EE-2025 Spring-2000

Lecture 15

Continuous-Time Signals and Systems

17-Mar-00

Info: Web-CT, Lab, HW

- Calendar:
  - Quiz #3 is 7–April (Friday)
- Get NEW CHAPTERS
  - PDF or Bookstore
- Prob Set #8 is due next week
- LAB QUIZ next week
  - Lab #8 on DTMF (Touch–Tone)
    - Spans two weeks

READING ASSIGNMENTS

- This Lecture:
  - Chapter 10, pp. 1000–1022
- Other Reading:
  - Recitation: Ch. 10, pp. 1022–1029
  - Next Lecture: Chapter 10, all

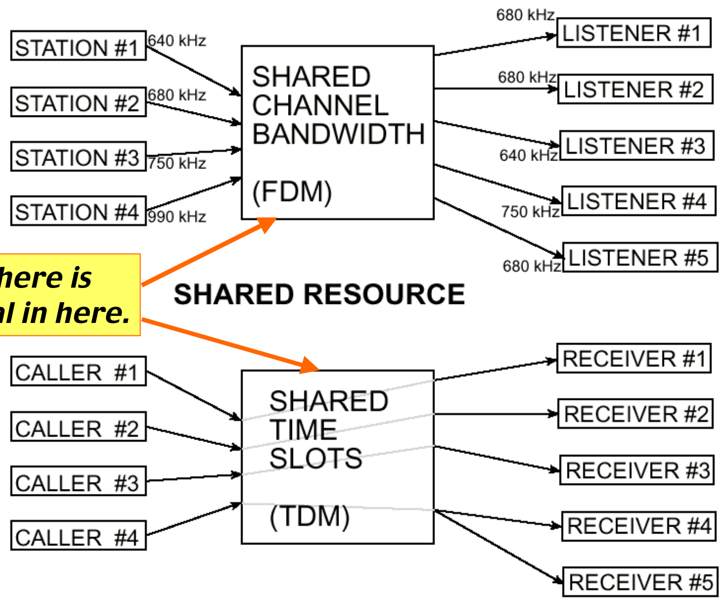
LECTURE OBJECTIVES

- Bye bye to D–T Systems for a while
- The UNIT IMPULSE signal
  - Definition
  - Properties
- Continuous–time signals and systems
  - Example systems
  - Review: Linearity and Time–Invariance
  - Convolution integral: impulse response

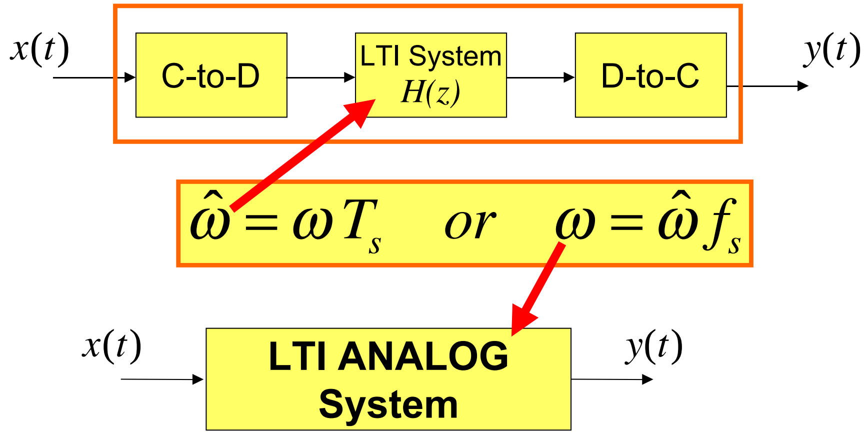
The way communication systems work

It's an analog world out here.

These days, there is A lot of digital in here.



