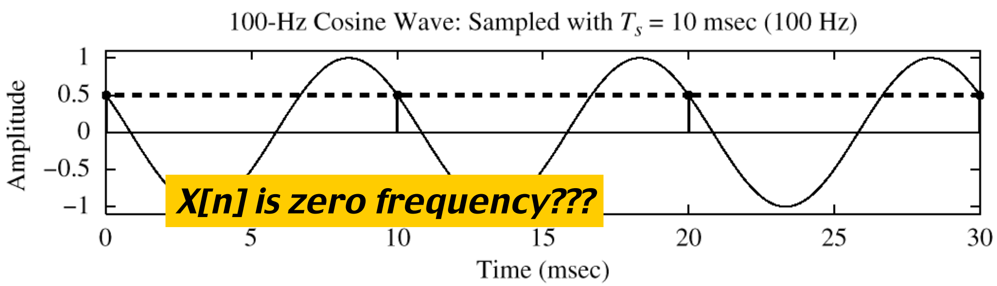


SPECTRUM (DIGITAL) ???

$$\hat{\omega} = 2\pi \frac{f}{f_s}$$

$f_s = 100\,\text{Hz}$





The REST of the STORY

- **Spectrum of x[n] has more than one line for each complex exponential**
  - **Called ALIASING**
  - **MANY SPECTRAL LINES**
- **SPECTRUM is PERIODIC with period = 2π**
  - **Because**

$$A\cos(\hat{\omega}n + \varphi) = A\cos((\hat{\omega} + 2\pi)n + \varphi)$$

ALIASING DERIVATION

- **Other Frequencies give the same $\hat{\omega}$**

If $x(t) = A\cos(2\pi(f + \ell f_s)t + \varphi)$

and we substitute: $t \leftarrow \frac{n}{f_s}$

then: $x[n] = A\cos(2\pi(f + \ell f_s)\frac{n}{f_s} + \varphi)$

or, $x[n] = A\cos(2\pi\frac{f}{f_s}n + 2\pi\ell n + \varphi)$

ALIASING DERIVATION–2

- **Other Frequencies give the same $\hat{\omega}$**

If $x(t) = A\cos(2\pi(f + \ell f_s)t + \varphi)$ $\quad t \leftarrow \frac{n}{f_s}$

and we want : $x[n] = A\cos(\hat{\omega}n + \varphi)$

then : $\hat{\omega} = \frac{2\pi(f + \ell f_s)}{f_s} = \frac{2\pi f}{f_s} + \frac{2\pi\ell f_s}{f_s}$

$$\hat{\omega} = \omega T_s = \frac{2\pi f}{f_s} + 2\pi\ell$$

ALIASING CONCLUSIONS

- ADDING f_s or $2f_s$ or $-f_s$ TO THE FREQ of x(t) gives exactly the same x[n]
- The samples, x[n] = x(n/ f_s) are EXACTLY THE SAME VALUES
- GIVEN x[n], WE CAN'T DISTINGUISH f_o FROM ($f_o + f_s$) or ($f_o + 2f_s$)

NORMALIZED FREQUENCY

- DIGITAL FREQUENCY

Normalized Radian Frequency

$$\hat{\omega} = \omega T_s = \frac{2\pi f}{f_s} \boxed{+2\pi\ell}$$

Normalized Cyclic Frequency

$$\hat{f} = \hat{\omega}/(2\pi) = f T_s = f/f_s$$

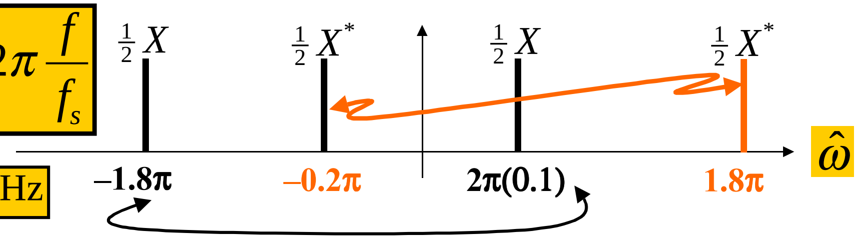
SPECTRUM for x[n]

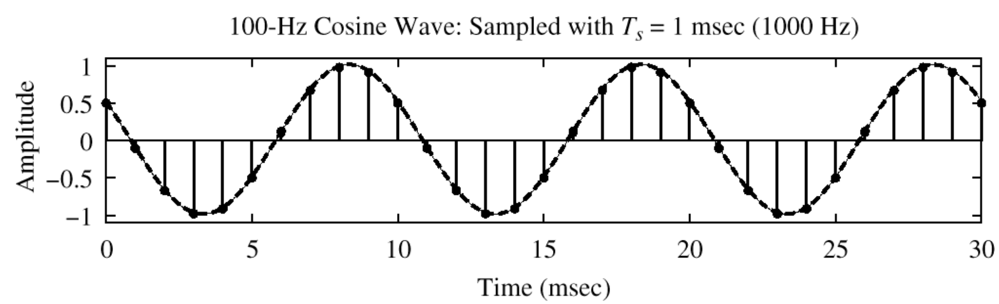
- PLOT versus NORMALIZED FREQUENCY
- INCLUDE ALL SPECTRUM LINES
  - ALIASES
    - ADD MULTIPLES of 2π
    - SUBTRACT MULTIPLES of 2π
  - FOLDED ALIASES
    - (to be discussed later)
    - ALIASES of NEGATIVE FREQS

