

EE-2025

Spring-2000

Lecture 12

Digital Filtering of Analog Signals

25-Feb-00

Info: Web-CT, Lab, HW

- **Quiz #2 on 3-March (Friday)**
 - ! Coverage: HW #4, #5, #6, and #7
 - ! **REVIEW SESSION** next Thursday @ 7:10pm
- **MATLAB & Help on Monday & TUESDAY**
 - ! 6 PM, VL-456
- **Prob Set #7 due **NEXT WEEK****
 - ! **Solution will be posted on Thursday**
 - ! **Next HW (#8) AFTER Spring Break !**

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READING ASSIGNMENTS

- **This Lecture:**
 - ! Chapter 6, pp. 188–194
- **Other Reading:**
 - ! **Recitation: Ch. 6, pp. 176–188**
 - ! **FREQUENCY RESPONSE EXAMPLES**
 - ! **Next Lecture: Chapter 7, start**

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FREQ. RESPONSE PLOTS

- **DENSE GRID (**ww**) from $-\pi$ to $+\pi$**
 - ! **ww = -pi:(pi/100):pi;**
- **yy = freqz(bb, 1, ww)**
 - ! **VECTOR bb** contains Filter Coefficients
 - ! **DSP-First: yy = freekz(bb, 1, ww)**

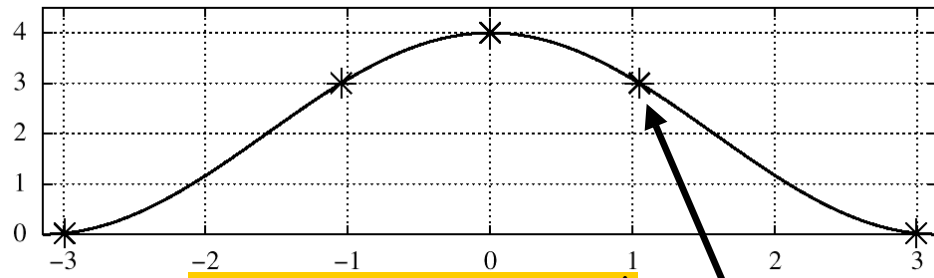
$$\mathcal{H}(\hat{\omega}) = \sum_{k=0}^M b_k e^{-j\hat{\omega}k}$$

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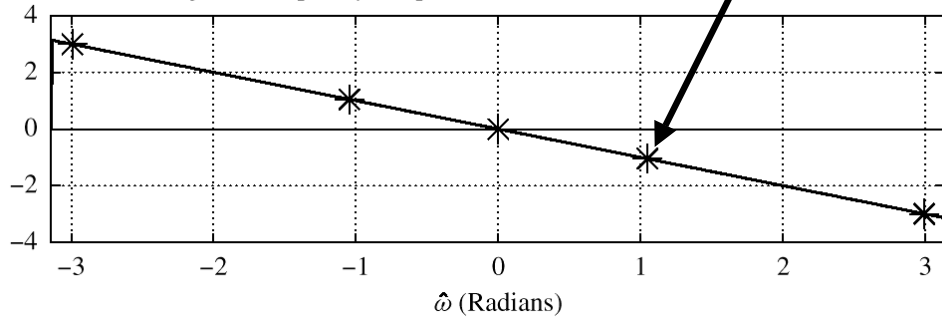
Magnitude of Frequency Response of FIR Filter with Coefficients [1, 2, 1]



$$H(\hat{\omega}) = (2 + 2 \cos \hat{\omega}) e^{-j\hat{\omega}}$$

RESPONSE at $\pi/3$

Phase Angle of Frequency Response of FIR Filter with Coefficients [1, 2, 1]



COURSE OBJECTIVE

- Students will be able to:
- Understand **mathematical** descriptions of signal processing **algorithms** and express those algorithms as computer **implementations** (MATLAB)

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LECTURE OBJECTIVES

- Two Domains: Time & Frequency
- Track the spectrum of $x[n]$ thru an FIR Filter.
- UNIFICATION:** How does Frequency Response affect $x(t)$ to produce $y(t)$?



