EE-2025 Spring-2000

Lecture 9

FIR Filtering Intro

14-Feb-00

Information

- Music Listening this week
  - Survey this week !!!!!!!
- Lab Quiz NEXT week
- Problem Set #5 due THIS WEEK
  - More On-Line Self Tests available
- MATLAB help:
  - Mon & Thurs @ 6pm, VL-456
- Quiz #2 on 3-March (Friday)

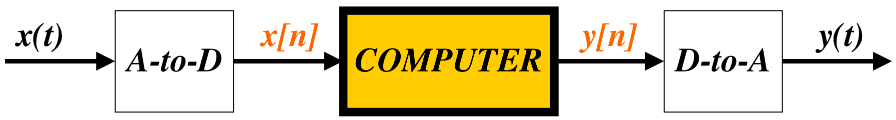
READING ASSIGNMENTS

- This Lecture:
  - Chapter 5, pp. 119-131
- Other Reading:
  - Recitation: Ch. 5, pp. 127-133, 142-146
    - CONVOLUTION
  - Next Lecture: Chapter 5, pp. 133-152

LECTURE OBJECTIVES

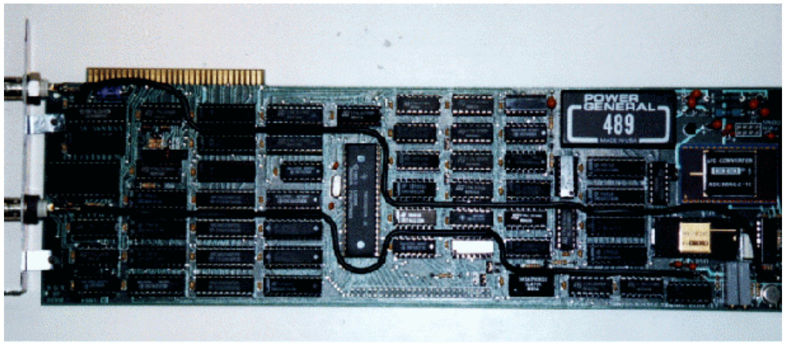
- INTRODUCE FILTERING IDEA
  - Weighted Average
  - Running Average
- FINITE IMPULSE RESPONSE FILTERS
  - FIR Filters
  - Show how to compute the output y[n] from the input signal, x[n]

DIGITAL FILTERING



- CONCENTRATE on the COMPUTER
  - PROCESSING ALGORITHMS
  - SOFTWARE (MATLAB)
  - HARDWARE: DSP chips, VLSI
- DSP: DIGITAL SIGNAL PROCESSING

The TMS32010, 1983

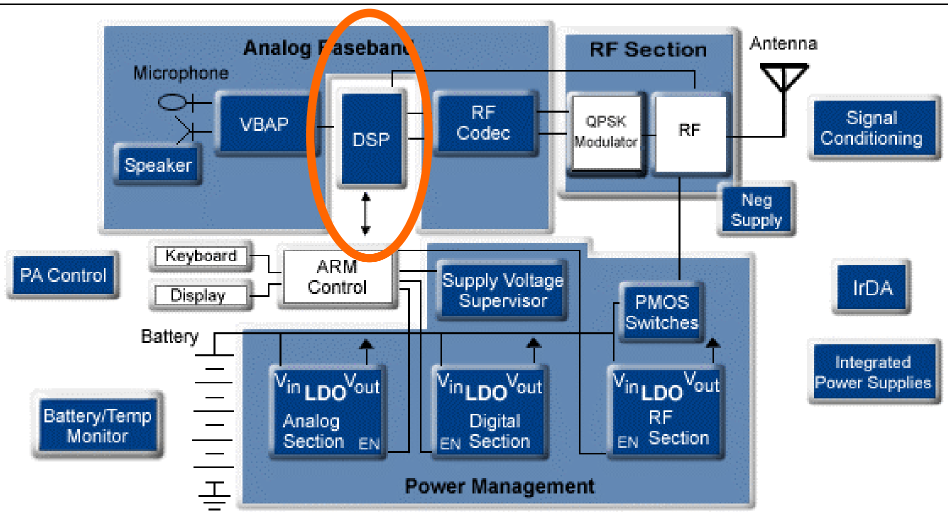


First PC plug-in board from Atlanta Signal Processors Inc.