SPECTRUM (MORE LINES)

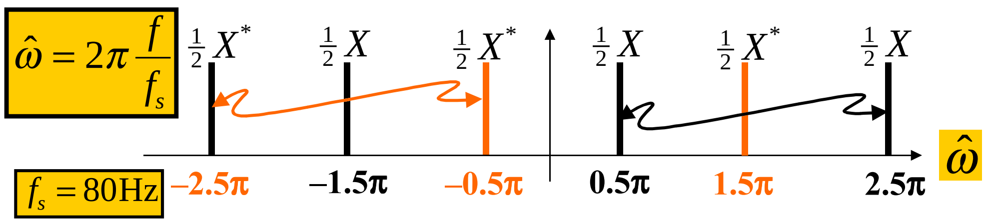
$$\hat{\omega} = 2\pi \frac{f}{f_s}$$

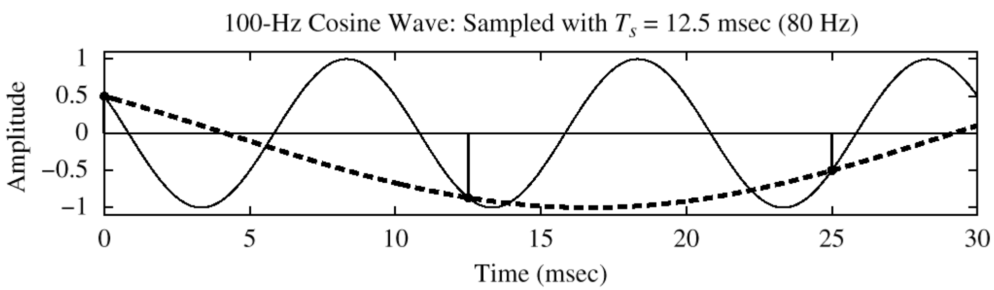
$f_s = 1\text{kHz}$



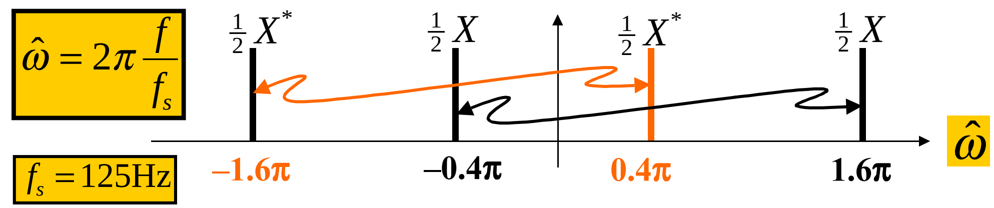


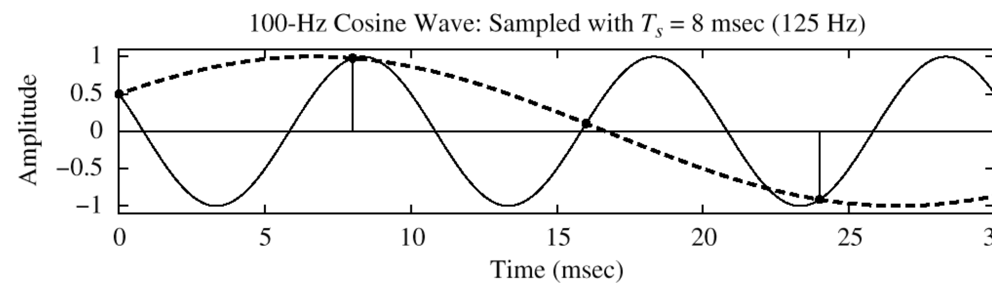
SPECTRUM (ALIASING CASE)



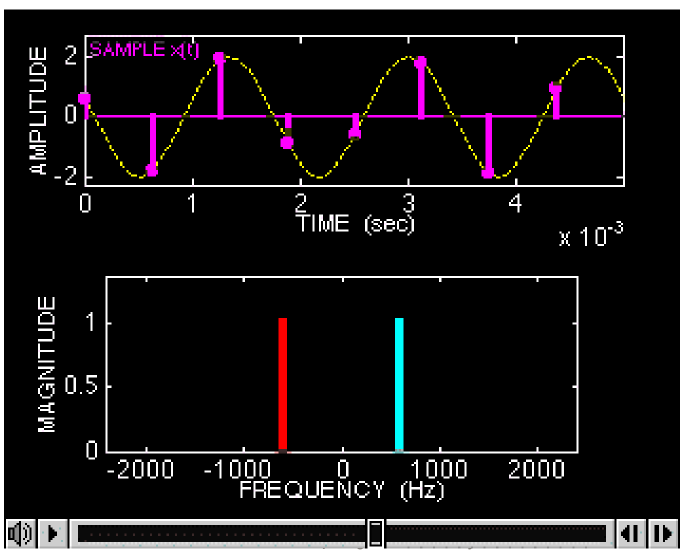


SPECTRUM (FOLDING CASE)





SAMPLING DEMO (Chap. 4)



STROBE DEMO (Synthetic)