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TIME & FREQ DOMAINS

- LTI: Linear & Time-Invariant
 - COMPLETELY CHARACTERIZED** by:
 - IMPULSE RESPONSE $h[n]$ (time domain)
 - FREQUENCY RESPONSE



- Two DOMAINS: time & frequency
 - Go back and forth QUICKLY

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TIME & FREQUENCY

$$y[n] = \sum_{k=0}^M b_k x[n - k] = \sum_{k=0}^M h[k] x[n - k]$$

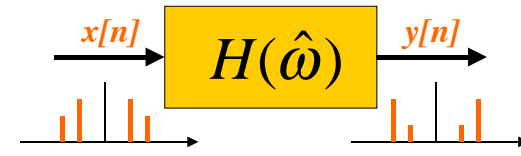
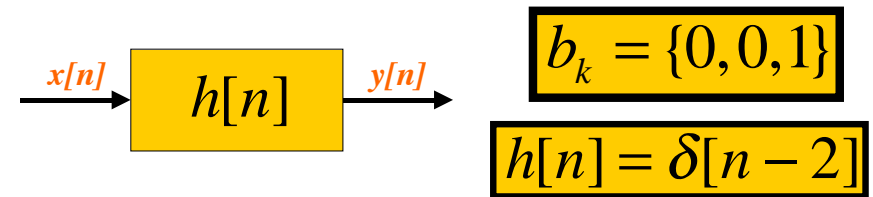
FIR DIFFERENCE EQUATION is the TIME-DOMAIN

$$H(\hat{\omega}) = \sum_{k=0}^M h[k] e^{-j\hat{\omega}k}$$

$$H(\hat{\omega}) = h[0]e^{-j\hat{\omega}0} + h[1]e^{-j\hat{\omega}1} + h[2]e^{-j\hat{\omega}2} + h[3]e^{-j\hat{\omega}3} + \dots$$