Rockland Digital Filter, 1971

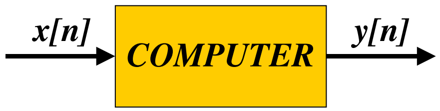
For the price of a small house, you could have one of these.

Digital Cell Phone



Free (?) with 2 year contract

DISCRETE-TIME SYSTEM



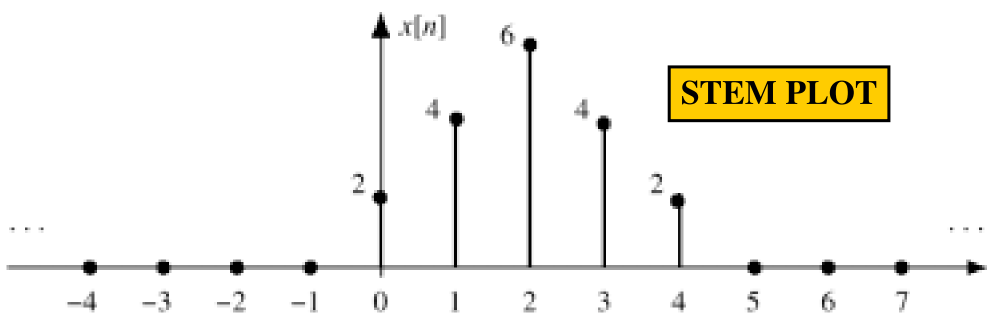
- OPERATE on x[n] to get y[n]
- WANT a GENERAL CLASS of SYSTEMS
  - ANALYZE the SYSTEM
    - TOOLS: TIME–DOMAIN & FREQUENCY–DOMAIN
  - SYNTHESIZE the SYSTEM

D-T SYSTEM EXAMPLES

- EXAMPLES:
  - POINTWISE OPERATORS
    - SQUARING: $y[n] = (x[n])^2$
  - RUNNING AVERAGE
    - RULE: "the output at time n is the average of three consecutive input values"

DISCRETE-TIME SIGNAL

- x[n] is a LIST of NUMBERS
  - INDEXED by "n"



3-PT AVERAGE SYSTEM

- ADD 3 CONSECUTIVE NUMBERS
  - Do this for each "n"

the following input–output equation **Make a TABLE**

$$y[n] = \tfrac{1}{3}(x[n] + x[n+1] + x[n+2])$$

n	$n < -2$	-2	-1	0	1	2	3	4	5	$n > 5$
$x[n]$	0	0	0	2	4	**6**	**4**	**2**	0	0
$y[n]$	0	$\frac{2}{3}$	2	4	$\frac{14}{3}$	**4**	2	$\frac{2}{3}$	0	0

n=0 $y[0] = \tfrac{1}{3}(x[0] + x[1] + x[2])$

n=1 $y[1] = \tfrac{1}{3}(x[1] + x[2] + x[3])$

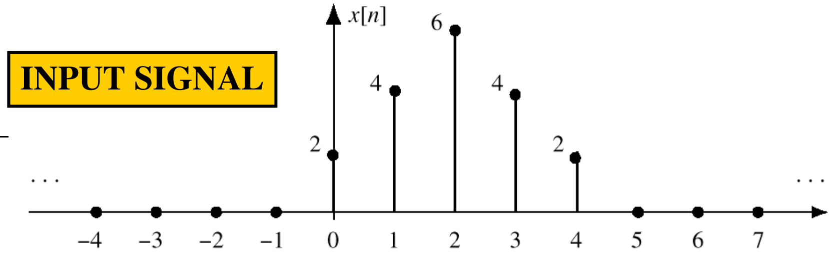


Figure 5.2 Finite-length input signal, $x[n]$.

