EE-2025 Spring-2000

Lecture 26

Review: DIGITAL FILTERING of ANALOG SIGNALS

28-April-00

Info: Web-CT, Lab, HW

- Final Exam, 2–May @ 11:30 AM
  - Calculator, 2 pages <u>Handwritten</u> Notes
  - Review: Monday, 1–May @ <u>6:45 pm</u>
- HW #13 due TODAY (now)
- Labs DEADLINE
  - ALL Lab Reports due by 28–April, no later than Friday at 5 pm.

FINAL EXAM

- FORMULA PAGES ?
  - Students bring TWO pages <u>HAND–WRITTEN</u>
- COVERAGE / EMPHASIS?
  - Fourier Transform
    - Sampling & Spectrum
  - Digital Filters: IIR & FIR & H(z)
  - Hard problems from Quizzes #2, #3.
  - Homework & Old Quizzes

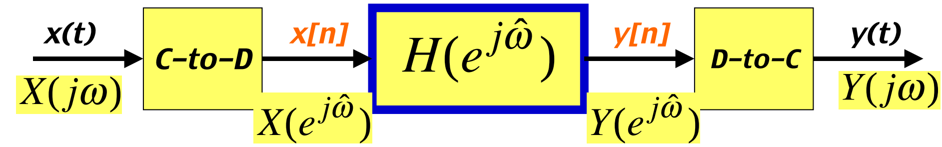
LECTURE OBJECTIVES

- THREE–DOMAIN APPROACH
  - EXHIBIT BANDPASS FILTERS
- RE–UNIFICATION:
  - How does Frequency Response affect x(t) to produce y(t) ?

Education

- Ambrose Bierce:
- Education, noun: "That which discloses to the wise and disguises from the foolish their lack of understanding"

DT Filtering of CT Signals

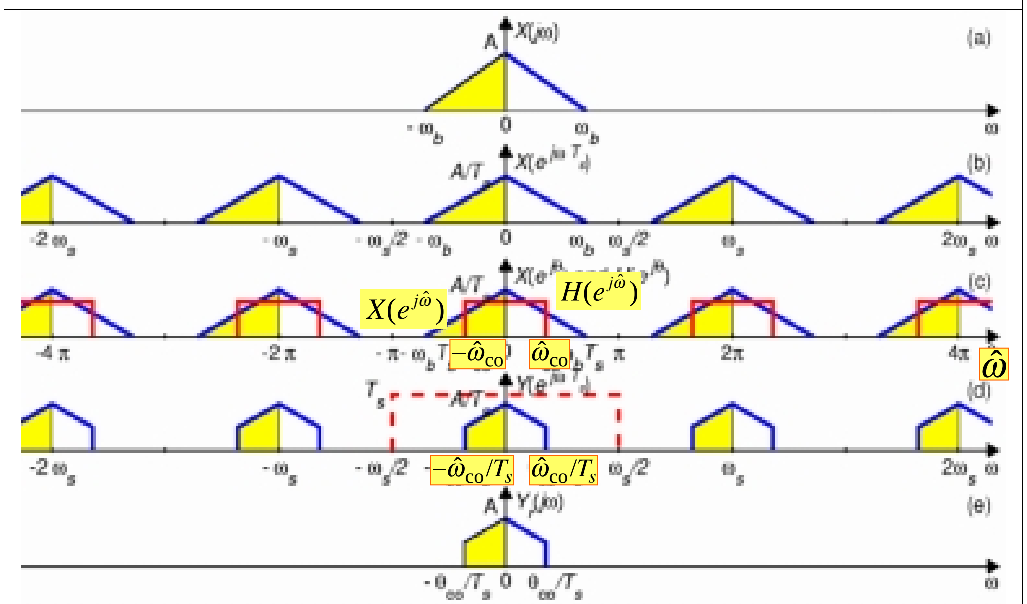


If no aliasing occurs in sampling x(t), then it follows that

$$Y(j\omega) = H_{\text{eff}}(j\omega)X(j\omega)$$

$$H_{\text{eff}}(j\omega) = \begin{cases} H(e^{j\omega T_s}) & |\omega| < \frac{1}{2}\omega_s \\ 0 & |\omega| > \frac{1}{2}\omega_s \end{cases}$$