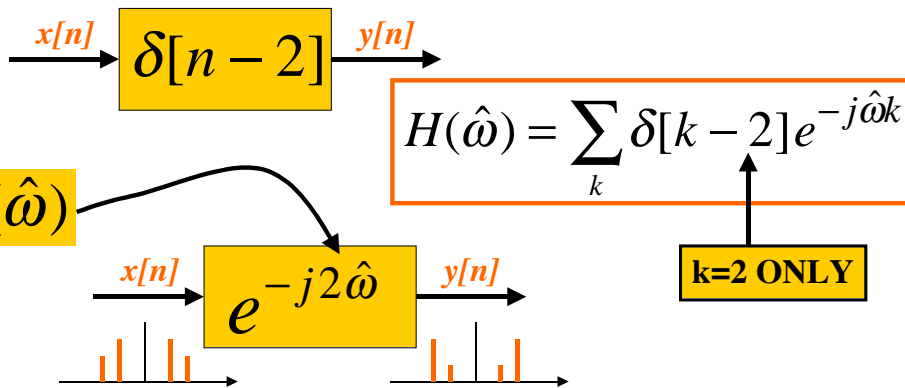
Ex: DELAY by 2 SYSTEM

Find $h[n]$ and $H(\hat{\omega})$ for $y[n] = x[n - 2]$



DELAY by 2 SYSTEM

Find $h[n]$ and $H(\hat{\omega})$ for $y[n] = x[n - 2]$



GENERAL DELAY PROPERTY

Find $h[n]$ and $H(\hat{\omega})$ for $y[n] = x[n - n_d]$

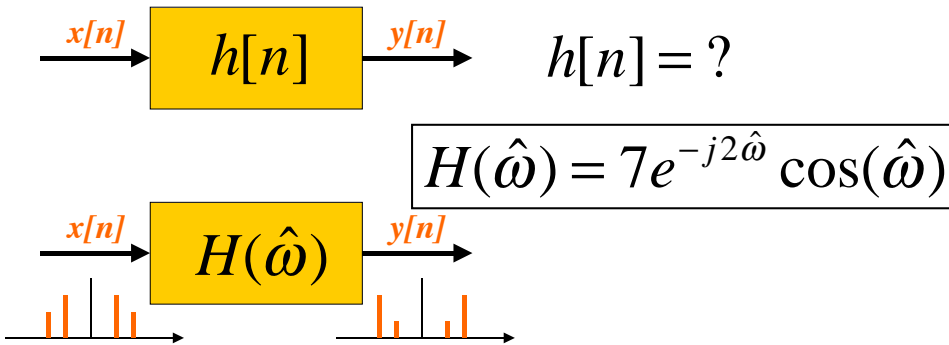
$$h[n] = \delta[n - n_d]$$

$$H(\hat{\omega}) = \sum_k \delta[k - n_d] e^{-j\hat{\omega}k} = e^{-j\hat{\omega}n_d}$$

ONLY ONE non-ZERO TERM for k at k = n_d

FREQ DOMAIN --> TIME ??

■ **START** with $H(\hat{\omega})$ and find $h[n]$ or b_k



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FREQ DOMAIN --> TIME

$$H(\hat{\omega}) = 7e^{-j2\hat{\omega}} \cos(\hat{\omega}) \quad \text{EULER's Formula}$$

$$= 7e^{-j2\hat{\omega}} (0.5e^{j\hat{\omega}} + 0.5e^{-j\hat{\omega}})$$

$$= (3.5e^{-j\hat{\omega}} + 3.5e^{-j3\hat{\omega}})$$

$$h[n] = 3.5\delta[n-1] + 3.5\delta[n-3]$$

$$b_k = \{0, 3.5, 0, 3.5\}$$

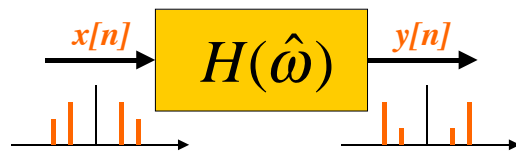
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EXAMPLE 6.2

Find $y[n]$ when $H(\hat{\omega})$ is known
& $x[n] = 2e^{j\pi/4} e^{j(\pi/3)n}$



$$H(\hat{\omega}) = (2 + 2\cos \hat{\omega})e^{-j\hat{\omega}}$$

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EXAMPLE 6.2 (answer)

Find $y[n]$ when $x[n] = 2e^{j\pi/4} e^{j(\pi/3)n}$

Answer: Eval $H(\hat{\omega})$ at $\hat{\omega} = \pi/3$.

$$H(\hat{\omega}) = (2 + 2\cos \hat{\omega})e^{-j\hat{\omega}}$$

$$H(\hat{\omega}) = 3e^{-j\pi/3} \quad @ \hat{\omega} = \pi/3$$

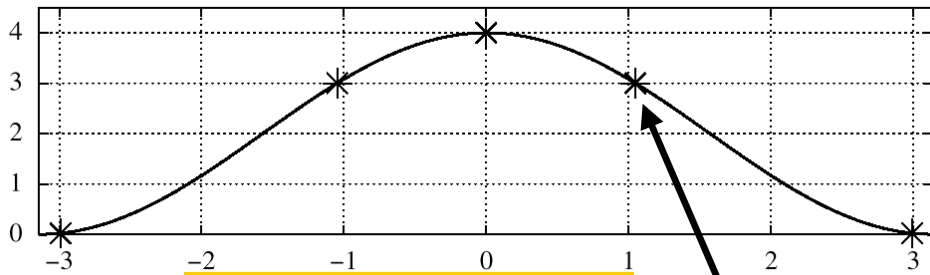
$$y[n] = (3e^{-j\pi/3}) \times 2e^{j\pi/4} e^{j(\pi/3)n} = 6e^{-j\pi/12} e^{j(\pi/3)n}$$