D-T Filtering of C-T Signals



$$\hat{\omega} = \omega T_s \quad or \quad \omega = \hat{\omega} f_s$$

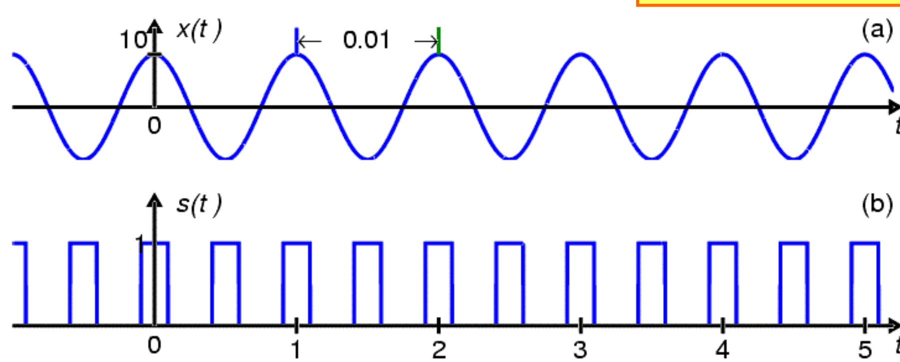
ANALOG SIGNALS x(t)

- INFINITE LENGTH
  - SINUSOIDS: (t = time in secs)
  - PERIODIC SIGNALS
- ONE-SIDED, e.g., for t>0
  - UNIT STEP: u(t)
- FINITE LENGTH
  - SQUARE PULSE
- IMPULSE SIGNAL: δ(t)
- DISCRETE-TIME: x[n] is list of numbers