Illustration of DT Filtering of a CT Signal



EFFECTIVE Freq. Response

- Assume NO Aliasing, then
  - ANALOG FREQ <---> DIGITAL FREQ

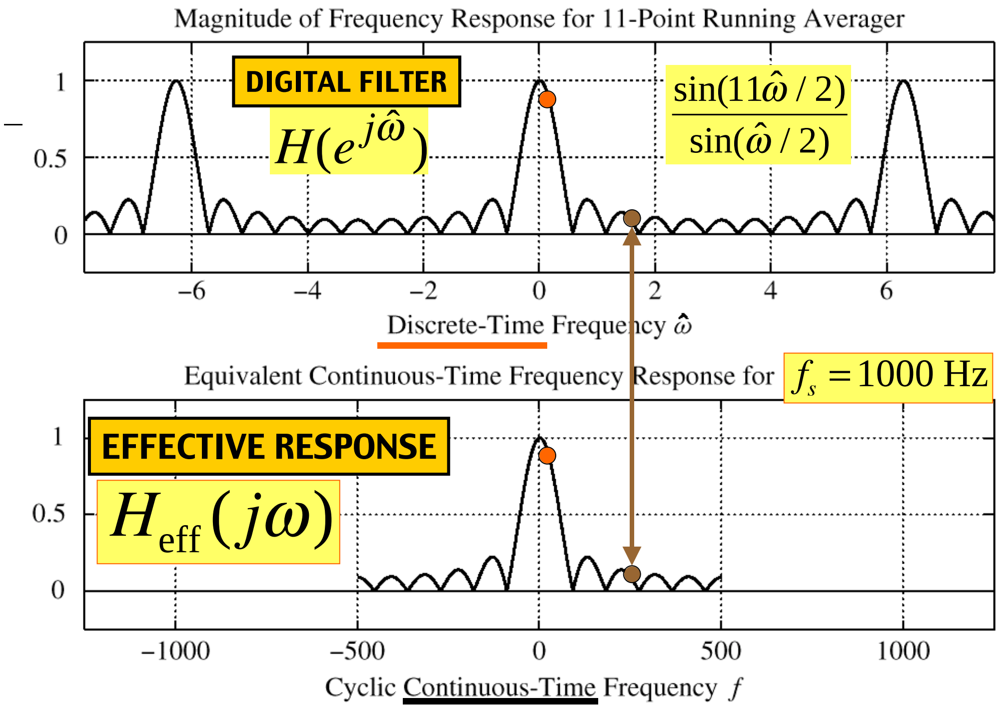
$$\hat{\omega} = \omega T_s = \frac{\omega}{f_s}$$