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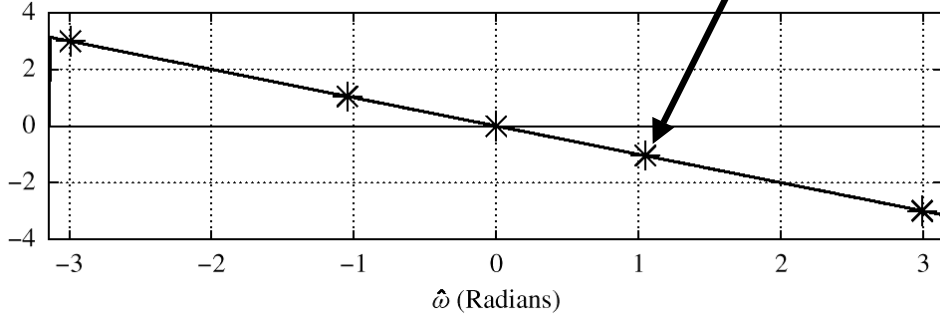
Magnitude of Frequency Response of FIR Filter with Coefficients [1, 2, 1]



$$H(\hat{\omega}) = (2 + 2 \cos \hat{\omega}) e^{-j\hat{\omega}}$$

RESPONSE at $\pi/3$

Phase Angle of Frequency Response of FIR Filter with Coefficients [1, 2, 1]



SINUSOID thru FIR

$$x[n] = X_0 + \sum_{k=1}^N \left(\frac{X_k}{2} e^{j\hat{\omega}_k n} + \frac{X_k^*}{2} e^{-j\hat{\omega}_k n} \right)$$

$$= X_0 + \sum_{k=1}^N |X_k| \cos(\hat{\omega}_k n + \angle X_k)$$

MULTIPLY MAGS

ADD PHASES

if $\mathcal{H}(-\hat{\omega}) = \mathcal{H}^*(\hat{\omega})$, the corresponding output is

$$y[n] = \mathcal{H}(0)X_0 + \sum_{k=1}^N \left(\mathcal{H}(\hat{\omega}_k) \frac{X_k}{2} e^{j\hat{\omega}_k n} + \mathcal{H}(-\hat{\omega}_k) \frac{X_k^*}{2} e^{-j\hat{\omega}_k n} \right)$$

$$= \mathcal{H}(0)X_0 + \sum_{k=1}^N |\mathcal{H}(\hat{\omega}_k)| |X_k| \cos(\hat{\omega}_k n + \angle X_k + \angle \mathcal{H}(\hat{\omega}_k))$$

LTI Demo with Sinusoids

LTI (Linear Time Invariant) System Demo ver 1.12

INPUT SIGNAL
 $x[n] = 1.5 \cos(0.2\pi n + 0.1\pi)$

OUTPUT SIGNAL
 $y[n] = 1.31 \cos(0.2\pi n - 0.1\pi)$

Magnitude of the Filter
 Magnitude vs Normalized Frequency ($\omega/2\pi$)

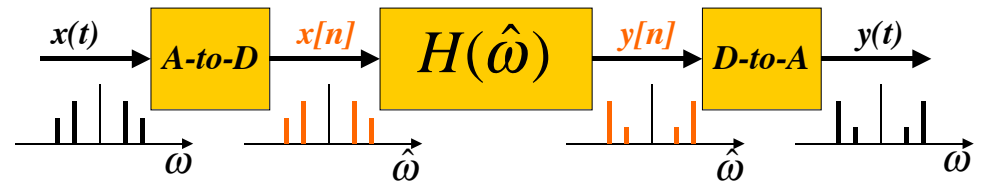
Phase of the Filter
 Phase (radians) vs Normalized Frequency ($\omega/2\pi$)

Filter Specifications:
 Length = 3 pts
 Filter Choice: Averaging Filter

Amplitude = 1.5
 Frequency = $2 \cdot \pi \cdot 0.1$
 Phase = $0.1 \cdot \pi$
 DC Level = 0

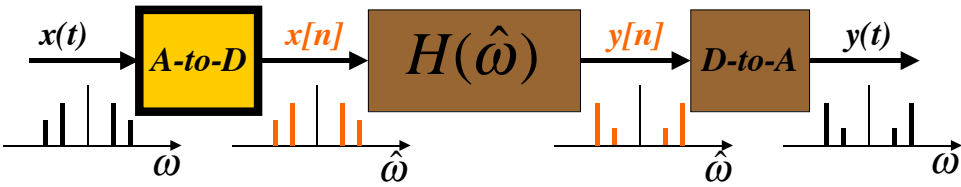
Theoretical Answer

DIGITAL "FILTERING"