$$y[n] = \tfrac{1}{3}(x[n] + x[n+1] + x[n+2])$$

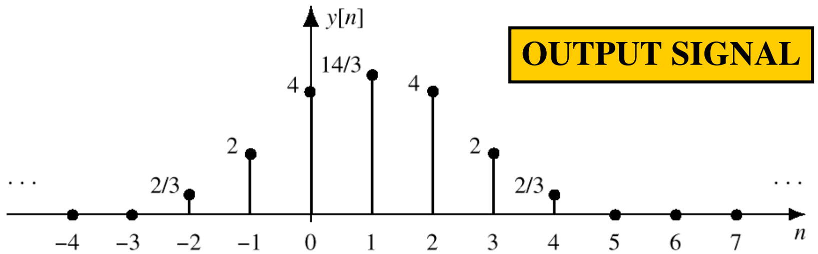


Figure 5.3 Output of running average, $y[n]$.

PAST, PRESENT, FUTURE

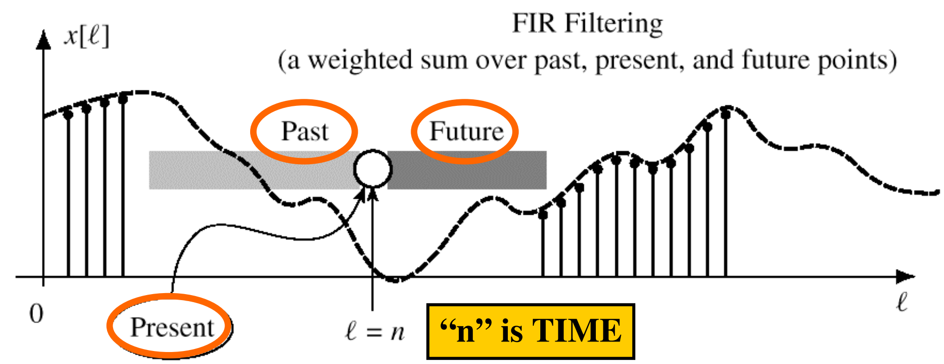


Figure 5.4 The running-average filter calculation at time index n uses values within a sliding window (shaded). Dark shading indicates the future ($\ell > n$); light shading, the past ($\ell < n$).

ANOTHER 3-pt AVERAGER

- Uses "PAST" VALUES of x[n]
  - IMPORTANT IF "n" represents REAL TIME
    - WHEN x[n] & y[n] ARE STREAMS

$$y[n] = \tfrac{1}{3}(x[n] + x[n-1] + x[n-2])$$

n	$n < -2$	-2	-1	0	1	2	3	4	5	6	7	$n > 7$
$x[n]$	0	0	0	2	4	6	4	2	0	0	0	0
$y[n]$	0	0	0	$\frac{2}{3}$	2	4	$\frac{14}{3}$	4	2	$\frac{2}{3}$	0	0