  - So, we can plot:
  - Scaled Freq. Axis

$$H(e^{j\omega T_s}) \text{ vs. } \omega$$

DIGITAL FILTER

ANALOG FREQUENCY



THREE DOMAINS

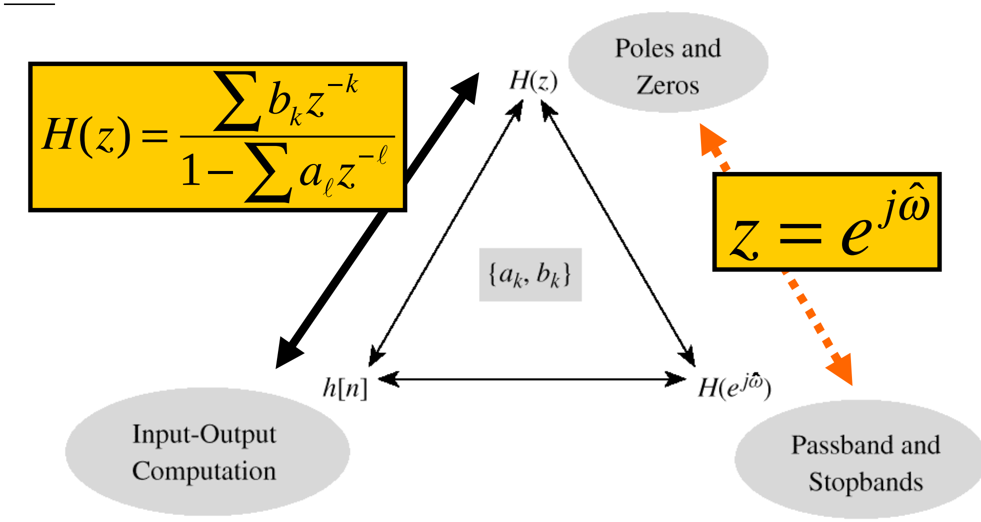
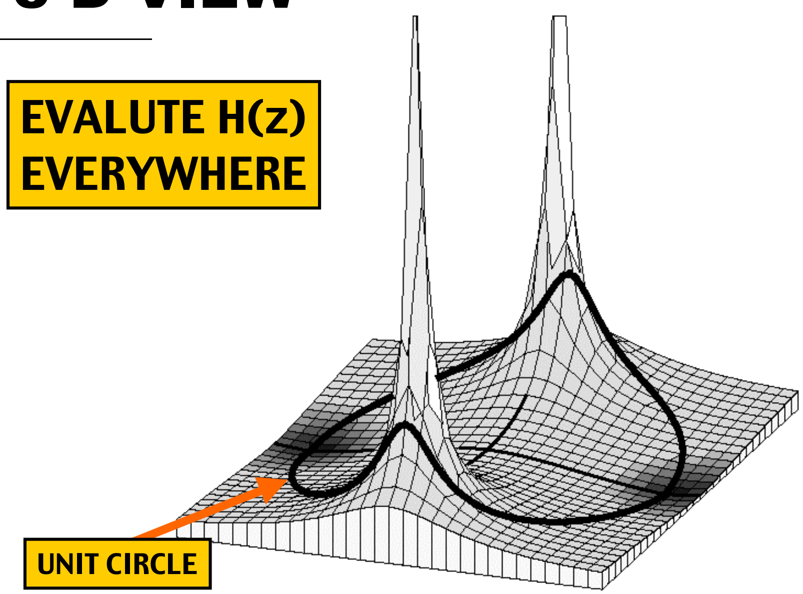


Figure 8.13 Relationship among the n-, z-, and $\hat{\omega}$-domains. The filter coefficients $\{a_k, b_k\}$ play a central role.

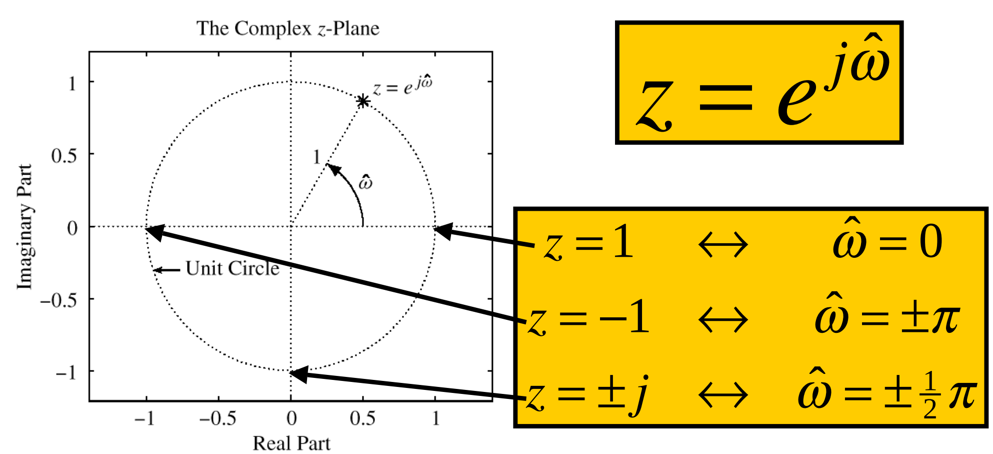
3-D VIEW



The poles are at $z = 0.85e^{\pm j\pi/2}$ and the zeros at $z = \pm 1$.