- ω SPECTRUM of $x(t)$ (SUM of SINUSOIDS)
- $\hat{\omega}$ SPECTRUM of $x[n]$
 - | Is ALIASING a PROBLEM?
- ω SPECTRUM $y[n]$ (FIR Gain or Nulls)
- ω Then, OUTPUT $y(t) = \text{SUM of SINUSOIDS}$

FREQUENCY SCALING



■ TIME SAMPLING:

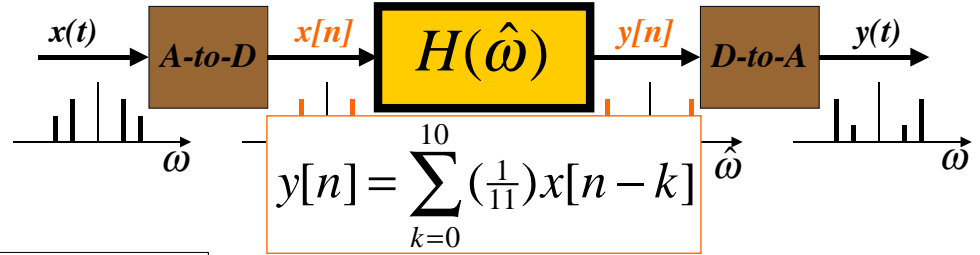
$$t = nT_s$$

■ IF **NO** ALIASING:

■ FREQUENCY SCALING

$$\hat{\omega} = \omega T_s = \frac{\omega}{f_s}$$

11-pt AVERAGER Example



$$y[n] = \sum_{k=0}^{10} \left(\frac{1}{11}\right) x[n-k]$$

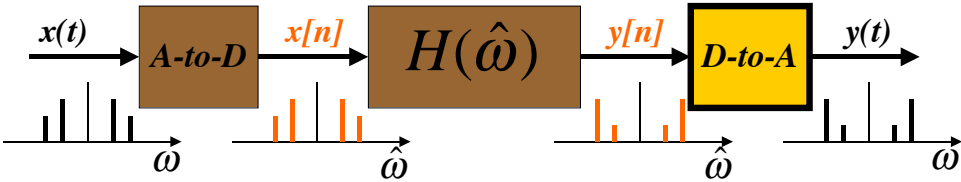
■ 250 Hz

■ 25 Hz

$$H(\hat{\omega}) = \frac{\sin(\frac{11}{2} \hat{\omega})}{11 \sin(\frac{1}{2} \hat{\omega})} e^{-j5\hat{\omega}} \quad ?$$

$$x(t) = \cos(2\pi(25)t) + \cos(2\pi(250)t - \frac{1}{2}\pi)$$

D-A FREQUENCY SCALING



■ TIME SAMPLING:

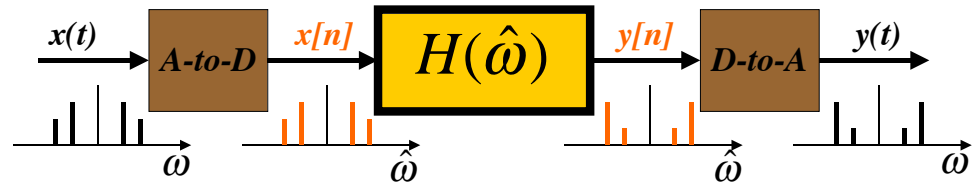
$$t = nT_s \Rightarrow n \leftarrow t f_s$$