CT Signals: PERIODIC

$x(t) = 10\cos(200\pi t)$

Sinusoidal signal



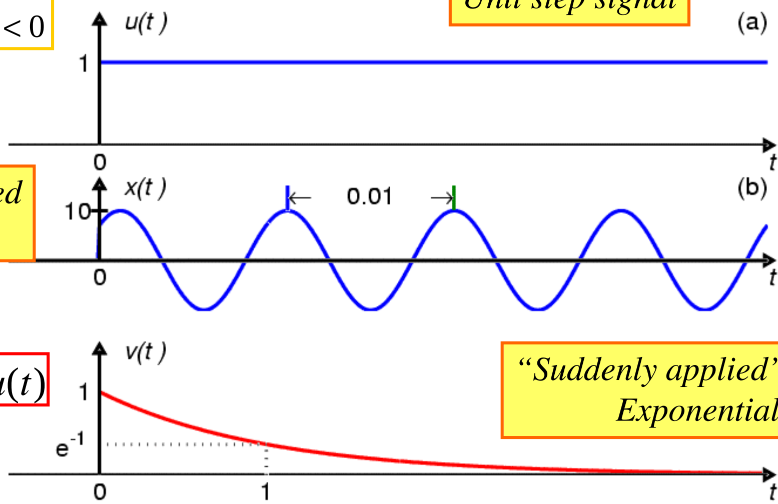
INFINITE DURATION

Square Wave

CT Signals: ONE-SIDED

$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$

Unit step signal



One-Sided Sinusoid

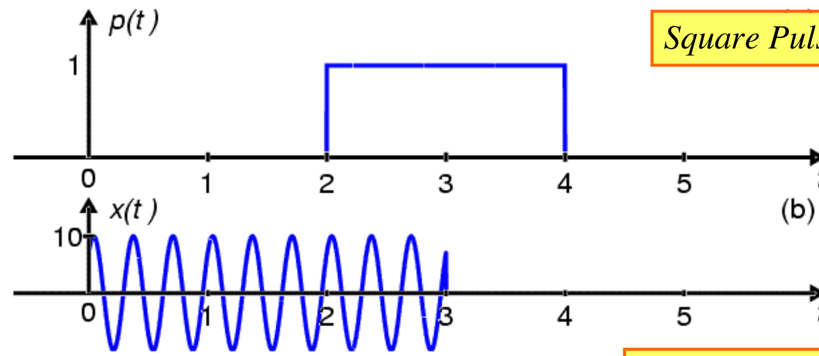
$v(t) = e^{-t}u(t)$

"Suddenly applied" Exponential

CT Signals: FINITE LENGTH

$p(t) = u(t-2) - u(t-4)$

Square Pulse signal

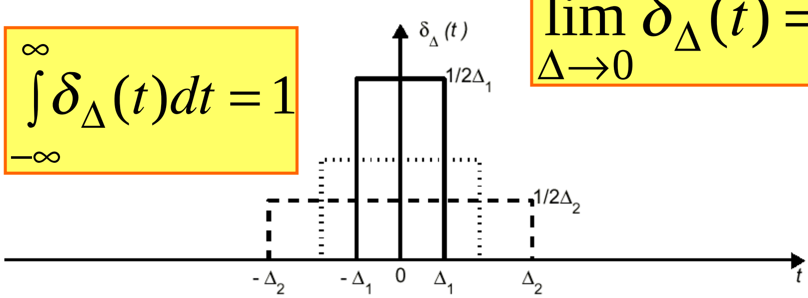


Sinusoid multiplied by a square pulse

What is an Impulse?

- A signal that is concentrated at one point.

$$\int_{-\infty}^{\infty} \delta_\Delta(t)dt = 1$$



$$\lim_{\Delta \to 0} \delta_\Delta(t) = \delta(t)$$

Defining the Impulse

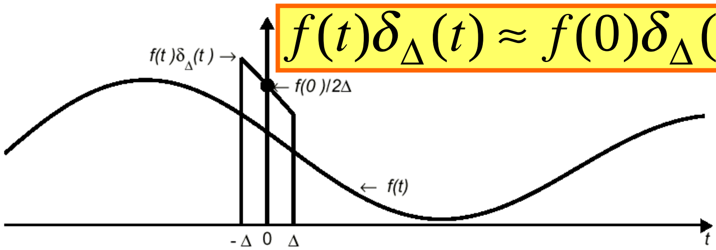
- Assume the properties apply to the limit: $\lim_{\Delta \to 0} \delta_\Delta(t) = \delta(t)$
- One INTUITIVE definition is:

$\delta(t) = 0, \quad t \neq 0$ ***Concentrated at t=0***

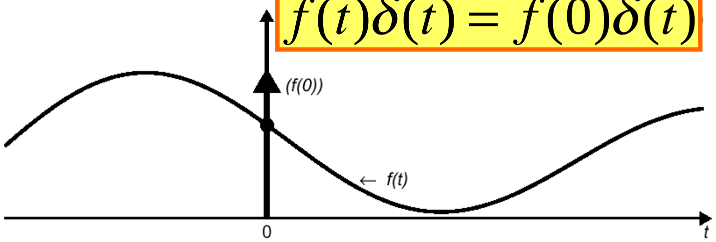
$\int_{-\infty}^{\infty} \delta(\tau)d\tau = 1$ ***Unit area***

Sampling Property

$$f(t)\delta_\Delta(t) \approx f(0)\delta_\Delta(t)$$

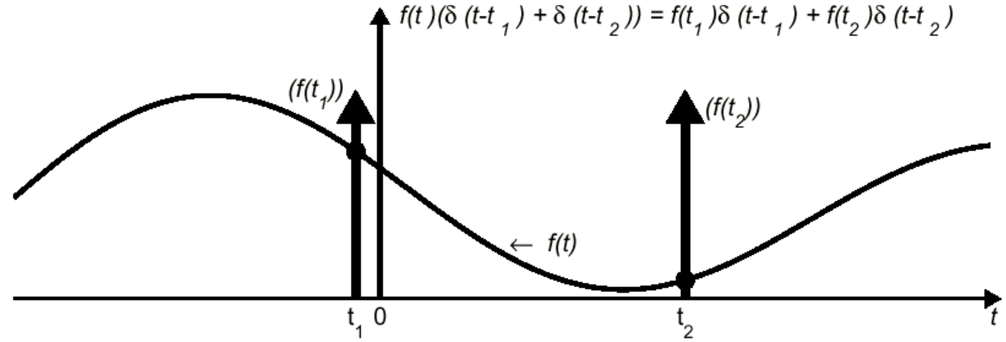


$$f(t)\delta(t) = f(0)\delta(t)$$



General Sampling Property

$$f(t)\delta(t-t_0) = f(t_0)\delta(t-t_0)$$



Properties of the Impulse

$$\delta(t-t_0) = 0, \quad t \neq t_0$$ ***Concentrated at one time***

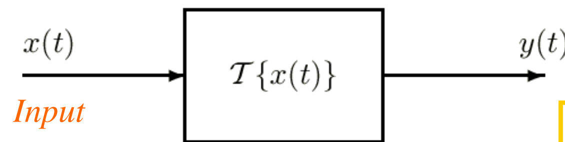
$$\int_{-\infty}^{\infty} \delta(t-t_0)dt = 1$$ ***Unit area***