UNIT CIRCLE

- **MAPPING BETWEEN z and $\hat{\omega}$**

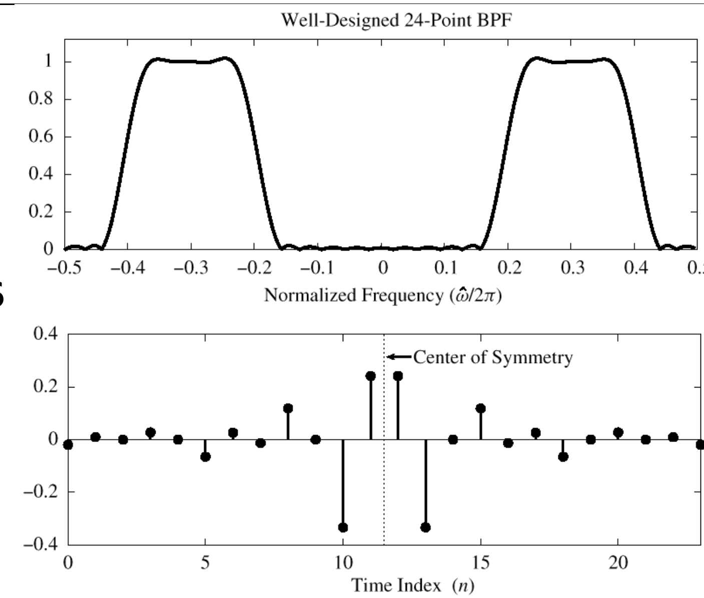


DIGITAL FILTER DESIGN

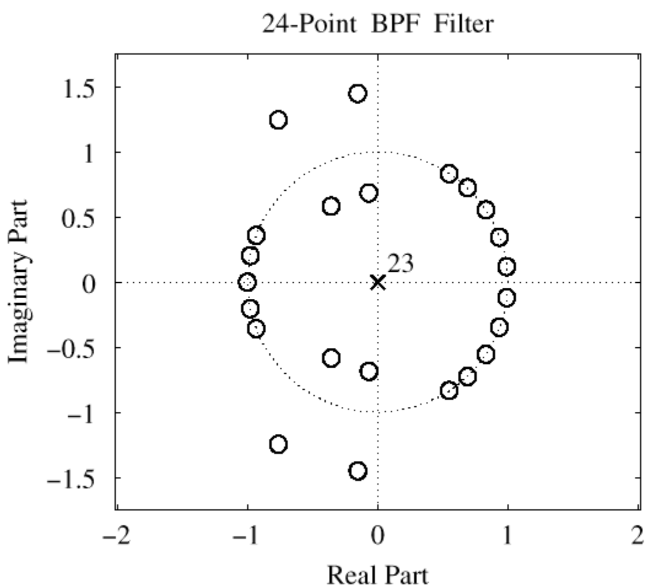
- Find the COEFFICIENTS to satisfy
  - PASSBAND & STOPBAND specifications
- FIR FILTERS
  - High Order: Many ZEROS. e.g., L=100
- IIR FILTERS
  - Poles & Zeros: 8-10 poles for a good filter
    - Implementation tricky with finite-precision