■ RECONSTRUCT up to $0.5f_s$

■ FREQUENCY SCALING

$$\omega = \hat{\omega} f_s$$

TRACK the FREQUENCIES



■ 250 Hz

■ 25 Hz

■ 0.5π

■ $.05\pi$

$H(0.5\pi)$

$H(0.05\pi)$

■ 0.5π

■ $.05\pi$

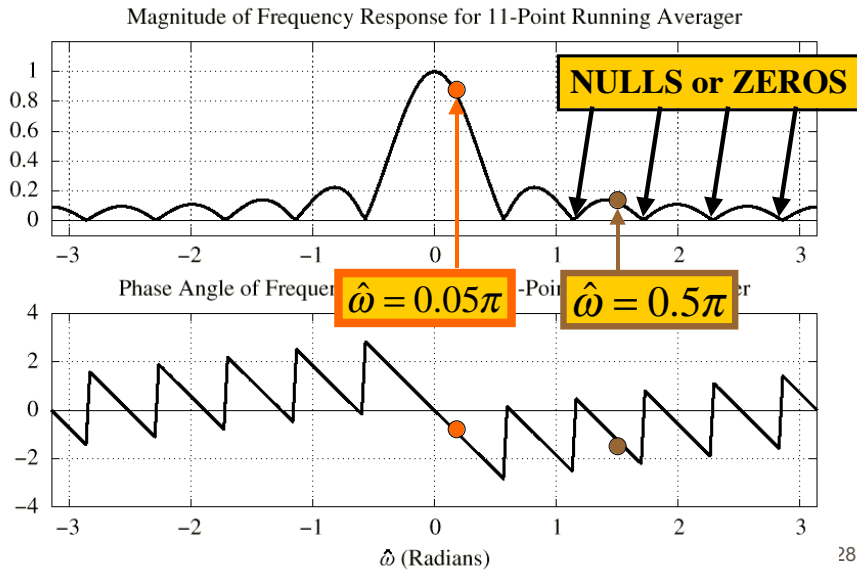
■ 250 Hz

■ 25 Hz

$F_s = 1000 \text{ Hz}$

NO new freqs

11-pt AVERAGER



EVALUATE Freq. Response

$$H(\hat{\omega}) = \frac{\sin(\hat{\omega}11/2)}{11 \sin(\hat{\omega}/2)} e^{-j\hat{\omega}5}$$

At $\hat{\omega} = 0.5\pi$

$$H(\hat{\omega}) = \frac{\sin((0.5\pi)11/2)}{11 \sin(0.5\pi/2)} e^{-j(0.5\pi)5}$$

$$= \frac{\sin(2.75\pi)}{11 \sin(0.25\pi)} e^{-j2.5\pi}$$

$$= 0.0909 e^{-j0.5\pi}$$

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EVALUATE Freq. Response

$$x(t) = \cos(2\pi(25)t) + \sin(2\pi(250)t)$$

evaluating at 25 and 250 Hz.

$$\mathcal{H}(2\pi(25)/1000) = \frac{\sin(\pi(25)(11)/1000)}{11 \sin(\pi(25)/1000)} e^{-j2\pi(25)(5)/1000}$$