$$f(t)\delta(t-t_0) = f(t_0)\delta(t-t_0)$$ ***Sampling Property***

$$\int_{-\infty}^{\infty} f(t)\delta(t-t_0)dt = f(t_0)$$ ***Extract one value of f(t)***

$$\frac{du(t)}{dt} = \delta(t)$$ ***Derivative of unit step***

Continuous-Time Systems



$$x(t) \mapsto y(t)$$

- **Examples:**
  - **Delay** $y(t) = x(t-t_d)$
  - **Modulator** $y(t) = [A + x(t)]\cos\omega_c t$
  - **Integrator** $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$

CT BUILDING BLOCKS

- INTEGRATOR (CIRCUITS)
- DIFFERENTIATOR
- DELAY by t_o
- MODULATOR (e.g., AM Radio)
- MULTIPLIER & ADDER

Ideal Delay:

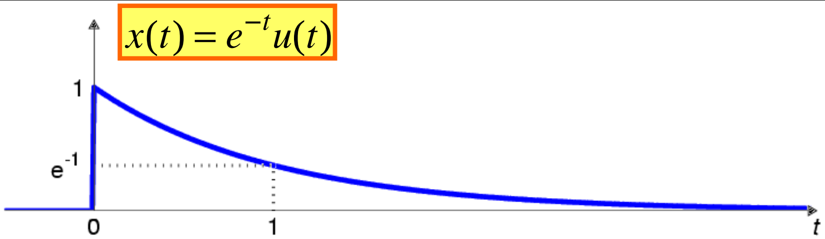
- Mathematical Definition:

$$y(t) = x(t - t_d)$$

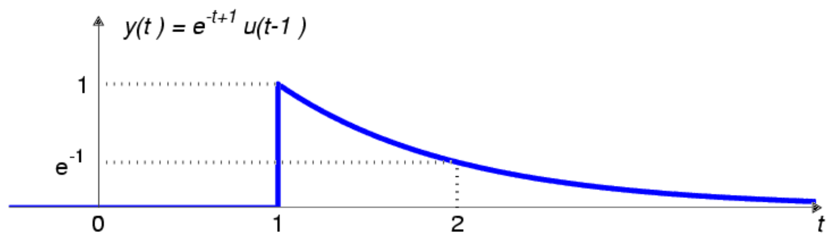
- To find the IMPULSE RESPONSE, ***h(t)***, let ***x(t)*** be an impulse, so

$$h(t) = \delta(t - t_d)$$

Output of Ideal Delay of 1 sec



$$y(t) = x(t-1) = e^{-(t-1)}u(t-1)$$



Integrator:

- Mathematical Definition:

$$y(t) = \int_{-\infty}^{t} x(\tau)d\tau$$

Running Integral

- To find the IMPULSE RESPONSE, ***h(t)***, let ***x(t)*** be an impulse, so

$$h(t) = \int_{-\infty}^{t} \delta(\tau)d\tau = u(t)$$

Integrator: $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$

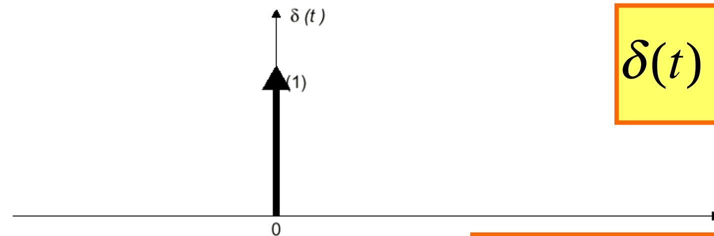
- Integrate the impulse

$$\int_{-\infty}^{t} \delta(\tau)d\tau = u(t)$$

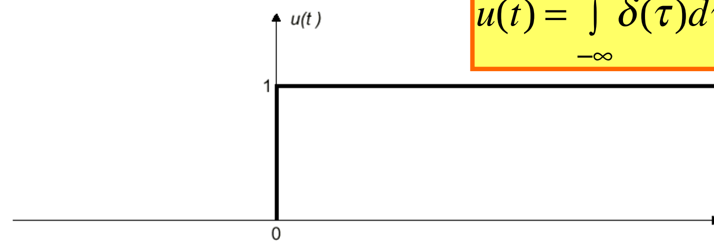
- IF t<0, we get zero
- IF t>0, we get one
  - Thus we have h(t) = u(t) for the integrator

Graphical Representation

$$\delta(t) = \frac{du(t)}{dt}$$



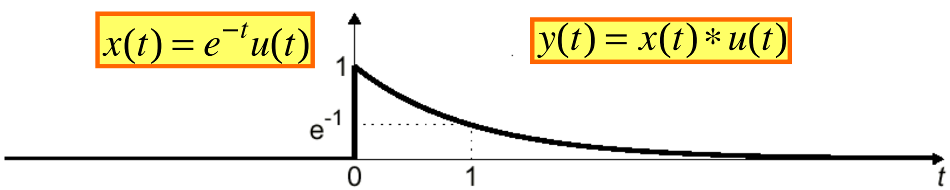
$$u(t) = \int_{-\infty}^{t} \delta(\tau)d\tau = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$



Output of Integrator

$x(t) = e^{-t}u(t)$

$y(t) = x(t) * u(t)$



$$y(t) = \int_{-\infty}^{t} e^{-\tau}u(\tau)d\tau = \begin{cases} 0 & t < 0 \\ \int_{0}^{t} e^{-\tau}d\tau & t \geq 0 \end{cases}$$

$$= (1 - e^{-t})u(t)$$

Differentiator:

- Mathematical Definition:

$$y(t) = \frac{dx(t)}{dt}$$

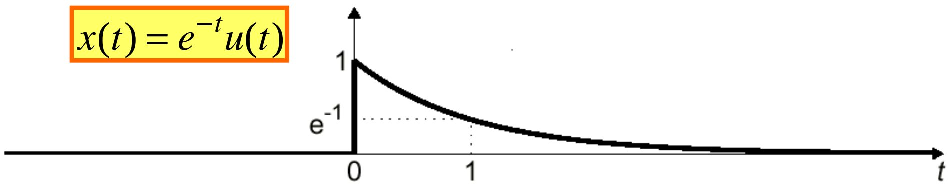
- To find ***h(t),*** let ***x(t)*** be an impulse, so

$$h(t) = \frac{d\delta(t)}{dt} = \delta^{(1)}(t)$$

Doublet

Differentiator Output: $y(t) = \frac{dx(t)}{dt}$

$x(t) = e^{-t}u(t)$



$$y(t) = \frac{d}{dt}\left(e^{-t}u(t)\right) = \frac{d}{dt}\left(e^{-t}\right)u(t) + e^{-t}\frac{d}{dt}\left(u(t)\right)$$