REALISTIC FIR BANDPASS

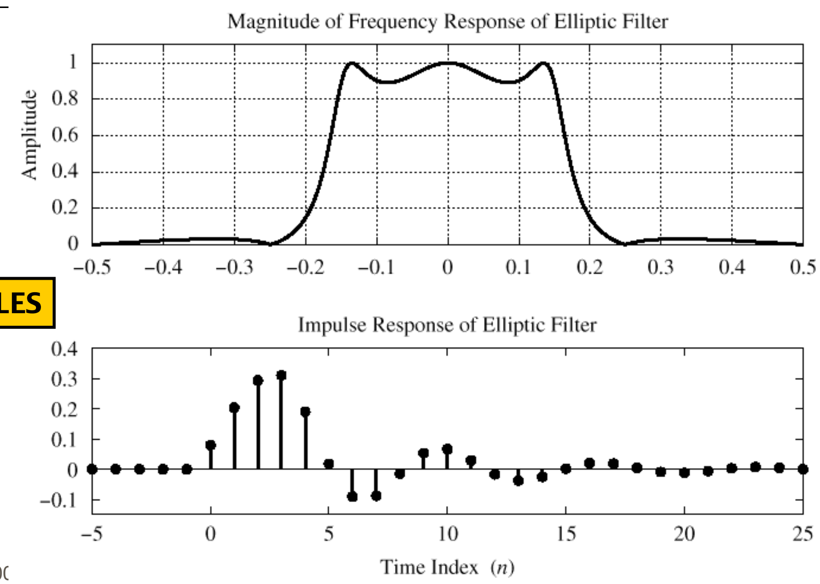
- FIR
- L = 24
- M=23
- 23 zeros



FIR BPF: 23 ZEROS

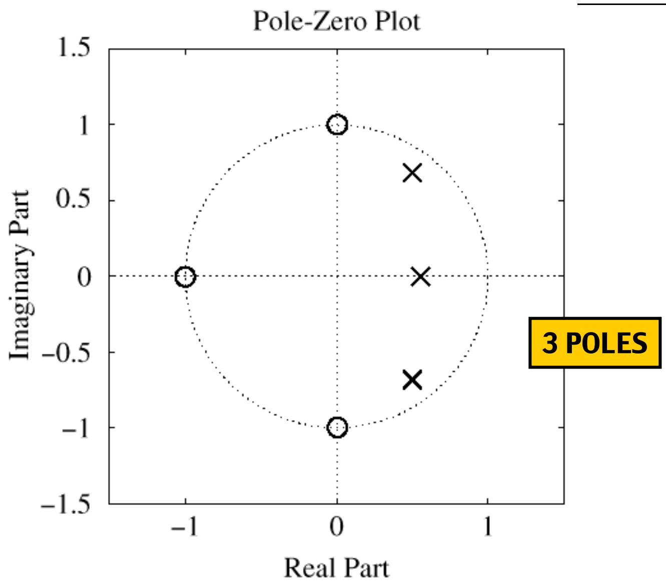


IIR Elliptic LPF (N=3)



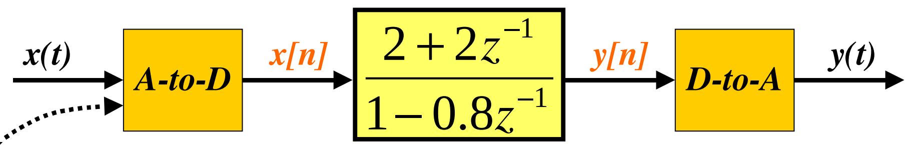
3 POLES

POLES & ZEROS of IIR



POP QUIZ

- Given:



- Find the output, y(t)
  - When $x(t) = \cos(2000\pi t)$

$f_s = 5000\,\text{Hz}$

POP QUIZ BECOMES

- Given: $H(z) = \dfrac{2+2z^{-1}}{1-0.8z^{-1}}$
- Find the output, y[n]
  - When $x[n] = \cos(0.4\pi n)$
  - Because $\omega T_s = 2000\pi / 5000 = 0.4\pi$

NO Aliasing

SINUSOIDAL RESPONSE

- x[n] = SINUSOID => y[n] is SINUSOID
- Get MAGNITUDE & PHASE from H(z)

$$\text{if } x[n] = e^{j\hat{\omega}n}, \text{ then}$$

$$y[n] = \mathcal{H}(\hat{\omega})e^{j\hat{\omega}n}$$

$$\mathcal{H}(\hat{\omega}) = H(e^{j\hat{\omega}}) = H(z)\Big|_{z=e^{j\hat{\omega}}}$$

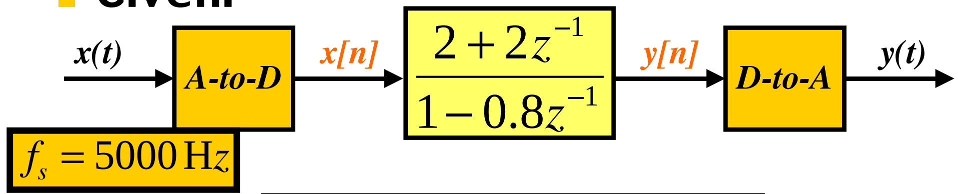
POP QUIZ INSIDE ANSWER

- **Given:** $H(z) = \dfrac{2+2z^{-1}}{1-0.8z^{-1}}$
- **The input:** $x[n] = \cos(0.4\pi n)$
- **Then y[n]** $y[n] = M\cos(0.4\pi n + \psi)$

$$H(e^{j0.4\pi}) = \frac{2+2e^{-j0.4\pi}}{1-0.8e^{-j0.4\pi}} = 3.02e^{-j0.452\pi}$$