MAG SCALE

$f_s = 1000$

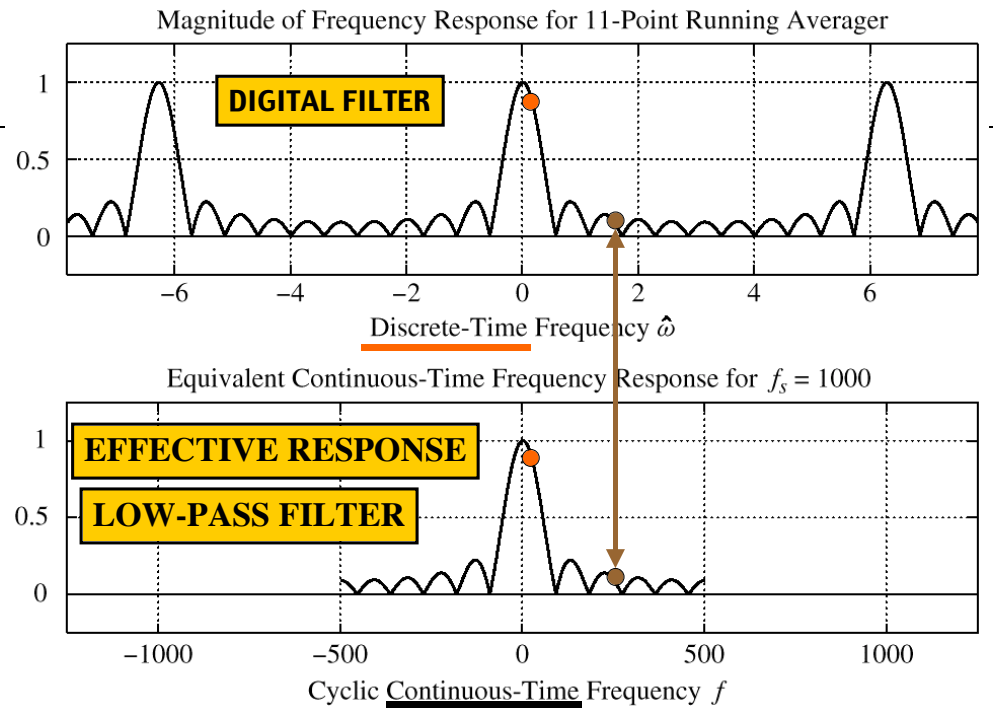
$$= 0.8811 e^{-j\pi/4}$$

PHASE CHANGE

$$\mathcal{H}(2\pi(250)/1000) = \frac{\sin(\pi(250)(11)/1000)}{11 \sin(\pi(250)/1000)} e^{-j2\pi(250)(5)/1000}$$

$$= 0.0909 e^{-j\pi/2}$$

$$y(t) = 0.8811 \cos(2\pi(25)t - \pi/4) + 0.0909 \sin(2\pi(250)t - \pi/2)$$

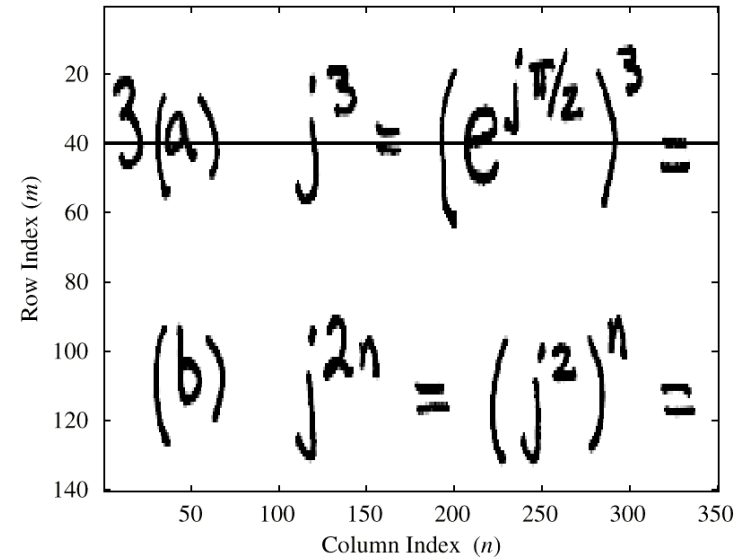


FILTER TYPES

- **LOW-PASS FILTER (LPF)**
 - BLURRING
 - ATTENUATES HIGH FREQUENCIES
- **HIGH-PASS FILTER (HPF)**
 - SHARPENING for IMAGES
 - BOOSTS THE HIGHS
 - REMOVES DC
- **BAND-PASS FILTER (BPF)**