$$= -e^{-t}u(t) + e^{-t}\delta(t)$$

$$= -e^{-t}u(t) + e^{-0}\delta(t) = -e^{-t}u(t) + \delta(t)$$

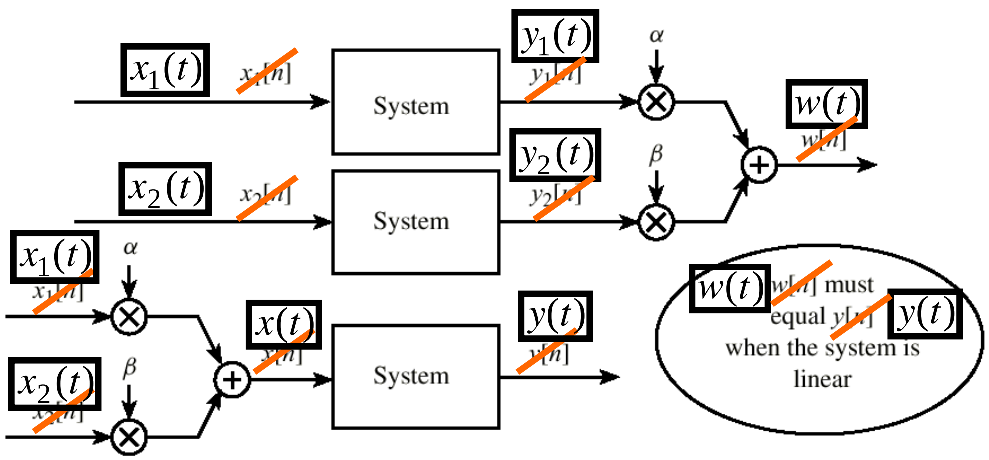
Linear and Time-Invariant (LTI) Systems

- If a continuous-time system is both linear and time-invariant, then the output *y(t)* is related to the input *x(t)* by a ***convolution integral***

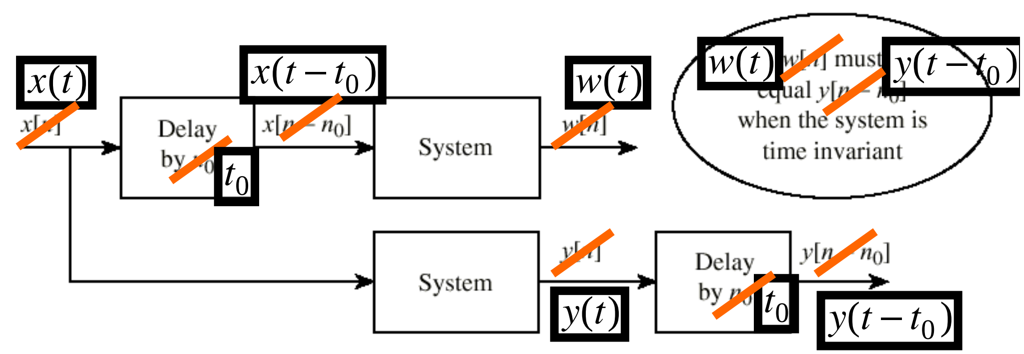
$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = x(t) * h(t)$$

where *h(t)* is the ***impulse response*** of the system.

Testing for Linearity



Testing Time-Invariance



Ideal Delay: $y(t) = x(t - t_d)$

- **Linear**

$$a x_1(t - t_d) + b x_2(t - t_d) = a y_1(t) + b y_2(t)$$

- **and Time–Invariant**

$$\begin{aligned} w(t) &= x((t - t_0) - t_d) \\ y(t - t_0) &= x((t - t_0) - t_d) \end{aligned}$$

Integrator: $y(t) = \int_{-\infty}^{t} x(\tau) d\tau$

- **Linear**

$$\int_{-\infty}^{t} [a x_1(\tau) + b x_2(\tau)] d\tau = a y_1(t) + b y_2(t)$$

- **And Time–Invariant**

$$w(t) = \int_{-\infty}^{t} x(\tau - t_0) d\tau \qquad \text{let } \sigma = \tau - t_0$$

$$\Rightarrow \quad w(t) = \int_{-\infty}^{t - t_0} x(\sigma) d\sigma = y(t - t_0)$$

Modulator: $y(t) = [A + x(t)]\cos\omega_c t$

- ***Not*** **linear--obvious because**

$$A + a x_1(t) + b x_2(t)] \neq$$
$$[A + a x_1(t)] + [A + b x_2(t)]$$

- ***Not*** **time–invariant**

$$w(t) = [A + x(t - t_0)]\cos\omega_c t \neq y(t - t_0)$$

Linear and Time-Invariant (LTI) Systems

- **If a continuous–time system is both linear and time–invariant, then the output *y(t)* is related to the input *x(t)* by a *convolution integral***

$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t - \tau) d\tau = x(t) * h(t)$$

where *h(t)* is the *impulse response* of the system.