POP QUIZ ANSWER

- **Given:**

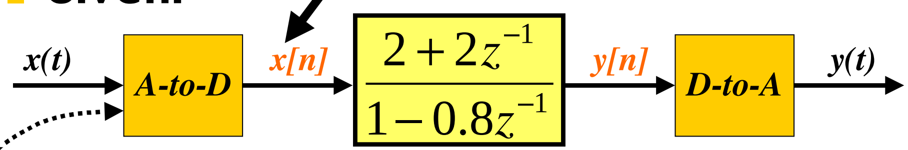


- **When** $x(t) = \cos(2000\pi t)$
- **The output is**

$$y(t) = 3.02\cos(2000\pi t - 0.452\pi)$$

ANOTHER POP QUIZ

- **Given:** $\hat{\omega} = ?$



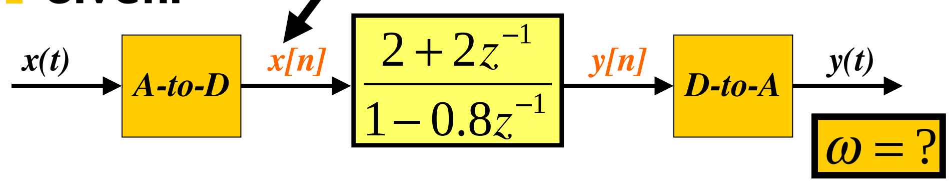
- **Find the output, y(t)**
  - **When** $x(t) = \cos(2\pi(6000)t)$

$f_s = 5000\,\text{Hz}$

$$\hat{\omega} = ?$$

2nd POP QUIZ ANSWER

- **Given:** $\hat{\omega} = 2\pi(6000/5000) = 2\pi(1.2)$



- **When** $x(t) = \cos(2\pi(6000)t)$

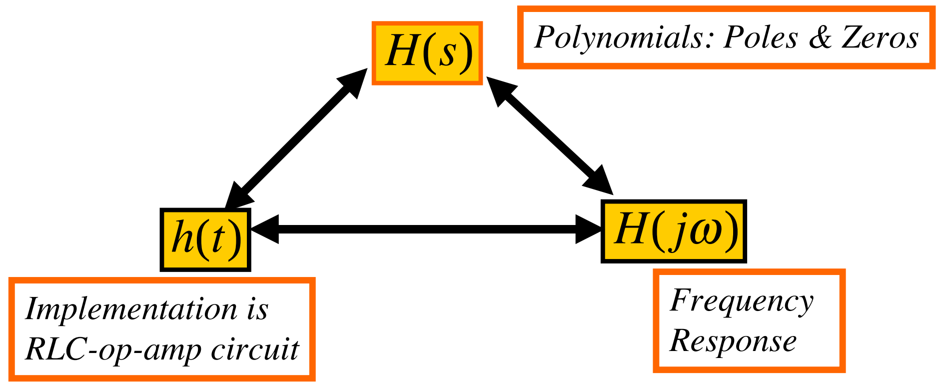
$f_s = 5000\,\text{Hz} \rightarrow \hat{\omega} = 2.4\pi \rightarrow y(t) = ?$

IMPORTANT CONCEPTS

- ALL Signals have **Frequency Content**
  - Sum of Sinusoids
  - Complex Exponentials
  - Impulses, Square Pulses
- **FILTERS** alter the **Frequency Content**
  - Image Processing Example: Blur
  - Linear Time-Invariant Processing
- **3 Domains** for Analysis

THE FUTURE

- Circuits & **Laplace** Transforms



Superficial Knowledge

- It depends how carefully you think about it. If you don't think very carefully it's obvious; but if you think about it in depth, you'll get confused and it won't be obvious.

Mathematical Elegance

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega$$

Fourier .
(Inverse

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

Fourier Analysis
(Forward Transform)

$$\text{Time-domain} \Leftrightarrow \text{Frequency-domain}$$
$$x(t) \Leftrightarrow X(j\omega)$$