B & W IMAGE

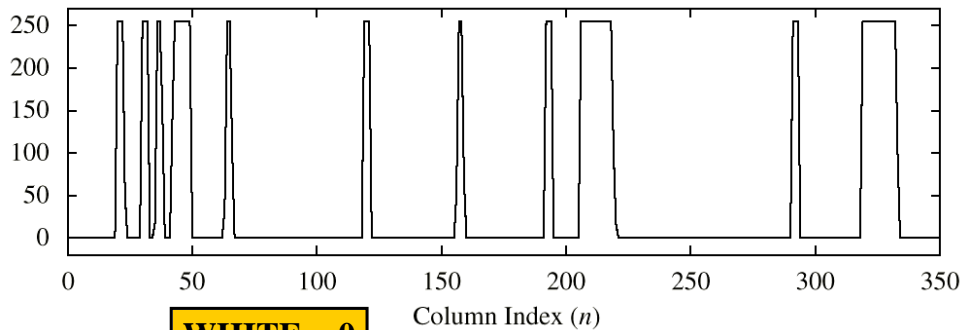
Original Black and White Image



ROW of B&W IMAGE

BLACK = 255

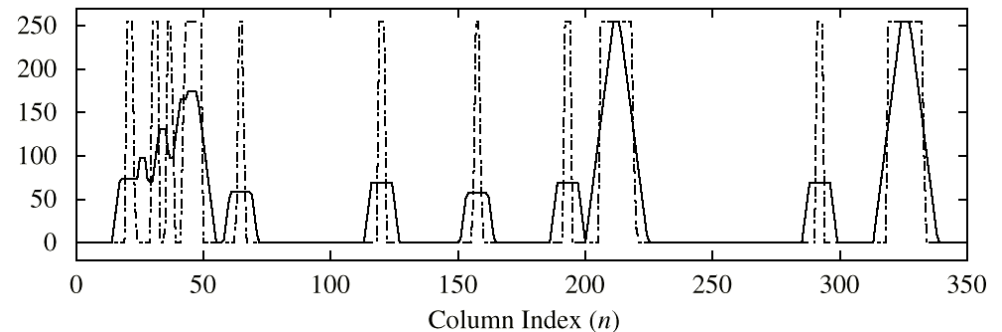
Row 40 of the Image



WHITE = 0

FILTERED ROW of IMAGE

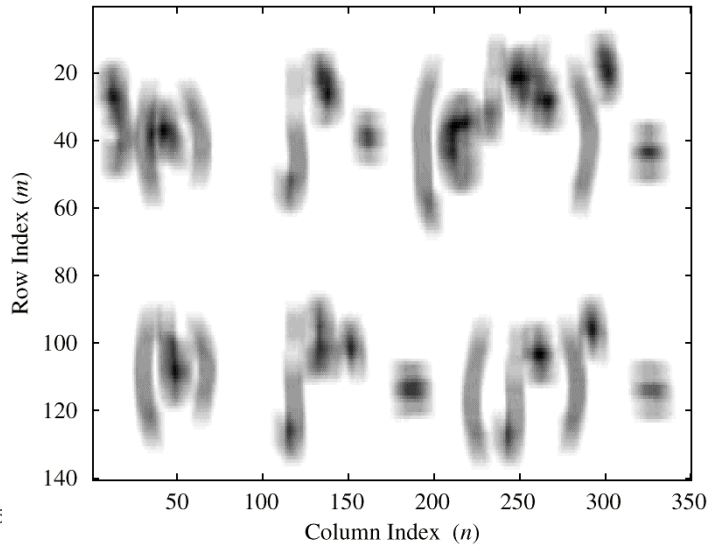
11-Point Averaging: 5-Sample Delay Equalization



ADJUSTED DELAY by 5 samples

FILTERED B&W IMAGE

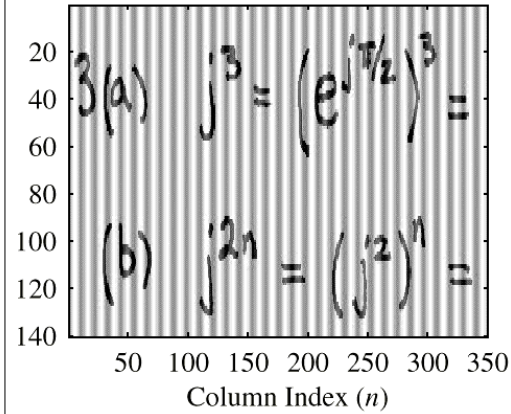
Row and Column Filtered Image



**LPF:
BLUR**

B&W IMAGE with COSINE

Homework plus Cosine



FILTERED: 11-pt AVG

Remove Cosine Stripe with Averaging Filter