GENERAL FIR FILTER

- FILTER COEFFICIENTS $\{b_k\}$
  - DEFINE THE FILTER

$$y[n] = \sum_{k=0}^{M} b_k\, x[n-k]$$

  - For example, $\{b_k\} = \{3, -1, 2, 1\}$

$$y[n] = \sum_{k=0}^{3} b_k x[n-k]$$

$$= 3x[n] - x[n-1] + 2x[n-2] + x[n-3]$$

GENERAL FIR FILTER

- **FILTER COEFFICIENTS $\{b_k\}$**

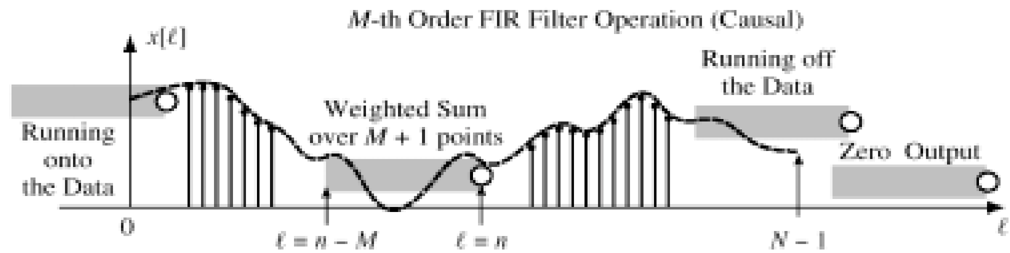
$$y[n] = \sum_{k=0}^{M} b_k \, x[n-k]$$

- **FILTER ORDER is M**
- **FILTER LENGTH is L = M+1**
  - **NUMBER of FILTER COEFFS is L**

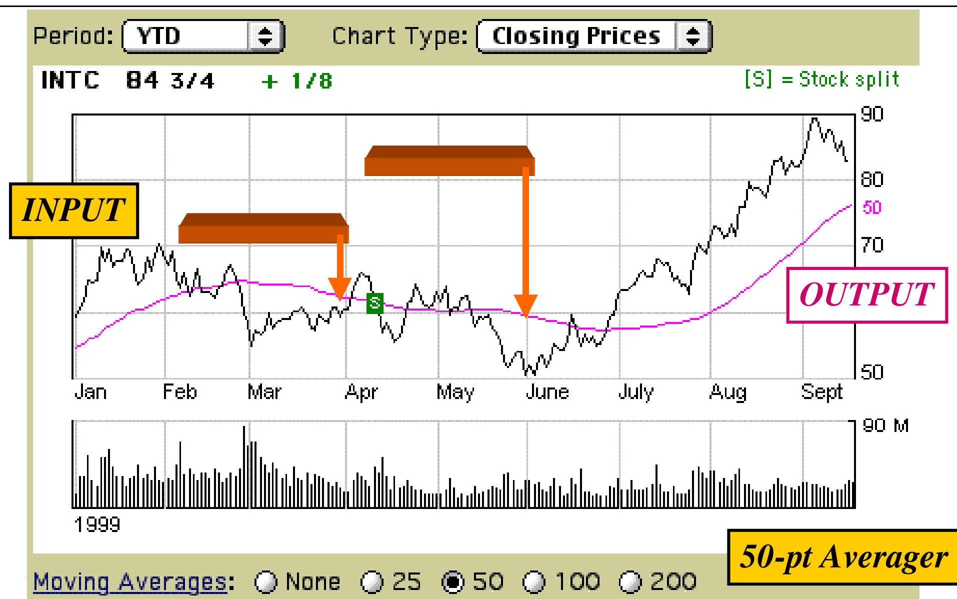
GENERAL FIR FILTER

- **SLIDE a WINDOW across x[n]**

$$y[n] = \sum_{k=0}^{M} b_k \, x[n-k]$$



FILTERED STOCK SIGNAL

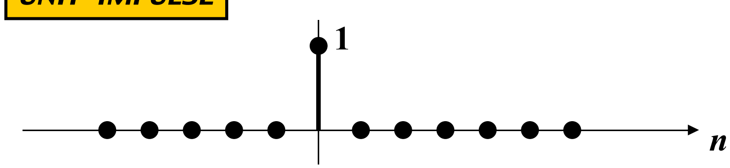


SPECIAL INPUT SIGNALS

- **x[n] = SINUSOID** FREQUENCY RESPONSE
- **x[n] has only one NON–ZERO VALUE**

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

UNIT–IMPULSE



UNIT IMPULSE SIGNAL δ[n]

n	...	-2	-1	0	1	2	3	4	5	6	...
$\delta[n]$	0	0	0	1	0	0	0	0	0	0	0
$\delta[n-3]$	0	0	0	0	0	0	1	0	0	0	0

δ[n] is NON-ZERO
When its argument
is equal to ZERO

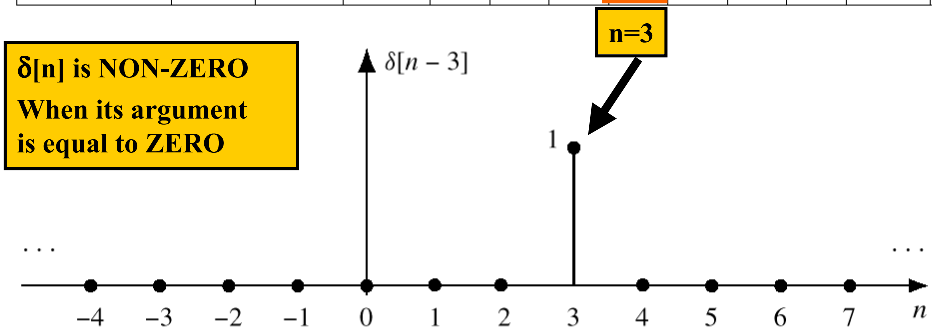


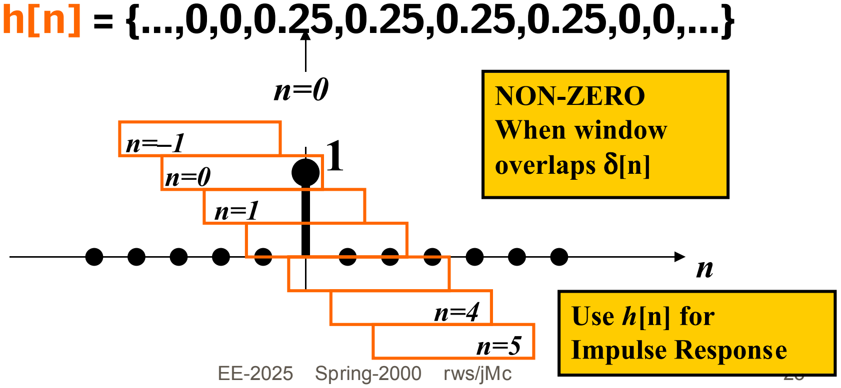
Figure 5.7 Shifted impulse sequence, $\delta[n-3]$.

4-pt AVERAGER

- CAUSAL SYSTEM: USE PAST VALUES
  - $y[n] = (x[n]+x[n-1]+x[n-2]+x[n-3])/4$
- INPUT = UNIT IMPULSE SIGNAL = δ[n]
  - $x[n] = \delta[n]$
  - $y[n] = 0.25\,\delta[n] + 0.25\delta[n-1] + 0.25\,\delta[n-2] + 0.25\delta[n-3]$
- OUTPUT is called **"IMPULSE RESPONSE"**
  - h[n] = {...,0,0,0.25,0.25,0.25,0.25,0,0,...}

4-pt Avg Impulse Response

- $y[n] = (x[n]+x[n-1]+x[n-2]+x[n-3])/4$
- **δ[n] "READS OUT" the FILTER COEFFICIENTS**
- **h[n]** = {...,0,0,0.25,0.25,0.25,0.25,0,0,...}



FIR IMPULSE RESPONSE

- Convolution = Filter Definition
  - Filter Coeffs = Impulse Response

n	$n<0$	0	1	2	3	...	M	$M+1$	$n>M+1$
$x[n]=\delta[n]$	0	1	0	0	0	0	0	0	0
$y[n]=h[n]$	0	b_0	b_1	b_2	b_3	...	b_M	0	0

$$y[n]=\sum_{k=0}^{M} b_k x[n-k]$$

$$y[n]=\sum_{k=0}^{M} h[k]x[n-k]$$

CONVOLUTION